Fuzzy Logic Modified Proportional-Integral-Derivative (MPID) Control for Flexible Manipulator

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Abstract: Control of flexible manipulator is considering one of the great challenges for control engineering. The flexibility of the manipulator converts the system to a non-minimum phase system. Classical controllers cannot cope with the vibration suppression of the tip in addition to achieving the desired position of the manipulator. A Modified Proportional-Integral- Derivative (MPID) control is proposed to control the flexible manipulator. The main difficulty with the proposed MPID controller lies on the tuning of the vibration gain of the controller. The vibration control gain has been determined in an empirical way so far. It is a considerable time consuming process because the vibration control performance depends not only on the vibration control gain but also on the other parameters such as the payload, references and PD joint servo gains. Hence, the vibration control gain must be tuned considering the other parameters. A fuzzy logic tuning scheme is used in this paper to find optimal vibration control gain for the MPID controller. The proposed fuzzy logic scheme finds an optimum vibration control gain that minimizes the tip vibration for the end effector of the flexible manipulator. The effectiveness of using the fuzzy logic appears in the ability to tune the gain with different loading condition and input parameters. The tuned gain response results are compared with results for other types of gains and show a good ability to suppress the tip vibration of the flexible manipulation in addition achieving accurate joint angle position.

Keywords– Flexible manipulator, Fuzzy logic, Gain tuning, Modified PID

I INTRODUCTION

The flexible manipulator started to play an important part in many engineering applications nowadays. Major advantages of flexible manipulators include small mass, fast motion, and large force to mass ratio, which are reflected directly in the reduced energy consumption, increased productivity, and enhanced payload capacity. Unlike the rigid manipulators, the difficulties facing the usage of flexible manipulators are numerous. The modeling of the flexibility of the manipulator is one of the challenges, the non-minimum phase problem, which appears from the modeling of the flexible manipulators, is also another challenge. The precise and availability of the measured variables used in the control is the third challenge. The control of flexible manipulators has been studied with great interest by many researchers over the past years due to its pronounced benefits. To find a controller that can achieve the end effector position of the flexible manipulator in a short time in addition to a suppression of its vibration to be able to achieve the tasks is the main goal of the control of flexible manipulator in the free space. Although significant progresses have been made in many aspects over the last two decades, many issues are not yet resolved yet, and simple, effective, and reliable controls of flexible manipulators remain open requests.

Using the approach of enhancement the measurements of the vibration variables was studied by Ge et al. [1], Sun et al. [2] while Etxebarria et al. [3] gives attention to the algorithms used in controlling the flexible manipulators. The enhancement of the traditional PD controller by adding a vibration control term is one of the most effective methods for the flexible manipulators. Lee et al. proposed PDS (proportional-derivative strain) control for vibration suppression of multi-flexible-link manipulators and analysed the Liapunov stability of the PDS control [4]. Maruyama et al. [5] developed a golf robot whose swing simulates human motion. They presented model accounting for golf club flexibility with all parameters identified in experiments and generated and implemented trajectories for different criteria such as minimizing total consumed work, minimizing summation of the squared derivative of active torque and maximizing impact speed. Matsuno and Hayashi applied the PDS control to a cooperative task of two one-link flexible arms [6]. They aimed to accomplish the desired grasping force for a common rigid object and the vibration absorption of the flexible arms. Some researchers tried to use the neural network (herein after abbreviated as NN) as a main controller like Talebi et al. [7]. In their research the controllers are designed by utilizing the modified output re-definition approach. The modified output re-definition approach requires only a priori knowledge about the linear model of the system but does not require a priori knowledge about the payload mass. Various NN schemes have been proposed so far such as a modified version of the "feedback error-learning" approach to learn the inverse dynamics of the flexible manipulator by kawato et al.[8]. On other proposed NN structure the controller is designed based on tracking the reference joint angle while controlling the elastic deflection at the tip. Mansour et al. [9] studies the use of neural networks as a tuning tool for the gain in Modified Proportional-Integral-Derivative (MPID) control used to control a flexible manip-

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ulator. Isogai et al. [10] proposed a fault-tolerant system using inverse dynamics constructed by NN for sensor fault detection and NN adaptive control for the actuator fault to reconfigure control to compensate for parameter changes due to actuator faults. Lianfang et al. [11] used the NN for the constrained motion control of flexible manipulators where both the contact forces exerted by the flexible manipulator and the position of the end-effectors contacting with a surface are controlled. Kan et al. [12] design of a fuzzy immune self-adjusting PID controller based on the differences among general PID control algorithm, fuzzy self-adjusting PID control algorithm and immune PID control algorithm. This kind of controller is used in the research of the selfheating system successfully. A fuzzy PID control method is introduced by Zheng et al. [13] to improve the overall performance of the electro-hydraulic position servo system. The relationships between the PID parameters and the response characteristics of electro-hydraulic position servo system are investigated. The fuzzy inference rules which enable adaptive adjustment of PID parameters are established based on the error and change in error. The simulations and experiments of step response and cosine tracking are carried out on the SRM direct drive hydraulic press. Huang and Lo [14] used two model-free auto-tuning PID and fuzzy PID control strategies to design a general temperature controller for different plants. The designed temperature control plants have heater control phase only without cooling control function. Piltan et al. [15] designed a mathematical tunable gain model free PID-like sliding mode fuzzy controller (GTSMFC) to rich the best performance. Sliding mode fuzzy controller is studied because of its model free, stable and high performance. They applied sliding mode controller in fuzzy logic theory to solve the limitation in fuzzy logic controller and sliding mode controller. One of the most important challenging in pure sliding mode controller and sliding mode fuzzy controller is sliding surface slope. By focusing on adjusting the gain updating factor and sliding surface slope in PID like sliding mode fuzzy controller they have the best performance and reduce the limitation. Geravand and Aghakhani [16] focused on experiments with the physical model of semi-active quarter-car suspension that using the method of fuzzy sliding mode which uses the fuzzy logic techniques to adjust the control gains that occur under the sliding mode.

The paper is organized as follows. A brief introduction about the flexible manipulators and the control algorithms used in control them is shown in section 1. The mathematical model of the flexible manipulator is discussed in section 2. In Section 3, the MPID controller algorithm is explained. The basic of fuzzy logic like the rules which related the inputs to the outputs of the system, the membership function for each variable is exposed. Also,the using the fuzzy logic tuning method is highlighted in section 4, while the simulation results of the proposed tuning method are shown in section 5. Conclusions and future research issues are described in section 6.

II INT

Before discussing the Fuzzy logic gain tuning method, the mathematical model of the flexible link and the MPID controller [17] is briefly introduced in sections and . From the analysis of the single-link flexible arm shown in Fig. 1, the flexible link is approximated by a continuous clampedfree beam. The flexible arm is rotating in the horizontal plane with a rotational angle $\theta(t)$ and the effect of gravity is not taken into consideration. Frame O-XY is the fixed base frame and frame O-xy is the local frame rotating with the hub. The tip deflection $\delta(L, t)$ is the difference between the actual tip position and the rotating frame O-xy. The deflection $\delta(x,t)$ is assumed to be small compared to the length of the arm. Let p(x,t) represents the tangential position of a point on the flexible arm with respect to frame O-xy. From the assumption of the deflection of the flexible arm, the tangential position is expressed as:

$$p(x,t) = x\theta(t) + \delta(x,t).$$
(1)

The flexible arm is treated as Euler-Bernoulli beam with uniform cross-sectional area and constant characteristics. Then, the Euler-Bernoulli equation for the link is given as follows :

$$EI\frac{\partial^4 p(x,t)}{\partial x^4} + \rho \frac{\partial^2 p(x,t)}{\partial t^2} = 0,$$
(2)

where ρ is the sectional density, E is the Young (elastic) modulus, and I is the second moment of area. Substituting (1) into (2) the following equation is obtained :

$$EI\frac{\partial^4 \delta(x,t)}{\partial x^4} + \rho \frac{\partial^2 \delta(x,t)}{\partial t^2} = -\rho x \ddot{\theta}(t).$$
(3)



Figure 1: Single-link flexible manipulator.

The flexible arm is clamped at its base, so both the deflection and slope of the deflection curve must be zero at the clamped end. Bending moment at the free end also equals zero. Making force balance at the tip obtains the following boundary conditions:

$$\frac{\delta(x,t)|_{x=0}}{2} = 0, \tag{4}$$

$$\frac{\partial \delta(x,t)}{\partial x}\Big|_{x=0} = 0,$$
(5)

$$EI\left.\frac{\partial^2 \delta(x,t)}{\partial x^2}\right|_{x=L} = 0, \tag{6}$$

$$EI \left. \frac{\partial^3 \delta(x,t)}{\partial x^3} \right|_{x=L} = m_t \left[x \ddot{\theta}(t) + \frac{\partial^2 \delta(x,t)}{\partial t^2} \right]_{x=L} \tag{7}$$

where L is the arm length. The dynamic equation describing the system is written as follows:

$$T(t) = \left(I_h + \frac{1}{3}\rho L^3\right)\ddot{\theta}(t) + \rho \int_0^L x\ddot{\delta}(x,t)dx + m_t L\left(L\ddot{\theta}(t) + \ddot{\delta}(L,t)\right).$$
(8)

A flexible manipulator simulator is built in MATLAB Simulink software using the mathematical model shown before to study the performance of the MPID control with different loading and gains conditions.

III CONTROLLER

A Modified PID controller (MPID) is proposed for controlling the tip position of the single-link flexible manipulator [9]. This controller used three measurements to generate the control signal, the hub rotational angle $\theta(t)$, the tip deflection $\delta(L,t)$, and the velocity of the hub $\dot{\theta}(t)$. If we choose the tip position as the output from the system then the error includes two components. The first component $e_i(t)$ is a result of the joint motion and is equal to $L(\theta_{ref} - \theta(t))$ which is identical with the rigid arm error where θ_{ref} is the reference joint angle. The second one is much more important and is due to the flexibility of the arm and equals $\delta(L, t)$. These two error components are coupled to each other. The Modified PID (MPID) controller replaces the classical integral term of a PID controller with a vibration feedback term to affect the flexible modes of the beam in the generated control signal. The MPID controller is formed as follows:

$$u(t) = K_p e_j(t) + K_d \dot{e_j}(t) + K_{vc} g(t) \operatorname{sgn}(\dot{e_j}(t)) \int_0^t g(t) dt, \qquad (9)$$

where u(t) is the control signal, K_p , K_d are the proportional and derivative gains for the joint control, respectively, K_{vc} is the vibration feedback gain, $e_j(t)$ is the tangential position error and g(t) is a vibration variable such as strain, deflection, shear force or acceleration under a single condition that the vibration variable value equal zero when the flexible manipulator is static and under goes no deformation. The stability of the proposed controller had been studied previously in [9]. It was proved that the system is stable as long as $K_d \ge 0$. The flexible manipulator simulator is used to validate the MPID controller given by (9) and the results are shown in Figs. 2 and 3. We found from the simulation results that the response of the flexible manipulator is sensitive to the change of the controller gains. In addition to that, the change



Figure 2: Effect of changing tip load.

in the tip payload have a noticeable influence on the response of the flexible manipulator end effector. If the controller gain is not tuned well, the response with the new loading condition will suffer a performance degradation. As shown in Fig. 2, a change in the tip load of the flexible manipulator has an undesirable effect for the vibration of the end effector.

Not only the change of the environment parameters like the tip payload causes an undesirable effect on the response as shown in Fig. 2, but also changing the system configuration like joint angle causes the same effect. Unlike industrial manipulators, both the environment parameter (i.e. tip payload) and the system configuration (i.e. joint angle) are always changeable in the case of space manipulators. This highlights the importance of optimization the gain used with this controller.

Another important point is that the change of the vibration control gain K_{vc} has seriously affects on the response of the single-link flexible manipulator. This is completely noticeable from the results in Fig. 3. This fact is the main motivation to find out a good way for tuning K_{vc} that brings the minimum vibration for the tip as well as a fast response for the joint position. It is predicted from Fig. 3 that the damping effect becomes stronger as the vibration control gain K_{vc} increases to a certain limit. However if the gain K_{vc} exceeds the limits it start to create an overshoot in the joint response.

The most difficulty of using the MPID controller is the adjustment of the vibration control gain. Ge et al. tried to use the genetic algorithm optimization process to find the suitable gain for the controller [18]. In their research they consider the fixed tip payload of the flexible manipulator and generate a set of gains for this configuration using the genetic



Figure 3: Effect of changing vibration control gain.

algorithm. However in general, the tip payloads and the joint angle are not the same in each operation but it varies from one task to another. Hence the tuning of the vibration control gain K_{vc} becomes the most importance issue to achieve the required position with a minimum vibration. To overcome the lake of consideration with the changing of tip payload and joint angle in the tuning of the MPID we proposed to use the fuzzy logic algorithm in the tuning of the MPID.

In this research the vibration control gain K_{vc} for the MPID controller given by equation (9) is tuned using the fuzzy logic for the environment parameter (i.e. tip payload), the system configuration (i.e. joint angle) and for both the other controller gains (i.e. K_p , K_d). By this way the controller gives the best response with respect to all the parameters related to the flexible manipulator.



Figure 4: Block diagram for the flexible manipulator with fuzzy logic tuning.

IV FUZZY LOGIC TUNING

Since Lotfi Zadeh had published his first research [19] in the field of using the fuzzy sets and its link with the human logic, the fuzzy logic had been used in many fields like classification of data by Sanford and Sztandera [20], document clustering by Hsieh et al. [21] and also in predictive control like the research done by Zhu and Li [22].

One important application of the fuzzy logic is the ability of finding the relationships of multiple input single output system MISO as shown previously by Pedrycz and Reformat [23]. The fuzzy logic is used in fuzzy modeling of multipleinput single-output nonlinear relationships in this research. From previous researches like [18], [9], [17] it was found that the relationship between the environment parameter (i.e. tip payload M_t), the system configuration (i.e. joint angle θ_{ref}) and for both the other controller gains (i.e. K_p , K_d) is nonlinear. The block diagram which represents the system during utilizing the fuzzy logic in the process of tuning the vibration control gain k_{vc} is shown in Fig. 4.

The inputs for the fuzzy logic tuning algorithm shown in Fig. 4. are the (i.e. tip payload M_t), the system configuration



Figure 5: Membership for input angle.

(i.e. joint angle θ_{ref}) and PD controller gains (i.e. K_p , K_d) while the output is the vibration control gain k_{vc} . The inputs is divided into two groups to find the rule base. The first group consists of joint angle θ_{ref} and proportional gain K_p , while the output from this group is k_{vc1} .

The membership function which defines a fuzzy set by mapping the actual input and output values from its domain to the sets associated degree of membership is shown in Fig. 5 for the reference input angle as one of the input to the fuzzy logic system.

To form the rule base each input in the first group (i.e. θ_{ref} and K_p) is divided based on its value into either Small which is indicated by S; Medium which is indicated by M and Big which is indicated by B. The limit values for the inputs are determined from the physical limitation of the system. Also the output is classified to the same three categories. The rule base summary for the first group is shown in Table 1. A sample of the rules for the first group is shown below

Table 1: The rule base for K_{vc1} using input angle θ_{ref} and proportional gain K_p

K_p							
		S	Μ	В			
θ_{ref}	S	S	Μ	Μ			
	M	M	В	В			
	В	M	В	В			

Rule 1 If θ_{ref} is *Small* and K_p is *Big* then k_{vc1} is *Medium*. **Rule 2** If θ_{ref} is *Big* and K_p is *Medium* then k_{vc1} is *Big*. **Rule ...** If θ_{ref} is ... and K_p is ... then k_{vc1} is

The second group of inputs consists of tip payload M_t and differential gain K_d , while the output from this group is k_{vc2} . Also to form the rule base each input in this group (i.e. M_t and K_d) is divided based on its value into either Small; Medium and Big. The limit values for the inputs are



Figure 6: Membership for tip payload.

determined from the physical limitation of the system. Also the output is classified into the same three categories S, M and B.

The membership function which is used in the fuzzification process for the tip payload is exposed in Fig. 6 while the membership function for the output from the fuzzy logic system-which is the tunned vibration gain K_{vc} is shown in Fig. 7. The rule base summary for the second set is shown in Table 2 followed by a sample of the rules for the sets of input group.

Table 2: The rule base for K_{vc2} using tip payload angle M_t and differential gain K_d

K_d							
		S	M	В			
M	S	S	M	В			
	M	S	M	Μ			
	В	M	Μ	В			

Rule 1 If M_t is *Small* and K_d is *Big* then k_{vc2} is *Big*.

Rule 2 If M_t is *Big* and K_d is *Big* then k_{vc2} is *Big*.

Rule ... If M_t is ... and K_2 is ... then k_{vc2} is

Finally the last set of rules is generated using k_{vc1} and k_{vc2} as inputs and the output is the tuned vibration control gain k_{vc} . An illustration for the rules base is publicized in Table 3

To demonstrate the change of vibration control gain K_{vc} with the changing of the input parameters to the fuzzy logic



Figure 7: Membership for vibration control gain.

Table 3: The rule base for K_{vc} using K_{vc1} and K_{vc2}

Γ	K_{vc1}						
Γ			S	М	В		
Γ	K_{vc2}	S	S	Μ	Μ		
		Μ	M	Μ	В		
		В	В	В	В		

system i.e. the environment parameter (i.e. tip payload M_t), the system configuration (i.e. joint angle θ_{ref}) and both the other PD controller gains (i.e. K_p , K_d), the fuzzy surface is drawn. In Fig. 8 the changing of the vibration control gain K_{vc} with both tip payload M_t and , the derivative gain K_d is highlighted while the effect of changing the joint angle θ_{ref} and proportional gain K_p is shown in Fig. 9





Figure 8: Fuzzy surface for tip payload and derivative gain with vibration control gain

Figure 9: Fuzzy surface for reference input and proportional gain with vibration control gain

V RESULTS

In this section, the tuned vibration control gain k_{vc} obtained by using fuzzy logic is used in the simulation of the flexible link manipulator system. The tip response of the flexible manipulator is plotted in Fig. 10. At the beginning the environment parameter (i.e. tip payload M_t), the system configuration (i.e. joint angle θ_{ref}) and both the other PD controller gains (i.e. K_p , K_d) is passed to the fuzzy logic module as input and tuned vibration control gain k_{vc} is received from it as an output. In Fig. 10., the tip payload M_t equals quarter Kg and the reference input angle θ_{ref} equals 13 degree while the proportional gain K_p and differential gain K_d equals 1000 and 250 respectively. The obtained vibration control gain k_{vc} equals 18000. The response with the tuned gain is shown with blue line while the response without tuning the gain is shown in red. As it appears from the tip response in Fig. 10., the tuned vibration control gain k_{vc} has the ability to drive the flexible manipulator to its reference value without overshoot and with minimal oscillation on the tip.



Figure 10: Tip response with tunned gain for 13 degree input and 0.25 Kg tip payload.

Another set of result is shown in Fig. 11. to verify the efficiency of the fuzzy logic in finding the best tuned vibration gain. A 0.5 kg tip payload was used with a reference angle equals 24 degree while the proportional gain K_p equals 600 and differential gain K_d equals 400. The response without tuning the gain is shown in red dashed line while the response with the tuned vibration gain appears in continuous blue line. It is clear from Fig. 11. that the tuned vibration control gain enables the flexible manipulator to reach its final position without noticeable vibration compared with the response with the tuning the vibration control gain. The deflection response for the tip of the flexible manipulator illustrated in Fig. 11. emphasizes the superiority of the tuned gain in damping the vibration of the end effector of the flexible manipulator.

The last set of result represents the joint response and the deflection response of the flexible manipulator with a considerably small input angle (i.e. five degree) and maximum physical load for the flexible manipulator (i.e. 0.5 Kg). The response is shown in Fig. 12, from the response we notice that the MPID with the fuzzy tuned gain was able to deliver the flexible manipulator faster to the final position of the joint and on the same time it has a remarkable vibration suppression on the deflection of the tip.

VI CONCLUSION

This paper discusses a fuzzy logic gain tuning method for the modified PID (MPID) controller of a single-link flexible manipulator. The fuzzy logic algorithm finds the optimum tuned



Figure 11: Tip response and deflection response with tunned gain for 24 degree input and 0.5 Kg tip payload.

vibration control gain of the MPID corresponding to the tip payload, reference input and the two PD control gains of the MPID. A dynamic simulator is used to produce the teacher signals. The main advantage of using fuzzy logic is the ability to include the effect of changing the system configuration, environment parameter in addition to the PD control gains to find best tuned gain. Simulation results with the obtained tuned gain validate the proposed method and it recommended to start implement this method experimentally.

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Figure 12: Tip response with tunned gain for 5 degree input and 0.5 Kg tip payload.

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