The Analysis of the Error Estimation and Ambiguity of 2-D Time Difference of Arrival Localization Method

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***Abstract*—**This article is focused on the analysis of the error estimation and the ambiguity of the TDOA (Time Difference of Arrival) localization method in the 2-D space. First, the algorithm of hyperbolic equations solution is presented as a background for a derivation of a covariance matrix. Next, the covariance matrix is derived in analytical form. Finally, the conditions of the covariance matrix solvability are shown. The analytical solution of the covariance matrix is the main contribution of this article.

***Keywords*—**Covariance Matrix, Localization Method, Time Difference of Arrival.

I. Introduction

The TDOA (Time Difference of Arrival) method is well-known signal source localization technique. It is used in many both civil and military applications, for example in EW (Electronic Warfare) systems. This method is based on extraction of TOA (Time of Arrival) *ti* of the received signals that are processed by generally *N* receiving stations. It can be expressed, in this article only for 2-D application, as a set of non-linear equations. These equations are usually called hyperbolical equations and they can be given as

$$t_1 = t_0 + \frac{\sqrt{(x_1 - x_t)^2 + (y_1 - y_t)^2}}{cl}, \qquad (1)$$
$$t_i = t_0 + \frac{\sqrt{(x_i - x_t)^2 + (y_i - y_t)^2}}{cl},$$
$$t_N = t_0 + \frac{\sqrt{(x_N - x_t)^2 + (y_N - y_t)^2}}{cl}$$

where t_i is measured TOA at receiving station with coordinates $\{x_i, y_i\}$, t_0 is initial transmit time of the signal, *cl* is speed of light and $\{x_t, y_t\}$ are coordinates of signal source (target).

The solution of the equations (1), i.e. the finding of signal source coordinates, is not trivial because these equations are non-linear ones. Generally, there are two ways to solve this set of equations. First, there are the linearization methods that are based on the linearization of equations (1) via Taylor expansion and following solution of the set of linear equations. See [1], [2]. Second, there are the analytical methods that are based on transformation equations (1) to solvable ones. See [3], [4], [5].

II. The Analytical Solution of Hyperbolical Equations

Figure 1 shows a localization system with $N = 3$ receiving stations and a target that are placed in special coordinate system $\{x', y'\}$. This coordinate system is selected to simplify the solution. Coordinates of the receiving stations are S_1 [0,0], S_2 [a,0], and S_3 [b,c] and coordinates of the target are [x_t', y_t'].

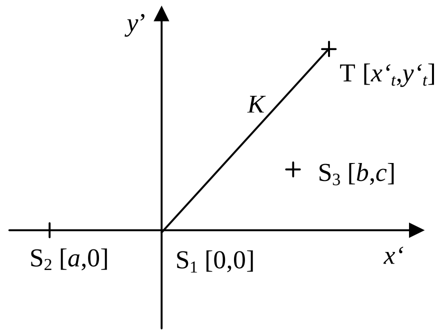

Fig.1 the configuration of passive system

In general, the coordinates of receiving stations and target can be arbitrary in the standard coordinate system $\{x,y\}$ but we can always transform them to the special coordinate system $\{x_t', y_t'\}$ via following terms

$$x' = (x - x_1).\cos\alpha + (y - y_1).\sin\alpha, \qquad (2)$$
$$y' = -(x - x_1).\sin\alpha + (y - y_1).\cos\alpha$$

where $\alpha = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$.

Now, let us define the distance between the target and the coordinate origin $\{x',y'\}$ by

$$K^2 = x_t'^2 + y_t'^2. \qquad (3)$$

In this case, the TOA equations (1) can be expressed by

$$t_1 = \frac{\sqrt{(0-x_t')^2 + (0-y_t')^2}}{cl} + t_0 = \frac{\sqrt{x_t'^2 + y_t'^2}}{cl} + t_0 = \frac{K}{cl} + t_0,$$
$$t_2 = \frac{\sqrt{(a-x_t')^2 + (0-y_t')^2}}{cl} + t_0, \qquad (4)$$
$$t_3 = \frac{\sqrt{(b-x_t')^2 + (c-y_t')^2}}{cl} + t_0.$$

If we use time delay instead TOA we can rearrange equations (4) to following form

$$\tau_1 = (t_2 - t_1) = \frac{1}{cl}\left(\sqrt{(a-x_t')^2 + y_t'^2} - K\right), \qquad (5a)$$
$$\tau_2 = (t_3 - t_1) = \frac{1}{cl}\left(\sqrt{(b-x_t')^2 + (c-y_t')^2} - K\right) \qquad (5b)$$

where τ_1 and τ_2 are time delay (i.e. time difference of arrival) between receiving of signal at stations S_1,S_2 and S_1,S_3. Substituting $L = \tau_1.cl$ into the equation (5a) yields

$$L + K = \sqrt{(a-x_t')^2 + y_t'^2}\,, \qquad (6)$$
$$L^2 + 2.L.K + K^2 = a^2 - 2.a.x_t' + x_t'^2 + y_t'^2 = a^2 - 2.a.x_t' + K^2.$$

Then, the target x_t' coordinate can be written as

$$x_t' = \frac{a^2 - L^2 - 2.L.K}{2.a} = A + B.K \qquad (7)$$

where

$$A = \frac{a^2 - L^2}{2.a}, \quad B = \frac{-L}{a}.$$

Similarly, the target y_t' coordinate can be written (with substituting $R = \tau_2.cl$ into the equation (5b)) as

$$y_t' = C + D.K \qquad (8)$$

where

$$C = \frac{b^2 + c^2 - 2.b.A - R^2}{2.c} \text{ and } D = \frac{-R - b.B}{c}.$$

Substituting equations (7) and (8) into equation (3) yields

$$K^2 = x_t'^2 + y_t'^2 = (A + B.K)^2 + (C + D.K)^2. \qquad (9)$$

The roots of quadratic equation (9) $K_{1,2}$ are

$$K_{1,2} = \frac{-N \pm \sqrt{N^2 - 4.M.P}}{2.M} \qquad (10)$$

where $N = A.B+C.D$, $M = B^2+D^2-1$ and $P = A^2+C^2$.

Finally, we can determine the target coordinates x_t', y_t' by substituting roots of equation (10) back into equations (7) and (8). The back transformation of computed target coordinates to the standard coordinate system $\{x, y\}$ can be determined by following terms

$$x_t = (x_t').\cos\alpha - (y_t').\sin\alpha + x_1, \qquad (11)$$
$$y_t = (x_t').\sin\alpha + (y_t').\cos\alpha + y_1.$$

III. The accuracy of TDOA method

The solution of equations (1) is only one part of troubleshooting area that is connected with TDOA theory. The evaluation of signal source position error presents another problem that is coupled with TDOA method. The covariance matrix seems to be a powerful tool for solution of accuracy TDOA method problem. The derivation of this matrix is following.

First, assume that the covariance matrix can be generally written as

$$\mathbf{C}(\mathbf{x}_t) = \left[\frac{\partial f(\mathbf{t}_i)}{\partial \mathbf{t}_i}\right].\mathbf{I}^{-1}(\mathbf{t}_i).\left[\frac{\partial f(\mathbf{t}_i)}{\partial \mathbf{t}_i}\right]^T \qquad (12)$$

where $\mathbf{I}^{-1}(\mathbf{t}_i)$ is Fisher information matrix, $\mathbf{t}_i = [t_1,\ldots,t_N]$ is vector of TOA's and $\mathbf{x}_t = [x_t, y_t]$ is vector of the target coordinates. In TDOA case, the inverted Fisher information matrix is equal to the covariance matrix of TOA's $\mathbf{C}(\mathbf{t}_i)$. In practical case, when the measured TOA's at particular receiving stations are independent, this matrix can be written as

$$\mathbf{C}(\mathbf{t}_i) = \begin{bmatrix} \sigma_{t1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{t2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \sigma_{tN}^2 \end{bmatrix} \qquad (13)$$

where σ_{ti}^2 is variance of TOA measured at receiving station S_i. Thus, we must only compute partial derivations of function $\mathbf{x}_i = f(\mathbf{t}_i)$, which is represented by above mentioned algorithm of analytical solution of TDOA equations for derivation of the covariance matrix $\mathbf{C}(\mathbf{x}_t)$. These partial derivations are generally expressed as matrix $\mathbf{J}$

$$\mathbf{J} = \left[\frac{\partial f(\mathbf{t}_i)}{\partial \mathbf{t}_i}\right] = \begin{bmatrix} \frac{\partial x_t(\mathbf{t}_i)}{\partial t_1} & \cdots & \frac{\partial x_t(\mathbf{t}_i)}{\partial t_N} \\ \frac{\partial y_t(\mathbf{t}_i)}{\partial t_1} & \cdots & \frac{\partial y_t(\mathbf{t}_i)}{\partial t_N} \end{bmatrix}. \qquad (14)$$

As demonstrated earlier, the derivation of analytical solution of the TDOA equations is realized with using of many substitutions. These substitutions are applied to derivation of matrix $\mathbf{J}$, too. So, we derive following terms:

$$\frac{\partial L}{\partial t_1} = LC = -cl, \quad \frac{\partial L}{\partial t_2} = LL = cl \text{ and } \frac{\partial L}{\partial t_3} = LR = 0 \qquad (15)$$

and

$$\frac{\partial R}{\partial t_1} = RC = -cl, \quad \frac{\partial R}{\partial t_2} = RL = 0 \text{ and } \frac{\partial R}{\partial t_3} = RR = cl. \qquad (16)$$

In the same way

$$\frac{\partial A}{\partial t_1} = AC = -\frac{L}{a}\cdot LC, \quad (17)$$
$$\frac{\partial A}{\partial t_2} = AL = -\frac{L}{a}\cdot LL,$$
$$\frac{\partial A}{\partial t_3} = AR = -\frac{L}{a}\cdot LR = 0,$$

$$\frac{\partial B}{\partial t_1} = BC = -\frac{1}{a}\cdot LC, \quad (18)$$
$$\frac{\partial B}{\partial t_2} = BL = -\frac{1}{a}\cdot LL,$$
$$\frac{\partial B}{\partial t_3} = BR = -\frac{1}{a}\cdot LR = 0,$$

$$\frac{\partial C}{\partial t_1} = CC = -\frac{b.}{c}\cdot AC - \frac{R}{c}\cdot RC, \quad (19)$$
$$\frac{\partial C}{\partial t_2} = CL = -\frac{b.}{c}\cdot AL,$$
$$\frac{\partial C}{\partial t_3} = CR = -\frac{R}{c}\cdot RR,$$

$$\frac{\partial D}{\partial t_1} = DC = \frac{-RC - b.BC}{c}, \quad (20)$$
$$\frac{\partial D}{\partial t_2} = DL = -\frac{b}{c}\cdot BL,$$
$$\frac{\partial D}{\partial t_3} = DR = -\frac{1}{c}\cdot RR,$$

$$\frac{\partial M}{\partial t_1} = MC = 2.(B.BC + D.DC), \quad (21)$$
$$\frac{\partial M}{\partial t_2} = ML = 2.(B.BL + D.DL),$$
$$\frac{\partial M}{\partial t_3} = MR = 2.D.DR,$$

$$\frac{\partial N}{\partial t_1} = NC = 2.(A.BC + B.AC + C.DC + D.CC), \quad (22)$$
$$\frac{\partial N}{\partial t_2} = NL = 2.(A.BL + B.AL + C.DL + D.CL),$$
$$\frac{\partial N}{\partial t_3} = NR = 2.(C.DR + D.CR),$$

$$\frac{\partial P}{\partial t_1} = PC = 2.(A.AC + C.CC), \quad (23)$$
$$\frac{\partial P}{\partial t_2} = PL = 2.(A.AL + C.CL),$$
$$\frac{\partial P}{\partial t_3} = PR = 2.C.CR\cdot$$

If the term $N^2 - 4.M.P$ is substituted by *DET*, then

$$\frac{\partial DET}{\partial t_1} = DETC = 2..N.NC - 4.(M.PC + P.MC), \quad (24)$$
$$\frac{\partial DET}{\partial t_2} = DETL = 2..N.NL - 4.(M.PL + P.ML),$$
$$\frac{\partial DET}{\partial t_3} = DETR = 2..N.NR - 4.(M.PR + P.MR)\cdot$$

Next, assume that $SD = \sqrt{DET}$, then

$$\frac{\partial SD}{\partial t_1} = SDC = DETC\cdot\frac{1}{2.\sqrt{DET}}, \quad (25)$$
$$\frac{\partial SD}{\partial t_2} = SDL = DETL\cdot\frac{1}{2.\sqrt{DET}},$$
$$\frac{\partial SD}{\partial t_3} = SDR = DETR\cdot\frac{1}{2.\sqrt{DET}}\cdot$$

Consider that $NOM = -\frac{(N \pm SD)}{2}$, then

$$\frac{\partial NOM}{\partial t_1} = NOMC = -\frac{(NC \pm SDC)}{2}, \quad (26)$$
$$\frac{\partial NOM}{\partial t_2} = NOML = -\frac{(NL \pm SDL)}{2},$$
$$\frac{\partial NOM}{\partial t_3} = NOMR = -\frac{(NR \pm SDR)}{2}\cdot$$

Finally, if $K = \frac{NOM}{M}$, then

$$\frac{\partial K}{\partial t_1} = KC = \frac{(NOMC.M - NOM.MC)}{M^2}, \quad (27)$$
$$\frac{\partial K}{\partial t_2} = KL = \frac{(NOML.M - NOM.ML)}{M^2},$$
$$\frac{\partial K}{\partial t_3} = KR = \frac{(NOMR.M - NOM.MR)}{M^2}.$$

Thus, the partial derivations of function $\mathbf{x}_i = f(\mathbf{t}_i)$, in coordinate system $\{x', y'\}$ can be written as

$$\frac{\partial x'_t}{\partial t_1} = XC' = AC + B.KC + BC.K, \quad (28)$$
$$\frac{\partial x'_t}{\partial t_2} = XL' = AL + B.KL + BL.K,$$
$$\frac{\partial x'_t}{\partial t_3} = XR' = B.KR,$$

$$\frac{\partial y_t'}{\partial t_1} = YC' = CC + D.KC + DC.K\,,$$
$$\frac{\partial y_t'}{\partial t_2} = YL' = CL + D.KL + DL.K\,,$$
$$\frac{\partial y_t'}{\partial t_3} = YR' = CR + D.KR + DR.K\,\cdot$$

In standard coordinate system $\{x,y\}$ these derivations are given as

$$\frac{\partial x_t}{\partial t_1} = XC = XC'.\cos\alpha - YC'.\sin\alpha\,, \qquad (29)$$

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$$\frac{\partial y_t}{\partial t_3} = YR = XR'.\sin\alpha + YR'.\cos\alpha\,\cdot$$

Consequently, the covariance matrix $\mathbf{C}(\mathbf{x}_t)$ can be written as

$$\mathbf{C}(\mathbf{x}_t) = \begin{bmatrix} XC & XL & XR \\ YC & YL & YR \end{bmatrix} . \begin{bmatrix} \sigma_{t1}^2 & 0 & 0 \\ 0 & \sigma_{t2}^2 & 0 \\ 0 & 0 & \sigma_{t3}^2 \end{bmatrix} . \begin{bmatrix} XC & XL & XR \\ YC & YL & YR \end{bmatrix}^T . \qquad (30)$$

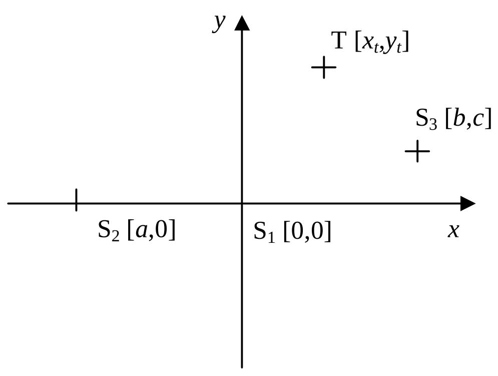
The covariance matrix $\mathbf{C}(\mathbf{x}_t)$ describes the error of target position localization and it represents so called error ellipse. In practical case, the *CEP* (Circular Error Probability) is used as parameter that can describe the error of target position. This parameter can be expressed by following term

$$CEP \cong 0.75\sqrt{\left(\frac{\sqrt{\lambda_1}}{2}\right)^2 + \left(\frac{\sqrt{\lambda_2}}{2}\right)^2}\,, \qquad (31)$$

where $\lambda_{1,2}$ are eigenvalues of covariance matrix.

IV. The Analysis of the Covariance Matrix

Generally, the set of equations (1) express the mapping of an arbitrary point from $\{x,y\}$ plane onto the hyperbolic plane $\{\tau_1,\tau_2\}$. This mapping is unambiguous. Figure 2 shows an example of this mapping.



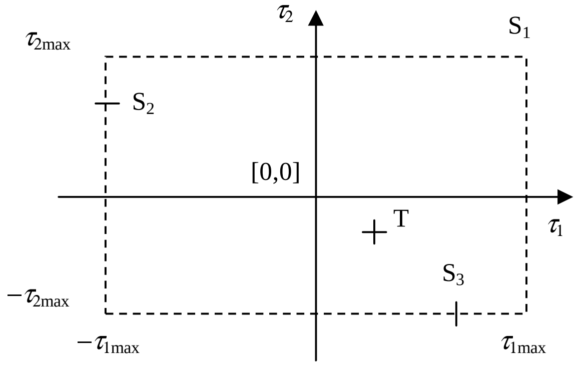


Fig. 2 the example of the mapping from the Cartesian coordinate system onto the hyperbolic coordinate system

The limiting values of both delays are restricted by receiving station coordinates and they can be following

$$\tau_{1\max} = \frac{\sqrt{a^2}}{cl} = \frac{a}{cl}\,,$$
$$-\tau_{1\max} = -\frac{\sqrt{a^2}}{cl} = -\frac{a}{cl}\,,$$
$$\tau_{2\max} = \frac{\sqrt{b^2+c^2}}{cl}\,,$$
$$-\tau_{2\max} = -\frac{\sqrt{b^2+c^2}}{cl}\,.$$

However, the back mapping from the hyperbolic plane onto $\{x,y\}$ plane is ambiguous. It is clear from mathematical notation of the analytical solution of TDOA equations where the quadratic term appears, see (10). Thus, the algorithm can generally have tree different solutions:

- the roots of quadratic equation are two real positive numbers, i.e. we can compute coordinates of two different real targets,
- the roots of quadratic equation are two identical real positive numbers, i.e. we can compute coordinates of one real target,
- the roots of quadratic equation are two complex numbers, i.e. we can not determine coordinates of target. It is example of non-real target.

Next, we will analyse the covariance matrix from conditions of solvability point of view. If we take into consideration, the above derived algorithm of the covariance matrix computation does not have solution under the conditions:

a) $a = 0$, in equations (17) and (18),
b) $c = 0$, in equations (19) and (20),
c) $M = 0$, in equation (27),
d) $DET = 0$, in equation (25).

The condition a) is satisfied just in case when the receiving station S_2 has the same coordinates as the receiving station S_1.

Of course, it means that the TDOA system has only two receiving stations and coordinates of target can not be computed, i.e. the covariance matrix can not be computed, too.

The condition b) is satisfied just in case when the receiving station S_3 has coordinates [b,0]. It means that all three receiving stations lie on the same line.

The analysis of the conditions c) and d) is more complicated than study of previous conditions. Thus, we chose following approach. The condition $M = B^2+D^2-1 = 0$ can be expressed, with using equation (8) and substitutions $L = \tau_1.cl$, $R = \tau_2.cl$ as function $R = f(L)$. Then, this function is given as

$$aa.R^2 + bb.R + cc = 0 \qquad (32)$$

where

$aa = U^2$, $bb = U.W$, $cc = B^2 + W^2 -1$, $U = 1/c$, and $W = -b.B/c$.

Figure 3 shows the curve (solid line) that includes all points which satisfy the function (32) in coordinate system $\{\tau_1,\tau_2\}$, resp. $\{L,R\}$ as result of numerical analysis of (32).

The condition $DET = N^2-4.M.P = 0$ was analyzed with using of similar idea. The dotted line in Fig 3 is graphic representation of analysis results. The border of unambiguous target position computation is the physical meaning of the $M = 0$ condition. It means that the TDOA equations have just one real solution ($K>0$) there. Figure 4 shows this border in coordinate system $\{x,y\}$ as boundary lines between light and dark grey areas. In dark grey area the TDOA equations have one real solution and one non-real solution ($K<0$). In light grey area these equations have two real solutions, i.e. the TDOA method is ambiguous.

The condition $DET = 0$ represents the points (in hyperbolic coordinate system) where the TDOA equations have only one solution and it is real. The boundary lines between black and light grey areas just represent this condition in $\{x,y\}$ coordinate system. From accuracy point of view, the covariance matrix does not have any solution here.

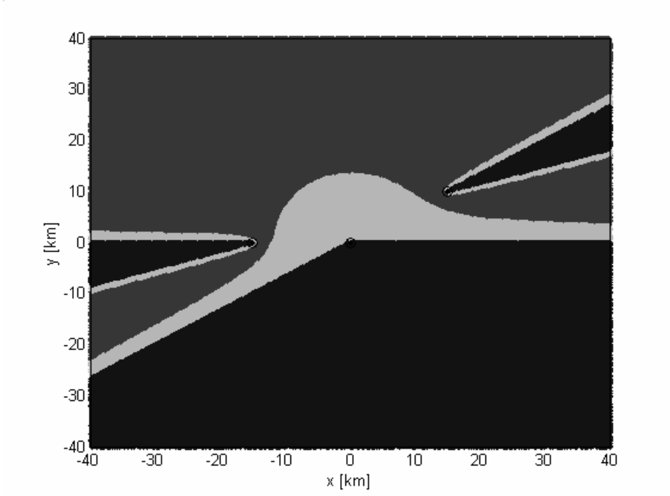


Fig 4 the graphic representation of conditions $M = 0$ and $DET = 0$ in $\{x,y\}$ coordinate system

V. The test of validity of the covariance matrix solvability conditions

In this part of this article we present some results of validity testing of the derived solvability conditions of the covariance matrix. The simulation was performed in MATLAB software environment. In coordinate system $\{x,y\}$ there are some areas where the covariance matrix does not have any solution which was demonstrated by analysis of a, b, c, and d conditions in previous part of this paper. Figure 5 shows a detail of *CEP* parameter, which represents the covariance matrix here, in surrounding of receiving site S_3. In this case the coordinates of the receiving sites are S_1 [-25 km,0], S_2 [0,-10 km] and S_3 [25 km,0] and the *CEP* is computed for 2500 targets with coordinates in intervals x = (-50 km,50 km) and y = (-50 km, 50 km). It is clear that the value *CEP* is close to infinity for targets that lie on lines that connect particular receiving sites. This result exactly correlates to the covariance matrix analysis.

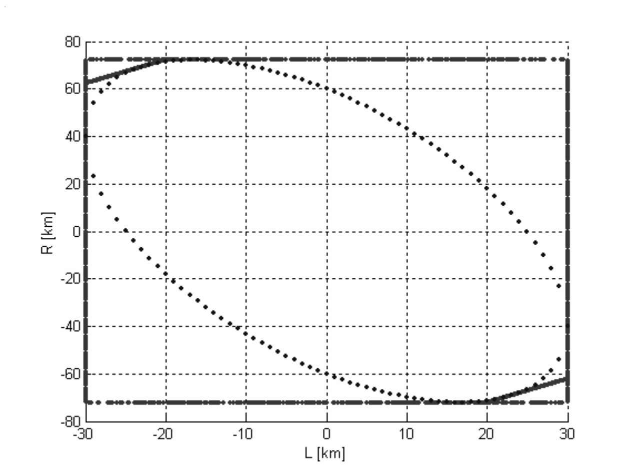


Fig. 3 the graphic representation of conditions c) and d)

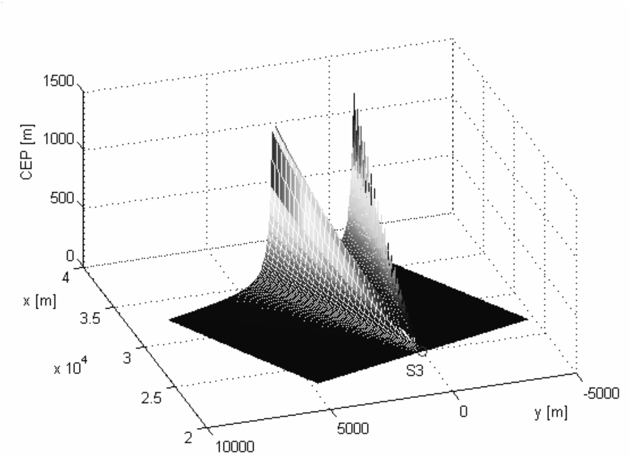


Fig. 5 the detail of *CEP* parameter values

It is clear that the TDOA system measures the target positions with large error just in these areas and it have very strong impact on the planning of the TDOA system arrangement in practical utilization.

To suppression this problem the TDOA system can have more receiving sites than three or the TDOA system is intended only for localization of targets that are inside the receiving sites triangle. This case is shown in Fig. 6 where the receiving station coordinates are S_1 [-34 km,20 km], S_2 [0,-40 km], S_3 [34 km,20 km] and possible targets are inside of a circle with radius 10 km. The centre of this circle is situated to origin of coordinate system.

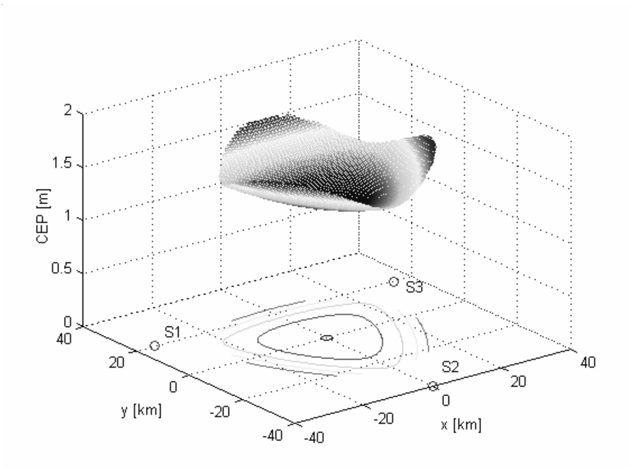


Fig. 6 the *CEP* parameter values

VI. The practical utilization of the covariance matrix analysis

The optimization of the TDOA system topology (i.e. number of receiving sites and their mutual arrangement) is one of the practical exploitation of the covariance matrix analysis. We designed an optimization algorithm. This optimization algorithm is based on the Monte-Carlo method and as the optimization parameter was chosen the *CEP* parameter. The principle of the algorithm operation is following. First, j TDOA systems with necessary number (for example with 3 stations) of receiving stations are randomly (with uniform distribution) deployed in defined area Π. The area Π is area of possible positions of receiving stations, for example it can be airport. Next, the CEP_k values are computed for all TDOA systems, i.e. for j TDOA systems, for k points from area Φ. The area Φ represents an area of possible targets and the number of targets positions is just k. It means k *CEP* values is computed for each TDOA system. The value *CEP* that satisfies following equation

$$CEP_j = \max\left(CEP_k\right) \tag{33}$$

is saved. Finally, the optimal topology of TDOA system is selected. The selection is accomplished according to following expression

$$CEP_{jopt} = \min\left(CEP_j\right). \tag{34}$$

The whole algorithm is repeated for situation when the number of receiving stations of TDOA systems is increased by 1. The results of all algorithm rundowns, represented by CEP_{jopt}, are then mutually compared and the topology of TDOA system with sufficient accuracy characteristics with regard to number of receiving stations is selected as the optimal topology.

VII. The optimization algorithm simulation

The result of simulation that tests looking for optimal topology of TDOA system with 3 receiving stations is shown in Figure 7. In this case the area Π is defined as circle centered at coordinate origin with radius 25 km. The area Φ is defined as circle too. This circle has radius 10 km and it is centered at coordinate origin, too. In this case 25000 TDOA systems are created and 10000 signal source positions are considered.

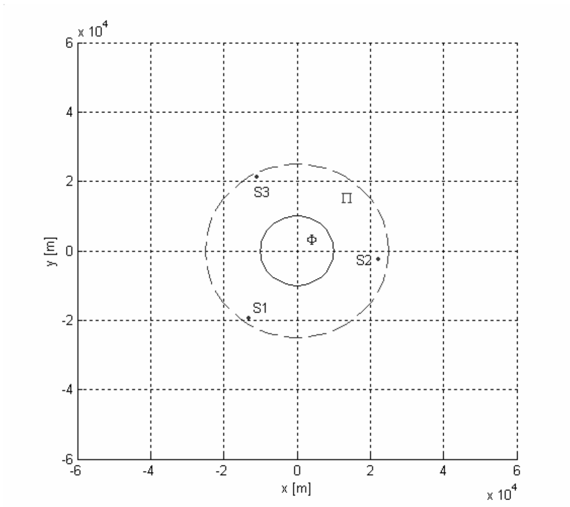


Fig. 7 the example of optimal topology of TDOA system with 3 receiving stations

VIII. The test of the optimization algorithm with real data

The operation of optimization algorithm is tested with real data here. The real TDOA system measured positions of airplane flight at approximately 8000 points. The topology of this TDOA system and direction of this flight are shown in Figure 8. This TDOA system has 4 receiving stations. In this case the map coordinate system is used.

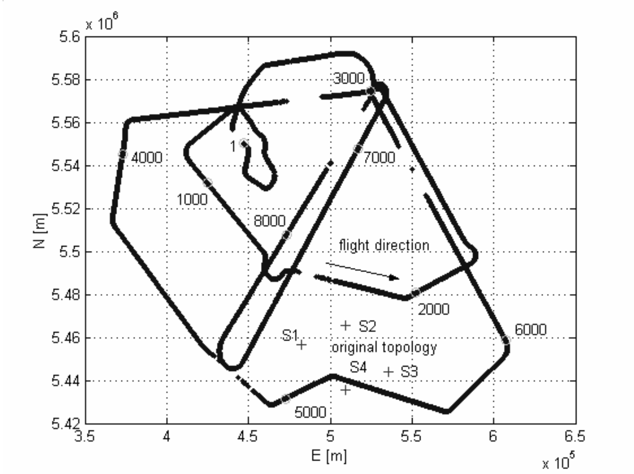


Fig. 8 the topology of real TDOA system and direction of flight

The new (optimal) topology of TDOA system is shown on Figure 9. The area Π is restricted by size of airport base.

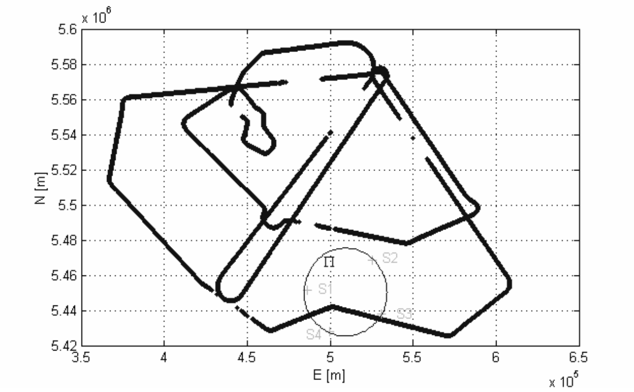


Fig. 9 the topology of the optimal TDOA system and direction of flight

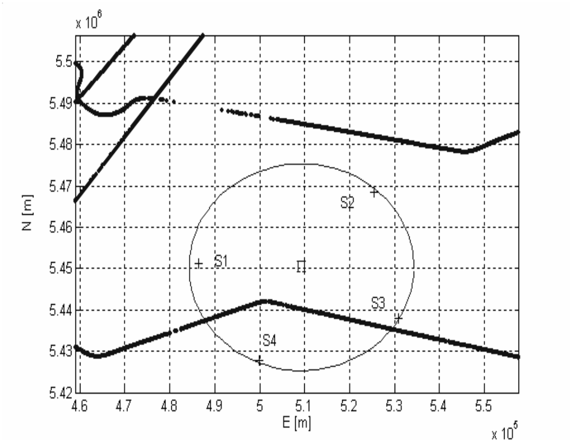


Fig. 10 the detail of topology of the optimal TDOA system

The comparison both topologies is shown in Figure 11. The parameter of comparison is value of *CEP*. We can see that new topology provides better accuracy of target position localization

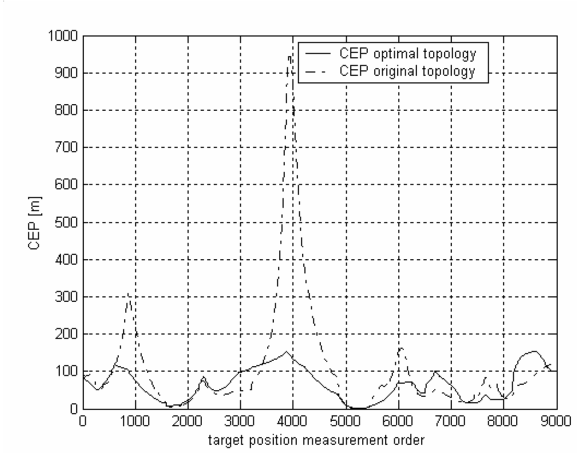


Fig. 11 the *CEP* comparison of optimal and original topology of real TDOA system

The results of comparison between original and optimal topology of TDOA system under real condition is described in Table 1. We can see that the average value of *CEP* dropped after optimization from 110 m to 66 m. The real TDOA system works with variation of TOA measuring $\sigma_{ti}^2 = (10 \text{ ns})^2$.

Table 1: The *CEP* comparison of optimal and original topology of real TDOA system

Original topology of TDOA system		Optimal topology of TDOA system	
μ_{CEP}	110,07 m	μ_{CEP}	66,33 m
CEP_{min}	944,90 m	CEP_{min}	153,61 m
CEP_{max}	1,15 m	CEP_{max}	1,14 m

IX. Conclusion

This article more detailed describes the derivation and the following analysis of the covariance matrix of the TDOA localization method. The derived algorithm of the covariance matrix computation is fully analytical. It means that it is a powerful tool for following solvability analysis of the covariance matrix that is main part of the article.

The example of practical using of this analysis is shown in last part of the paper. This is the simulation of the system topology optimization algorithm computation. The results of the simulation illustrated that the optimal topology of the TDOA system can be found whereas the criterion of optimization can be values of *CEP* parameter.

Generally, the same procedure can be applied to solution of 3-D case of a TDOA system using. The results of the covariance matrix analysis were used for innovation of current TDOA systems.

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