Application of the classical reliability to address optimization problems in mesh networks

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***Abstract*—** In this paper we address optimization problems in wireless networks, specifically tailored for mesh networks, via the application of a classical network reliability measure. In classical reliability theory, a communication network can be modeled as a directed graph $G=(V,E)$, composed of a finite set V of nodes, and a finite set E of links, where the links of the network underlying graph fail independently with known probabilities (nodes are perfectly reliable). Given a set K of *terminal* nodes and a source node s of K, the *Source-to-K-terminal* reliability, $R_{s,K}(G)$, is the probability of the event that the source s will be able to communicate with the terminal nodes thru operational directed paths. Each link represents a stochastic wireless communication channel connecting two nodes of a network whose probability of failure is based upon recent results in Information Theory. We present efficient algorithmic techniques to tackle optimization problems in communication networks such as *nodes' redundancy* and *areas' connectivity*.

***Keywords*—**Mesh networks, optimization, outage probability, reliability theory, simulation.

I. Introduction

Failures in communication networks may arise from natural catastrophes, component wear out, or action of intentional enemies. A communication network can be modeled by an undirected graph (or digraph) $G= (V,E)$, where V and E are the set of nodes and edges (links) of G, respectively. Moreover the failure probabilities of the network components are represented by assigning probabilities of failure to the nodes and/or edges (links) of its underlying graph (digraph). In this paper, we will discuss a reliability model called the *edge-reliability-model*, and we present a clear description of its applicability to assess performance objectives of real communication networks such as mesh networks.

Given a probabilistic undirected graph $G=(V,E)$ (in the remaining sections we will discuss networks modeled as directed graphs) with a distinguished set of terminal nodes K of V (also call *participating* nodes), and where the edges fail independently with known probabilities (nodes are always operational), a widely studied network reliability measure is the classical K-terminal reliability $R_K(G)$ of G, which is defined as the probability that the terminal nodes are connected thru operational paths composed of surviving edges. In particular, if we set $K=\{s,t\}$, $R_{\{s,t\}}(G)$ gives the probability that two nodes of the network, s and t, will be able to communicate thru operational paths (called the *two-terminal* reliability) [1]-[4].

For the classical reliability measure, computation of the K-terminal reliability had been shown to be NP-hard [5]. For the specific case when $|K|=2$, Valiant proved that the computation of the reliability is NP-hard as well [6]. The same computational complexity was determined when $K=V$ (called the *all-terminal reliability*) [7]-[8]. However efficient Monte Carlo techniques had been developed to estimate the classical reliability. In particular Monte Carlo Recursive Variance Reduction techniques (RVR for short) were implemented by Cancela and El Khadiri to estimate the classical reliability measure, showing excellent computational results [9].

The classical reliability model was extended to assess performance objectives of communication networks in which the quality of a communication depends on the length of the paths connecting the terminal nodes. In 2001, Petingi and Rodriguez introduced the K-terminal Diameter-constrained reliability of a communication network [10]. Given a graph $G=(V,E)$ whose edges fail independently with known probabilities, a set of terminal nodes K, and a diameter bound D, $R_K(G,D)$ is defined as the probability that the surviving edges allow the existence of a path of at most D edges for each pair of terminal nodes of G. This model is applicable to assess for example the reliability of networks in which the communication between terminal nodes (or participating nodes) must meet delay constraints, as for example is the case in broadcasting networks. As the length of any path in a network has at most $|V|$ - 1 edges, the Diameter-constrained reliability subsumes the classical reliability as $R_K(G,D) = R_K(G)$, when $D = |V|-1$. Further information concerning this reliability measure can be found in [11]-[13].

Although network reliability models have been studied for several years to evaluate the reliability as well as to address design and/or optimization problems of complex communication networks, the theoretical study of these models has taken an independent path from the applied one. Network Reliability Theory was developed in the 60s responding to the needs of measuring performance objectives of communication networks such as DARPA, and satellite communication networks.

A series of papers are now appearing recognizing the importance of these models to assess performance objectives of communication networks such as wireless networks [14]-[16]. In addition to the original intention of applying these measures to evaluate performance objectives of specific networks such as communication or electronic networks, these

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models are also applicable to a broader range of networks, such as for example biological networks; for example the reliability (the one's complement of the failure probability) of an interaction between two proteins in a Proteins Interaction Network (PIN), was defined in terms of the number of proteins in the common neighborhood of the adjacent proteins (the interaction of two proteins are represented as an edge whose endpoints are the nodes representing the two proteins) [17].

The reliability of an edge (link) of a network is the probability of a successful communication through the edge (link) and it is determined as a function of the network's operational characteristics (e.g., wireless networks, optical networks), and depends upon the performance objectives to be measured (e.g., determine the quality of communication between two access points in a mesh network).

This paper analyses two classes of mesh networks; planned networks and random networks. Typically in a planned network, the location of each transmitting/receiving node was carefully planned, as for example in urban areas, where the nodes are usually located on roof-tops. Unlike planned networks, random networks consist of nodes which are randomly distributed and the network is usually set up in a ad hoc fashion (e.g., tactical military operations).

In this paper we show how the behavior of mesh networks can be accurately simulated by random digraphs, and, consequently, a profound reliability analysis of these networks could be performed. Once the reliability of a network can be determined, several optimization and design problems in networks can be addressed as for example *nodes' redundancy* in planned mesh networks. As the reliability analysis of a network considers all possible paths connecting terminal nodes, no assumptions are made on routing protocols or overall throughput of the network.

In Section II we model wireless mesh networks as directed random graphs in which the probability of failure of a link connecting two different nodes of a topology is determined by the power outage of the wireless communication channel represented as a random variable using current results in Information Theory [14]; moreover we present a theoretical introduction of the Source-to-*K*-terminal reliability measure for digraphs. In Section III, we explain how this reliability measure could be applied to tackle optimization problems in planned mesh networks. In Section IV we study the problem of indicating locations among different geographical regions, in which optimal connectivity could be achieved, via an existing mesh network which was deployed in a ad hoc fashion. Finally, in Section V, we present conclusions and future research. An Appendix section is included with mathematical proofs of reliability results mentioned in the paper.

II. Wireless Network Modeling

A. Random Graphs and link failure representation

Wireless transmissions are degraded by a phenomenon known as *Path Loss*, which is known as the difference between the transmitted power p_o and the received power of a signal p_r. The relationship between p_r and p_o is given by the relation $p_r = p_0 / d^n$, where d is the distance between the transmitter and receiver, and n is a constant between 2 and 4, known as the *path loss exponent*, representing the degradation of the signal due to the physical characteristics of the terrain embracing the communication between the transmitter and receiver.

The determination and optimization of a channel capacity over a wireless link is also a topic of much research and some studies in this area. Shannon's law provides the theoretical maximum rate at which error free digits can be transmitted. Mathematically, the capacity of a communication channel C is defined by the relation $C = b\log_2(1+SNR)$, where b is the bandwidth in *Hz*, and the *SNR* is the signal to noise ratio at the receiver end [18].

A more representative model was recently introduced for the capacity of a wireless communication channel [14]. In this model, the instantaneous capacity of a wireless link is treated as a random variable and it is represented by the relation

$$C = \log_2(1 + |f|^2 / d^n \, SNR), \tag{1}$$

where f is the fading state of the channel and it is a Rayleigh random variable and the *SNR* is the signal to noise ratio. If η represents a zero mean additive white Gaussian noise with average power of σ_η^2, and assuming a transmitted input signal x, the *SNR* is then defined as $|x|^2/\sigma_\eta^2$. The probability of a link (communication channel) power outage, *Poutage* (or link failure probability), is defined as the probability of the event that the channel capacity is less than the transmission rate R, usually expressed in bits per channel use (i.e., *Poutage = Prob* $\{C<R\}$). From (1) and after some algebraic manipulation, the outage probability of a communication link is expressed as

$Poutage = Prob\{|f|^2 < \frac{d^n}{SNR'}\}$, where $SNR' = \frac{SNR}{2^R - 1}$. As fading is a Rayleigh random variable, *Poutage* is then the CDF of $|f|^2$, and by application of probability theory, one gets

$$Poutage = 1 - \exp(\frac{-d^n}{\mu SNR'}), \tag{2}$$

where $\mu = E(|f|^2)$. The probability of a successful reception, or equivalently the *link reliability*, for a Rayleigh fading link with fixed distance is then given by the simple expression

$$Psuccess = 1 - Poutage = \exp(-d^n / \mu SNR'). \tag{3}$$

The importance of this link reliability representation is that the communication of real wireless networks can be then accurately simulated using standard Graph Theoretical models, and therefore several communication optimization problems can be successfully tackled.

B. Mesh Networks

As an example of the application of the reliability consider Cisco Aironet Mesh technology. The access points are basically composed of *Maps* (Mesh Access Points) and *Raps* (Routed Access Points). The basic difference between these network nodes is that Raps have wired connections to their controllers while Maps use wireless connections to connect to their controllers.

Each node of the network can transmit as well as to receive signals, thus we could represent this mesh network as a digraph (directed graph). The communication between two nodes *i* and *j* is represented by two anti-parallel links, where the link (i,j) represents the transmission from transmitter *i* to receiver *j*; similarly we define the link (j,i) but interchanging the roles of the nodes (see Fig. 1).

Usually in standard Mesh Network analysis, the transmission power, transmission rate, as well as the *SNR* of the mesh nodes are assumed constant through the whole network (thus the range of the nodes' transmissions are depicted by circumscribed circles of identical radius around each node), thus our model offers a more realistic simulation; if a link $l=(i,j)$ has failure probability $q(l)$ as stated in (2), in this model is very possible that $q((i,j)) \neq q((j,i))$ since for example the adjacent nodes may have different receiving and/or transmitting specifications.

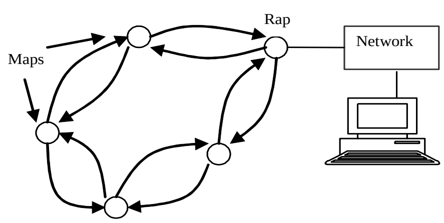

Fig. 1. An example of a mesh network modeled as a digraph.

Let $G=(V,E)$ be a digraph with node-set V and link-set E. Moreover let s be the predefined source node and K be the set terminal (or participating) nodes of G. For digraphs, $R_{s,K}(G)$ (called the *Source-to-K-terminal* reliability) is defined as the probability that the surviving links span a digraph such that an operational dipath from s to u exists, for each $u \in K$ (where the links from s to u of the dipath are all forward links). In other words, $R_{s,K}(G)$ gives the probability of the event that the source s will be able to send information (via directed paths) to every participating node of K. In particular $R_{s,u}(G)$ measures the probability that s will be able to send information to just another node u of G, and this notation will replace the standard notation $R_{s,K}(G)$, whenever $K=\{s,u\}$.

The sample space is composed of all possible subgraphs of G, called the *states* of G, and a state $H = (V(H), E(H))$ has probability of occurrence

$$P(H) = \prod_{e\in E(H)}(1-q(e)) \prod_{e\notin E(H)} q(e),$$

where $q(e)$ is the probability of failure of the link e. An *operating state H*, is a state in which there exists a directed path from s to each node u of K (i.e., s will be able to send information to all the terminal vertices of K), therefore it contributes to the reliability. Moreover let Φ be the set of all possible operating states of G. Thus we have

$R_{s,K}(G) = Prob$ {there exists a directed path from s to each node of K}= $\sum_{H\in\Phi} P(H)$. (4)

Using this reliability measure, several design and optimization problems could be addressed. In the next section we present the applicability of the Source-to-*K*-terminal reliability to tackle the problem of nodes' redundancy in planned networks.

III. Optimization Problems in Planned Networks

A. Nodes' redundancy in planned networks

One interesting optimization problem that could be undertaken by applying the Source-to-*K*-terminal reliability is nodes' redundancy in wireless networks. Usually in mesh networks (or wireless networks), if a set of nodes X fail, the other remaining nodes, V-X, will cover the areas previously covered by X.

Suppose that a node x fails (x is not a node of K), and we would like to be able to assess how the ability of s to communicate (i.e., send information) with other terminal nodes was diminished by the failure (or deletion from the network) of x. Indeed, $R_{s,K}(G\text{-}x)$ will give the probability of the event that s will be able to transmit to the remaining participating nodes of K, given that x has failed (or equivalently it doesn't belong to the network). In particular for $K=\{s,t\}$, then $R_{s,K}(G\text{-}x)$ is the probability that the transmitting node s will be able to deliver information to just another node t (called the *Source-to-terminal* reliability) and for ease of notation it will denoted as $R_{s,t}(G)$. It is important to emphasize here that from the definition of a link failure probability discussed in Section II-A, for most wireless networks (and in particular for ad hoc networks) $R_{s,t}(G) \neq R_{t,s}(G)$ whenever $K=\{s,t\}$, since the transmission between nodes depends on the interconnections between nodes as well as if the nodes play the role of transmitters or receivers; thus the model proposed in this paper can accurately simulate real wireless networks.

Even though our model works well for single source broadcasting networks, whenever every pair of anti-parallel channels connecting two nodes of a network have the same probability of failure, then $R_{s,K}(G) = R_K(G')$, that is the Source-to-*K*-terminal reliability of the digraph G is equal to the *K*-terminal reliability (discussed in the Introduction) of a undirected graph G' obtained from G by replacing two anti-parallel links having the same probability of failure between two nodes, by a single undirected edge with the same probability; in the Appendix we formally present this equivalence between these reliability models and we explain how our reliability analysis based on which node is the source and which nodes are the terminals can be then extended to the case where K terminal nodes can communicate independently of which nodes are transmitting and which nodes are receiving information. Taken into account the stochastic link representation introduced in Section II-A, this situation is realizable for example when the nodes of a network have identical transmitting and receiving characteristics as well as considerations pertaining the fading states and path loss exponents of the channels. Also, even if the anti-parallel channels have different probabilities of failure, our methodology yields a good approach to estimate the *K*-terminal reliability for the undirected case.

B. Determining the relevance of each redundant node in a given topology.

Once the locations of the redundant nodes have been determined, it is important to assign to each of these nodes a weighted value which individually assesses the role each plays in improving the communication of a network.

In our model we must first establish which node is the source and which nodes are the terminal ones, as the weighted importance of a redundant node in a network depends on which node is transmitting and which nodes are receiving information. Consider a random digraph $G=(V,E)$ representing a network, and let s and K be the source and set of terminal nodes of G, respectively. Moreover let $N \subseteq V\text{-}K$ be the set of redundant nodes of G. The redundancy weight of a node $x \in N$, $Red_x(G, s, K)$, is defined as

$$\mathrm{Re}\,d_x(G,s,K) = R_{s,K}(G) - R_{s,K}(G-x)\,, \qquad (5)$$

that is, the contribution of x to the reliability of G is determined by the difference between the reliability of the network G from the reliability of the network obtained from G by just deleting the node x. Here we are trying to identify the node whose removal from G affects more the reliability of the communication between s and the terminal nodes of K (i.e., $\{x \in N$: $Red_x(G, s, K)$ is the maximum among all nodes in $N\}$).

As an example consider the network exemplified in Fig. 2. In this example the terminal vertices are s and t, where s is the transmitting node and the set of redundant nodes is $N = \{1,2,3\}$. Each link connecting two nodes (refer to Section II-A for definition of a link failure probability in wireless networks) has been assigned a value corresponding to the Cartesian distance between them. However, as previously discussed, every two nodes are connected by anti-parallel directed links representing the two channels of communication between them, in which the nodes exchange their roles. For this example we assume that each node, when acting as a receiver, has SNR = 1000, or equivalently its SNR_{db}= 30; it should be noted that by just stating the SNR of each access point, we are then assuming the same transmission and noise powers for the nodes of G, otherwise transmission and noise powers should be specified for each node. Moreover let the transmission rate R = 1 bit per channel use for each of the nodes of G when each plays the role of a transmitter. In addition each channel has path loss exponent n = 2 (corresponding to open space) and the expected value of the fading state is μ =1.

Table I illustrates the corresponding probability of failure for each communication link of the network depicted in Fig. 2, as calculated by application of (2). When calculating the reliability (refer to (4)) one obtains $R_{s,K}(G)$ = 0.904, $R_{s,K}(G\text{-}1)$ = 0.763, $R_{s,K}(G\text{-}2)$ = 0.693, and $R_{s,K}(G\text{-}3)$ = 0.792 thus $Red_1(G, s, K)$ = 0.141, $Red_2(G, s, K)$ = 0.211, and $Red_3(G, s, K)$ = 0.112. Thus node 2 is the node whose removal will affect more the reliability of the communication between s and t.

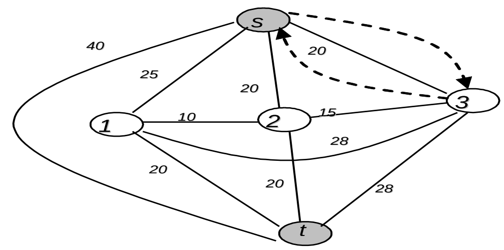

Fig. 2. A mesh network with terminal set $K=\{s,t\}$ (each node has SNR=1000, transmission rate R = 1 bit per channel use, and the fading state of each channel has expected value μ =1).

TABLE I
PROB. OF FAILURE FOR EACH LINK OF THE NETWORK DEPICTED IN FIG. 2

Directed link	Distance (meters)	Prob. of failure (*Poutage*)
(s,1), (1,s)	25	0.464
(s,2), (2,s)	20	0.330
(s,3), (3,s)	20	0.330
(s,t), (t,s)	40	0.800
(1,2), (2,1)	10	0.095
(1,3), (3,1)	28	0.543
(1,t), (t,1)	20	0.330
(2,3), (3,2)	15	0.201
(2,t), (t,2)	20	0.330
(3,t), (t,3)	28	0.543

Cancela and El Khadiri developed a Monte Carlo Recursive Variance Reduction (RVR) technique to efficiently estimate the reliability $R_K(G)$ of an undirected graph $G=(V,E)$ [9]. This technique takes at most $O(|E|^2)$ steps and it can be easily translated into an algorithm to evaluate $R_{s,K}(G)$ for a digraph G as well. Let $\mathrm{MCR}_{s,K}(G)$ be such algorithm.

Algorithm **Assign-Redundancy-Value** assigns a redundancy weight to each redundant node of the network $G=(V,E)$, with source node s, terminal node-set K, and returns the node (or the list of nodes) with maximum redundancy value in G as well as their corresponding unique redundancy weight.

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Algorithm Assign-Redundancy-Value ( )

Input: Digraph G=(V,E), source node s, terminal set K, and
redundant node-set N ⊆ (V-K).
maxL ← ∅;  /* list of nodes with max. redund. value.
max ← 0; /* initial redund. value.
R_{s,K}(G) ← MCR_{s,K}(G); /* calculate reliability via MC
for v ∈ N do
    R_{s,K}(G-v) ← MCR_{s,K}(G-v);
    Red_v(G, s, K) ← (R_{s,K}(G)- R_{s,K}(G-v));
    if Red_v(G, s, K) ≥ max do
        max ← Red_v(G, s, K);
    end if
end for
for v ∈ N do /* list of nodes with max. redund. value.
    if Red_v(G, s, K) = max do
        maxL = maxL ∪ {v};
    end if
end for
return (max, maxL)
```

As the number of redundant nodes is most $|V|$-2, the complexity of **Assign-Redundancy-Value is** $O(|V|\ |E|^2)$.

C. Determining the order of priority of redundant nodes.

When several nodes are scheduled to be deployed given that their locations are already planned ahead of time, it is important to determine the order of priority they should be installed as a function of how the reliability of the communication increases. This analysis corresponds to the case where the cost of deploying is equal for all possible locations. Because of budget constraints or because by just installing a subset of the redundant nodes a required threshold reliability value for the communication has been achieved, prioritizing the deployment of the nodes is then critical.

Let $G=(V, E)$ be a digraph and V' be a subset of the node-set V. The subgraph of G induced by the nodes V', denoted as $[V']_G$, is the digraph $G'=(V',E')$ were E' is the set of links of G that have all their end-points in V' (i.e., $E' = \{(u,v) \in E : u \in V'$ and $v \in V'\}$).

Let G the graph where all the redundant nodes have been placed at their corresponding locations. The set of nodes of G, V, is partitioned into the set C of core nodes, which includes the terminal set K, and the set of redundant nodes N (i.e., $V = C \cup N$). Starting with the subgraph of G induced by the node-set C of core nodes (i.e., $[C]_G$), build on this subgraph, by adding a node at the time (and corresponding induced links), according to which choice of node increases the reliability the most, and include each chosen node in a priority list.

As an example consider the network underlined by $G = (V, E)$ depicted in Fig. 2. Characteristics of the nodes and channels remain as stated in the previous sub-section. Here we are assuming that the core node-set is the terminal set $K=\{s,t\}$, with source node s, and the remaining nodes, 1, 2, and 3, are the redundant nodes. Let G_0 be the subgraph of G induced by the set $C=\{s,t\}$, $[C]_G$ (see Fig. 3); the ordered pair assigned to each communications link between s and t, correspond to the distance and link probability of failure (shown in Table I). Calculating the reliability one gets $R_{s,t}(G_o) = 0.2$.

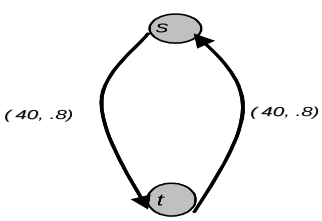

Fig. 3. The graph induced from G by the terminal set $K=\{s,t\}$.

Next consider the subgraphs of G induced by C and another node of N, that is, $[C \cup \{1\}]_G$, $[C \cup \{2\}]_G$, and $[C \cup \{3\}]_G$ (see Fig. 4), and where the probability of failure of their links are shown in Table I. We then calculate the Source-to-terminal reliability for each of these subgraphs (refer to (4)), and we obtain $R_{s,t}([C \cup \{1\}]_G) = 0.486$, $R_{s,t}([C \cup \{2\}]_G) = 0.555$, and $R_{s,t}([C \cup \{3\}]_G) = 0.443$, thus node 2 will be first in the priority list L, since, when added to G_o it has increased the reliability by the largest amount (i.e., from 0.2 to 0.555). Let $G_1 = [C \cup \{2\}]_G$.

Next we proceed to look at the subgraphs of G induced by $C \cup \{2\}$ and another node of N-{2}, that is, $[C \cup \{2\} \cup \{1\}]_G$ and $[C \cup \{2\} \cup \{3\}]_G$. Calculating the reliability of these graphs (refer to (4)) we obtain $R_{s,t}([C \cup \{2\} \cup \{1\}]_G) = 0.792$ and $R_{s,t}([C \cup \{2\} \cup \{3\}]_G) = 0.763$. Since when added to G_1 (and the corresponding induced links), node 1 has increased the reliability by the largest amount (i.e., from 0.555 to 0.792), then node 1 will be assigned the second priority in the list L. Finally node 3 is inserted in the third position of L. Thus $L=\{2,1,3\}$.

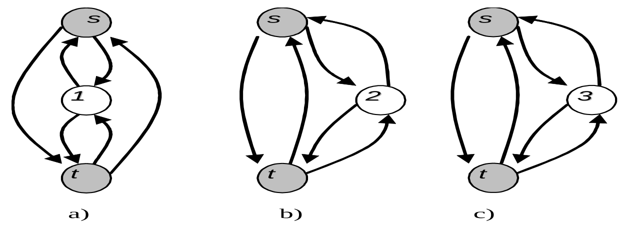

Fig. 4. a) $[C \cup \{1\}]_G$, b) $[C \cup \{2\}]_G$, c) $[C \cup \{3\}]_G$.

The following algorithm, **Assign-Redundancy-Priority**, creates a priority list of the order in which redundant nodes should be inserted to the original core nodes of G to maximize the reliability. Moreover, if at any moment the reliability value of the induced graph by the core nodes and a sub-list of the redundant nodes have reached a threshold value T, then only the sub-list of nodes will be returned. If $T = \infty$, a priority list of all redundant nodes will be returned.

Algorithm **Assign-Redundancy-Priority ( )**

Input: Digraph $G=(V,E)$, source node s, terminal set K, core-set C, redundant node-set $N \subseteq (V\text{-}K)$, threshold value T.

$L \leftarrow \varnothing$; /* priority list of nodes.
$Op \leftarrow C$; /* core nodes.
$R_{s,K}([Op]_G) \leftarrow \mathrm{MCR}_{s,K}([Op]_G)$; /* calc. rel. MC.
$N' \leftarrow N$; /* auxiliary list of redundant nodes N'.
$maxr \leftarrow 0$; /* max. reliability value achieved.
$maxv \leftarrow \varnothing$; /* node with max reliability.
while $R_{s,K}([Op]_G) < T$ and $N' \neq \varnothing$ **do**
  $maxr \leftarrow 0$;
  for $v \in N'$ **do**
    $R_{s,K}([Op \cup \{v\}]_G) \leftarrow \mathrm{MCR}_{s,K}([Op \cup \{v\}]_G)$;
    if $R_{s,K}([Op \cup \{v\}]_G) > maxr$ **do**
      $maxr \leftarrow R_{s,K}([Op \cup v]_G)$;
      $maxv \leftarrow \{v\}$;
    end if
  end for
  $Op \leftarrow Op \cup \{maxv\}$; /* update list of curr. nodes.
  $N' \leftarrow N' - \{maxv\}$; /* delete $maxv$ from N'.
  $L \leftarrow L \cup \{maxv\}$; /* add $maxv$ to the end of L.
  $R_{s,K}([Op]_G) \leftarrow \mathrm{MCR}_{s,K}([Op]_G)$; /* Rel. via MC
end while
return (L)

In line 1 of the algorithm **Assign-Redundancy-Priority**, the priority list L of nodes is initialized. The current set of nodes *Op* is set equal to the core nodes of G (line 2). In line 3, the reliability of the subgraph of G induced by the node-set *Op* (i.e., $[Op]_G$) is determined by application of a Monte Carlo technique (see Section III-B). If either the reliability value of $[Op]_G$ has reached a threshold value T, or no more redundant nodes ought be considered, the priority list L is returned, by the algorithm, with the ordered sequence of nodes to be added to the original core nodes (line 21); otherwise, from the remaining redundant nodes, we determine one, *maxv*, which when added to the current set of nodes *Op*, will induce a subgraph of G that yields the maximum possible reliability among the outstanding redundant nodes (i.e., for-loop). In lines 16 thru 18, the chosen node is added to *Op*, it is inserted at the end of the priority list L, and deleted from the set N' of remaining redundant nodes. Moreover, the reliability of subgraph of G induced by the updated set of nodes *Op*, now containing the new node, is then calculated (line 19).

With respect to the time complexity of the **Assign-Redundancy-Priority** algorithm, as explained in Section III-B, to estimate the reliability by applying a Monte Carlo technique $\text{MCR}_{s,K}$ (G) to a graph $G=(V, E)$, takes $O(|E|^2)$ steps, and this evaluation procedure is executed at most $|V|^2$ times (lines 10 and 19), yielding total complexity $O(|V|^2|E|^2)$.

IV. Optimization Problems in Random Networks

A. Selecting locations for optimal communication between regions.

Unlike planned networks where each node is carefully deployed, in random mesh networks, the nodes are randomly distributed through a geographical area, and the nodes transmitting as well as receiving characteristics do not follow a planned network's approach; this situation appears for example in networks that are set-up in a ad hoc manner (e.g., disaster recovery operations, tactical military operations, conferences, etc.).

Given an existing network $G= (V, E)$, one relevant problem is for example to determine within different surrounding geographical areas, locations where maximum communication capabilities could be attained using G as the backbone for the communication.

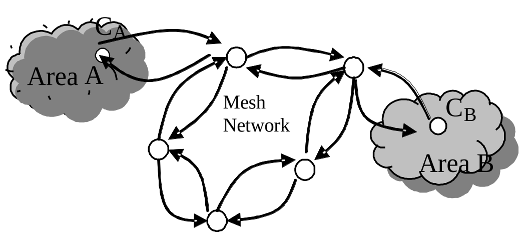

Fig. 5. Two different geographical areas and a mesh network.

For an area A, the center of area C_A (i.e., the area's centroid) with mean coordinates (x_A , y_A), is determined. Suppose that in area A we place a mesh access point M_A at location C_A. Similarly we consider another geographical area B and access point whose centroid has coordinates (x_B, y_B). Next we connect these access points to the backbone network G by adding the corresponding links representing the communication channels (see Fig. 5).

A geographical region R can be divided into sectors or areas having different physical (or tactical) characteristics. In many deployment or tactical situations, given the existence of a wireless mesh network whose nodes have been deployed in a ad hoc fashion, sometimes it is of relevance importance to identify locations of two different regions in which an optimal (or threshold) level of communication could be achieved. A region could represent a space within which movement of communication equipment (e.g., mobile wireless access point) is allowed (for tactical or strategic purposes). As the geographical areas composing the regions may have different transmitting and/or receiving capabilities due for example to terrain obstruction, we may be interested in determining within two regions, R_1 and R_2, two different areas (or sectors) $A \in \mathsf{R}_1$ and $B \in \mathsf{R}_2$ for which the communication is optimal (in comparison with the other areas) or the quality of the communication surpasses a given threshold level, under the assumption that an access point is located at coordinate C_A and another at coordinate C_B. In this case, in order to simulate a mobile device, we are assuming a unique access point for each region with fixed transmitting as well as receiving capabilities (e.g., transmission rate, transmission power, and noise average power). Suppose that region R_1 is composed of i areas $A(1,1)$, $A(1,2)$, $\ldots A(1,i)$ with centroid coordinates $C(1,1)$, $C(1,2)$, $\ldots C(1,i)$, respectively, and region R_2 comprises j areas $A(2,1)$, $A(2,2)$, $\ldots A(2,j)$ with centroid coordinates $C(2,1)$, $C(2,2)$, $\ldots C(2,j)$, respectively. Given an already existing network $G = (V,E)$ and two subordinate regions R_1 and R_2 with corresponding mobile access points M_1 and M_2, respectively, let $G\,[\mathsf{R}_1 ,\mathsf{R}_2\,]$ be the graph obtained from G by adding a node $v(1,l)$ located at the centroid coordinate $C(1,l)$ of the region R_1, for $1 \le l \le i$, and a node $v(2,p)$ located at the centroid coordinate of area $A(2,p)$ of the region R_2, for $1 \le p \le j$; let V_1 and V_2 represent the sets of such nodes. In order to simulate a mobile access point, we can think that the same mesh access point M_q of region R_q $(1 \le q \le 2)$ is positioned at each of the nodes of V_q (at different times), representing the movement the access point within the same region. Consider the links representing the channels of transmitting nodes of R_1 to nodes in G, $E_1^{\,t} = \{(u,v)\colon u = v(1,l),\ 1 \le l \le i;\ v \in V\}$, and the links representing the channels of transmitting nodes of G to receiving nodes of R_1, $E_1^{\,r} = \{(u,v)\colon u \in V;\ \ v = v(1,l),\ 1 \le l \le i\}$. Similarly we define $E_2^{\,t}$ and $E_2^{\,r}$, but involving nodes of R_2 instead. Moreover let $E_{1,2} = \{(u,v)\colon u = v(1,l),\ 1 \le l \le i;\ v = v(2,p),\ 1 \le p \le j\ \}$ (i.e., when nodes of R_1 are transmitting to nodes of R_2), and $E_{2,1}= \{(u,v)\colon u = v(2,p),\ 1 \le p \le j;\ v = v(1,l),\ 1 \le l \le i\}$. Thus the link-set of $G\,[\mathsf{R}_1,\mathsf{R}_2\,]$ is $E_G[\mathsf{R}_1 ,\mathsf{R}_2\,] = E \cup E_1^{\,t} \cup E_1^{\,r} \cup E_2^{\,t} \cup E_2^{\,r} \cup E_{1,2} \cup E_{2,1}$. Moreover the probability of failure of the links are calculated using (2) (see Section II-A). For ease of notation let $L(u)$ be the location of a node u of $G[\mathsf{R}_1,\mathsf{R}_2]$ (corresponding to a centroid point of one of the areas).

Consider the following optimization problem:

***O_G*(R$_1$, R$_2$)**: Find in $G[\mathsf{R}_1,\mathsf{R}_2]$ nodes u and v, $u \in V_1$ and $v \in V_2$, such as

$$R_{u,v}(G[\mathsf{R}_1,\mathsf{R}_2]) = \underset{\substack{x\in V_1\\ y\in V_2}}{Max}\ R_{x,y}(G[\mathsf{R}_1,\mathsf{R}_2]),$$

that is, we are trying to find a location $L(u)$ of a source node u of region R_1 and location $L(v)$ of a terminal node v of region R_2, for which information sent from u to v is the most reliable among all pair of points belonging to the two regions. As mentioned in Section III, when two nodes u and v switch their role from transmitter to receiver, and vice versa, the quality of the communication could change according to the characteristics of the mesh access points as well as the topology representing the communication network. Thus we can also formulate the complementary optimization problem of $O_G(\mathsf{R}_1, \mathsf{R}_2)$:

***O_G*(R$_2$, R$_1$)**: Find in $G[\mathsf{R}_1,\mathsf{R}_2]$ nodes u and v, $u \in V_1$ and $v \in V_2$, such as

$$R_{v,u}(G[\mathsf{R}_1,\mathsf{R}_2]) = \underset{\substack{x\in V_1\\ y\in V_2}}{Max}\ R_{y,x}(G[\mathsf{R}_1,\mathsf{R}_2]),$$

where in general $O_G(\mathsf{R}_1, \mathsf{R}_2) \neq O_G(\mathsf{R}_2, \mathsf{R}_1)$.

Procedure **Optimal-Regional-Reliability** solves optimization problem $O_G(\mathsf{R}_1, \mathsf{R}_2)$, when we are assuming a unique mobile access point for each region with known transmission power (i.e., $|x|^2$, where x is the input signal), transmission rate (bits per channel use), and noise power (i.e., σ_η^2, where η is a zero mean additive white Gaussian noise). For simplicity we assume that all the communication channels have a fading Rayleigh state with unique expected value μ, and the probability of failure of each link is calculated using (2). If threshold value $T \neq \infty$, then the algorithm returns (besides the maximum reliability value, and a pair achieving this reliability) a list of pair of nodes (u,v) for which $R_{u,v}(G[\mathsf{R}_1,\mathsf{R}_2]) \geq T$, otherwise it returns just the pair of nodes that maximizes (the first one is the source node, and the second is the receiving node) the reliability.

Algorithm **Optimal-Regional-Reliability( )**

Input: Digraph $G[\mathrm{R}_1,\mathrm{R}_2]$ is the composed of existing network $G=(V,E)$, and regions R_1 and R_2 with corresponding sets of nodes V_1 and V_2. Threshold value T. Ordered pair (i, j); i transmitting region, j receiving region, $1 \leq i, j \leq 2$, i not = j.

1: $L \leftarrow \emptyset$; /* list of pairs of nodes.
2: $maxr \leftarrow 0$; /* max. reliability value achieved.
3: $maxpair \leftarrow \emptyset$; /* pair of nodes achieving max rel.
4: **for** $u \in V_i$ **do**
5:   **for** $v \in V_j$ **do**
6:     $R_{u,v}(\mathrm{G}[\mathrm{R}_1,\mathrm{R}_2]) \leftarrow \mathrm{MCR}_{u,v}(\mathrm{G}[\mathrm{R}_1,\mathrm{R}_2])$;/* MC.
7:     **if** $R_{u,v}(\mathrm{G}[\mathrm{R}_1,\mathrm{R}_2]) > maxr$ **do**
8:       $maxr \leftarrow R_{u,v}(\mathrm{G}[\mathrm{R}_1,\mathrm{R}_2])$;
9:       $maxpair \leftarrow \{(u, v)\}$;
10:     **end if**
11:     **if** $R_{u,v}([\mathrm{G}[\mathrm{R}_1,\mathrm{R}_2]) \geq T$ **do**
12:       $L = L \cup \{(u, v)\}$;
13:     **end if**
14:   **end for**
15: **end for**
16: **return** (*maxr*, *maxpair*, *L*)

With respect to the time complexity of **Optimal-Regional-Reliability**, to estimate the reliability by application of the Monte Carlo technique $\mathrm{MCR}_{s,K}(G')$ for a digraph $G'=(V',E')$ takes $O(|E'|^2)$ steps (see Section III-B). In line 6, $\mathrm{MCR}_{s,K}(G')$ is applied to estimate the reliability of digraph $G[\mathsf{R}_1,\mathsf{R}_2]$ for source node u and terminal node v for every pair (u,v) of nodes, where u represents an area of R_1 and v an area of R_2. Thus line 6 is executed $|V_1|\,|V_2|$ times, where each execution to estimate the reliability takes $O(|E_G[\mathsf{R}_1,\mathsf{R}_2]|^2)$ steps. Thus algorithm **Optimal-Regional-Reliability** has time complexity $O(|V_1|\,|V_2|\,|E_G[\mathsf{R}_1, \mathsf{R}_2]|^2)$.

In the next sub-section we present an example of the applicability of algorithm **Optimal-Regional-Reliability** to solve optimization problem $O_G(\mathsf{R}_1, \mathsf{R}_2)$.

B. Example.

Consider the network $G=(V,E)$ and related regions R_1 and R_2, each composed of two areas (Fig. 6). We assume all the nodes of $G=(V,E)$ (i.e., $V=\{1,2,3,4\}$) have identical *SNR* ($SNR_{\mathrm{db}} = 30$), transmission rate (1 bit per channel use), and the fading state of the channels connecting the nodes of V, is a Rayleigh random variable with expected value $\mu=1$. Moreover we are assuming a path loss exponent equal to 2 for each of the communication channels of G.

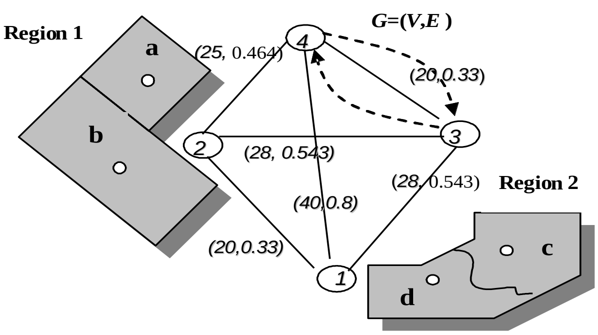

Fig. 6. Existing network $G=(V,E)$ and regions R_1 and R_2 with corresponding areas *A*, *B*, and *C*, *D* with centroid nodes *a*, *b*, and *c*, *d*, respectively. The first value of an ordered pair represents the distance between two nodes and the second value is the probability of failure of the anti-parallel links.

Using these parameters as well as the distances between nodes as depicted in Fig. 6, the probability of failure of each link of G is calculated using (2) (shown in Fig. 6 as the second value of the ordered pair representing a communication link).

Regarding the access point for each region, we assume that mobile access point M_1 (corresponding to R_1) has $SNR_{\mathrm{db}} = 30$ and transmission rate of 0.5 bits per channel use, and M_2 also has $SNR_{\mathrm{db}} = 30$ and transmission rate of 2 bits per channel use. It is important to note that by just stating the *SNR* of the access points, we are assuming identical transmission as well as noise powers for each of the nodes of $G[\mathsf{R}_1,\mathsf{R}_2]$; otherwise we need to state the transmission and noise powers for each node of $G[\mathsf{R}_1,\mathsf{R}_2]$, and then calculate the *SNR* of each communication channel. The probability of failure of each link of $E_G[\mathsf{R}_1,\mathsf{R}_2] - E$ is evaluated using (2), under the previous assumptions and by fixing the fading state of each communication channel as a

Rayleigh random variable with expected value μ = 1, and where the distance and the path loss exponent for each link are given by Table II.

TABLE II
PROBABILITY OF FAILURE OF THE LINKS IN $E_G[R_1,R_2]$ - E

Nodes	Distance (meters)	Chann. Path L. Expon.	Link	Failure Prob.	Link	Failure Prob.
a, 1	50	2.0	(*a*, 1)	0.645	(1, *a*)	0.918
a, 2	30	2.0	(*a*, 2)	0.311	(2, *a*)	0.593
a, 3	33	2.0	(*a*, 3)	0.363	(3, *a*)	0.663
a, 4	11	2.0	(*a*, 4)	0.049	(4, *a*)	0.114
a, *c*	50	3.0	(*a*, *c*)	1.000	(*c*, *a*)	1.000
a, *d*	58	2.6	(*a*, *d*)	1.000	(*d*, *a*)	1.000
b, 1	45	2.0	(*b*, 1)	0.568	(1, *b*)	0.868
b, 2	23	2.0	(*b*, 2)	0.197	(2, *b*)	0.411
b, 3	42	2.0	(*b*, 3)	0.518	(3, *b*)	0.829
b, 4	20	2.0	(*b*, 4)	0.153	(4, *b*)	0.330
b, *c*	58	2.8	(*b*, *c*)	1.000	(*c*, *b*)	1.000
b, *d*	64	2.4	(*b*, *d*)	1.000	(*d*, *b*)	1.000
c, 1	41	2.3	(1, *c*)	0.994	(*c*, 1)	1.000
c, 2	46	2.3	(2, *c*)	0.999	(*c*, 2)	1.000
c, 3	15	2.3	(3, *c*)	0.398	(*c*, 3)	0.782
c, 4	36	2.3	(4, *c*)	0.978	(*c*, 4)	1.000
d, 1	40	2.0	(1, *d*)	0.798	(*d*, 1)	0.992
d, 2	50	2.0	(2, *d*)	0.918	(*d*, 2)	0.999
d, 3	22	2.0	(3, *d*)	0.384	(*d*, 3)	0.766
d, 4	44	2.0	(4, *d*)	0.856	(*d*, 4)	0.977

Regarding optimization problem $O_G(R_1,R_2)$, one gets $R_{a,c}(G[R_1,R_2])$=0.601, $R_{a,d}(G[R_1,R_2])$= 0.744, $R_{b,c}(G[R_1,R_2])$= 0.597, and $R_{b,d}(G[R_1,R_2])$=0.739; thus information sent from region R_1 to region R_2 is most reliable when it is sent from node *a* (of area A) to node *d* (of area *D*), under the assumption of an unique mobile access point for each region.

When applying algorithm **Optimal-Regional-Reliability** to solve problem $O_G(R_2,R_1)$, we get $R_{c,a}(G[R_1,R_2])$=0.192, $R_{c,b}(G[R_1,R_2])$=0.180, $R_{d,a}(G[R_1,R_2])$=0.214, and $R_{d,b}(G[R_1,R_2])$ = 0.200, therefore information is most reliable when sent from node *d* (area *D*) and received at node *a* (area *A*). It is important to emphasize here the importance that plays the high transmission rates of the access point of region R_2, as well as the high path loss exponents of the communication channels connecting the different areas of the regions, for the low reliability values obtained for this problem.

V. CONCLUSIONS

In this paper we've modeled mesh networks as random digraphs in which a directed link of the graph represents a wireless communication channel between a transmitting node and a receiving node, and where this link is modeled using current results in Information Theory applied to wireless communications. Once a wireless network can be accurately simulated as a digraph, then network reliability measures can be applied to solve optimization problems in communication theory, and, in particular we've presented algorithms to tackle problems such as nodes' redundancy in planned mesh networks as well as the problem of locating in different geographical regions surrounding an already existing network assembled in a ad hoc fashion, points in which the communication is the most reliable.

Future research will consider more sophisticate link models of wireless channels to integrate concepts such as antenna power gain and nodes interference, in order to accurately simulate real wireless networks.

It is of relevant importance to mention here that the analysis presented in this article can be easily adapted to general wireless networks, and mesh networks were chosen, among other wireless networks, to clarify concepts.

APPENDIX

Even though the Source-to-*K*-terminal reliability is applicable to single source broadcasting networks, whenever every pair of anti-parallel channels connecting two nodes have the same probability of failure then $R_{s,K}(G) = R_K(G')$, that is, the Source-to-*K*-terminal reliability of the digraph *G* is equal to the *K*-terminal reliability of an undirected graph *G'* discussed in the Introduction, where *G*' is obtained from *G* by replacing two anti-parallel links having the same probability of failure between two nodes, by a single undirected edge with the same probability. This is an important assumption since the optimization algorithms presented in the paper, based on which node is the source and which nodes are the terminals, can be then extended to the case where *K* terminal nodes can communicate independently of which nodes are transmitting and which nodes are receiving information. For the stochastic link representation introduced in Section II-A, this situation is realizable for example when the nodes of a network have identical transmitting as receiving characteristics as well as considerations pertaining the fading states and path loss exponents of the channels.

Consider first the digraph *G* and the operating states Φ of *G* with respect to the Source-to-*K*-terminal reliability as defined in Section II-B; for the purpose of relating the reliabilities we will rename the set Φ as $\Phi_{s,K}(G)$. An operating state *H* of $\Phi_{s,K}(G)$ is a minpath if for any $x \in H$, $H - x$ is not in $\Phi_{s,K}(G)$. Satyanarayana called these minpaths *K-trees* [19]. Formally a *K*-tree T=(V',E') of *G* is a directed tree that contains all the nodes of *K*, there exists a unique dipath between the source *s* and each node of *T*, and each node of *u* of *T* not in *K* must have a link emanating from it (see Fig. 7).

For a digraph $G = (V, E)$ with terminal set *K* and node *s* of *K*, let $\mathbf{M} = \{M_1, M_{2\ldots}\ M_l\}$ be the set of all minpaths of $\Phi_{s,K}(G)$ (i.e., *K*-trees). The situation where all the links of M_i operate (survive), is a random event which will be denoted by E_i. By Inclusion-Exclusion we obtain

$$R_{s,K}(G) = Pr\{\cup_{i=1}^{l} E_i\} = \sum_i Pr(E_i) - \sum_{i<j} Pr(E_i E_j) + \ldots + (-1)^{l+1} Pr(E_1 E_2 \ldots . E_l), \qquad (6)$$

where the event $E_1 E_2 \ldots E_m$ is the event that all the links of the subgraph obtained by the union of M_1, M_2,…, and M_m are operating; thus (6) considers only subgraphs of *G* which are the union of *K*-trees.

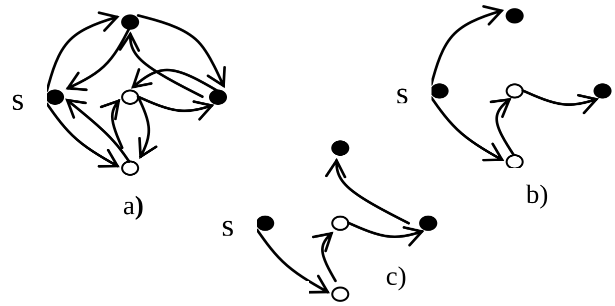

Fig. 7. a) Directed graph *G* with source node *s* and *K* terminal nodes (black nodes), b) - c) *K*-trees of *G*.

Given a probabilistic undirected graph $G=(V,E)$ with a distinguished set of terminal nodes K of V, the classical K-terminal reliability $R_K(G)$ of G, is defined as the probability that the terminal nodes are connected thru operational (undirected) paths composed of surviving edges. If we let $\Phi_K(G)$ be the operating states of G (i.e., connected subgraphs of G containing the terminal set K), the corresponding minpaths are undirected K-trees (i.e., Steiner trees) and a formulation similar to (6) can be derived to evaluate $R_K(G)$.
Let $\mathbf{M} = \{M_1, M_2 \ldots M_r\}$ be the set of Steiner trees of the undirected graph G. Using Inclusion-Exclusion we obtain

$$R_K(G) = Pr\{\cup_{i=1}^{r} E_i\} = \sum_i Pr(E_i) - \sum_{i<j} Pr(E_iE_j) + \ldots + (-1)^{r+1} Pr(E_1E_2 \ldots E_r), \qquad (7)$$

Let G' be the undirected graph obtained from a digraph G by replacing each pair of anti-parallel links having the same probability of failure between two nodes, by a single undirected edge with the same probability (under the assumption that is the case for each set of anti-parallel links of G). It is not difficult to prove that in that case there is a one-to-one correspondence between the directed K-trees of G and the undirected K-trees of G' (Steiner trees); then it follows that each term of (6) for the directed case is identical to each term of (7) for the undirected case, consequently $R_{s,K}(G) = R_K(G')$.

In this case each optimization algorithm presented in this paper based upon which node is the source and which nodes are the terminals, can be then extended to the case where K terminal nodes can communicate independently of which nodes are transmitting and which nodes are receiving information.

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