Quasi-Optimal and Optimal Generalized Mutually Orthogonal ZCZ Sequence Sets Based on an Interleaving Technique

Hideyuki Torii, Masaaki Satoh, Takahiro Matsumoto, and Makoto Nakamura

***Abstract*—** The present paper proposes methods for constructing quasi-optimal or optimal generalized mutually orthogonal zero-correlation zone (GMO-ZCZ) sequence sets. Zero-correlation zone (ZCZ) sequence sets have been studied as spreading sequences for approximately synchronized code-division multiple-access (AS-CDMA) systems. A mutually orthogonal ZCZ (MO-ZCZ) sequence set is composed of several small ZCZ sequence sets, and two arbitrary sequences that belong to different small ZCZ sequence sets are orthogonal. Moreover, in a GMO-ZCZ sequence set, two arbitrary sequences that belong to different small ZCZ sequence sets have a zero-correlation zone instead of being orthogonal. In the present paper, we propose two methods for constructing GMO-ZCZ sequence sets. One is a method for constructing quasi-optimal GMO-ZCZ sequence sets using perfect sequences and orthogonal codes, and this method is a generalized version of our previously proposed method. The other is a method for constructing optimal GMO-ZCZ sequence sets using discrete Fourier transform (DFT) matrices and orthogonal codes. The proposed methods can generate new GMO-ZCZ sequence sets that cannot be obtained from known methods.

***Keywords*—** GMO-ZCZ sequence sets, MO-ZCZ sequence sets, Optimal ZCZ sequence sets, Quasi-optimal ZCZ sequence sets, AS-CDMA systems

I. Introduction

CODE-division multiple-access (CDMA) has been widely applied in digital cellular systems. In CDMA systems, channel separation is provided by the correlation properties of pseudo-random codes referred to as spreading sequences. Therefore, spreading sequences with good autocorrelation and cross-correlation properties play an important part in CDMA systems.

In recent years, approximately synchronized CDMA (AS-CDMA) systems have attracted a great deal of attention because co-channel interference within a cell does not exist in some types of AS-CDMA systems [1]. In such AS-CDMA systems, zero-correlation zone (ZCZ) sequence sets are used as spreading sequences in order to realize this advantage. Various types of ZCZ sequence sets have been widely studied [1]–[29]. Generally, a ZCZ sequence set is characterized by the sequence period, the number of sequences, the ZCZ length, and the number of phases of the sequence elements. The ZCZ length is restricted by a mathematical upper bound determined by the sequence period and the number of sequences [7], [30], and ZCZ sequence sets that satisfy this mathematical upper bound are referred to as optimal ZCZ sequence sets. A number of studies have evaluated optimal ZCZ sequence sets [1], [7], [13], [23], [27], [29]. In addition, quasi-optimal ZCZ sequence sets have also been investigated extensively [10], [18], [25], [28]. Quasi-optimal ZCZ sequence sets have a ZCZ length that is equal to one less than the mathematical upper bound.

Recently, as one type of ZCZ sequence set, mutually orthogonal ZCZ (MO-ZCZ) sequence sets have been proposed by some researchers [31]–[33]. As mentioned above, the ZCZ length is restricted by the sequence period and the number of sequences. This means that the number of sequences in a ZCZ sequence set is also restricted by the sequence period and the ZCZ length. MO-ZCZ sequence sets have been studied in order to increase the number of available sequences for AS-CDMA systems. An MO-ZCZ sequence set is composed of several small ZCZ sequence sets, and two arbitrary sequences that belong to different ZCZ sequence sets are orthogonal. In addition, the authors have proposed generalized MO-ZCZ (GMO-ZCZ) sequence sets [34]. In a GMO-ZCZ sequence set, two arbitrary sequences that belong to different ZCZ sequence sets have a zero-correlation zone instead of being orthogonal. Therefore, GMO-ZCZ sequence sets are more suitable for reducing co-channel interference than conventional MO-ZCZ sequence sets. Since a GMO-ZCZ sequence set has a zero-correlation zone between different ZCZ sequence sets, the GMO-ZCZ sequence sets can be regarded as a single ZCZ sequence set. The GMO-ZCZ sequence set proposed in [34] becomes a single quasi-optimal ZCZ sequence set. Such GMO-ZCZ sequence sets are referred to as quasi-optimal GMO-ZCZ sequence sets.

In the present paper, we propose two methods for constructing GMO-ZCZ sequence sets based on interleaving technique. One is a method for constructing quasi-optimal GMO-ZCZ sequence sets using perfect sequences and orthogonal codes, and this method is a generalized version of our previously proposed method [34]. In the previous method, the number of small ZCZ sequence sets and the number of sequences in each ZCZ sequence set must be divisors of the period of a perfect sequence used for constructing a GMO-ZCZ sequence

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The material in this paper was presented in part at [37].

set. On the other hand, the proposed method does not need this condition. The other is a method for constructing GMO-ZCZ sequence sets using discrete Fourier transform (DFT) matrices and orthogonal codes. The proposed GMO-ZCZ sequence set can be regarded as a single optimal ZCZ sequence set. Such GMO-ZCZ sequence sets are referred to as optimal GMO-ZCZ sequence sets. That is, the proposed GMO-ZCZ sequence sets are optimal GMO-ZCZ sequence sets. These proposed methods can generate new GMO-ZCZ sequence sets that cannot be obtained from known methods.

II. Preliminaries

In this section, we define ZCZ sequence sets, MO-ZCZ sequence sets, GMO-ZCZ sequence sets, and related terms.

First, we explain ZCZ sequence sets. Let Z be a sequence set with N sequences of period P. Then, Z can be represented as

$$\begin{aligned} &Z = \left\{ Z_n \mid 0 \le n \le N-1 \right\}, \\ &Z_n = \left(z_0^{(n)}, z_1^{(n)}, \cdots, z_p^{(n)}, \cdots, z_{P-1}^{(n)} \right), \end{aligned} \tag{1}$$

where Z_n and $z_p^{(n)}$ represent a sequence and a sequence element, respectively. Let $R_{Z_{n_0},Z_{n_1}}(\tau)$ be the periodic correlation function between Z_{n_0} and Z_{n_1}. For the sake of simplicity, the modulo operator is represented as %, i.e.,

$$x\%P \stackrel{\text{def}}{=} x \bmod P. \tag{2}$$

Then, $R_{Z_{n_0},Z_{n_1}}(\tau)$ is defined as

$$R_{Z_{n_0},Z_{n_1}}(\tau) \stackrel{\text{def}}{=} \sum_{p=0}^{P-1} z_p^{(n_0)} z_{(p+\tau)\%P}^{(n_1)*}, \tag{3}$$

where the symbol $*$ denotes complex conjugation. When Z_{n_0} does not correspond to Z_{n_1}, $R_{Z_{n_0},Z_{n_1}}(\tau)$ is referred to as the periodic cross-correlation function between Z_{n_0} and Z_{n_1}. On the other hand, when Z_{n_0} corresponds to Z_{n_1}, $R_{Z_{n_0},Z_{n_1}}(\tau) = R_{Z_{n_0},Z_{n_0}}(\tau)$ is referred to as the periodic autocorrelation function of Z_{n_0}. If all of the sequences in Z satisfy the following conditions, Z is referred to as a ZCZ sequence set:

$$\begin{aligned} &\forall n_0, 1 \le |\tau| \le L, \\ &\qquad R_{Z_{n_0},Z_{n_0}}(\tau) = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &\forall n_0 \ne n_1, |\tau| \le L, \\ &\qquad R_{Z_{n_0},Z_{n_1}}(\tau) = 0. \end{aligned} \tag{5}$$

The integer L is referred to as a ZCZ length. The ZCZ sequence set is represented as $Z_{cz}(P, N, L)$ in order to illustrate the sequence period, the number of sequences, and the ZCZ length. The ZCZ length is restricted by the following mathematical upper bound [7], [30]:

$$L \le \frac{P}{N} - 1. \tag{6}$$

If a ZCZ sequence set satisfies the following condition, the ZCZ sequence set is referred to as an optimal ZCZ sequence set:

$$L = \frac{P}{N} - 1. \tag{7}$$

Although L is always an integer, P/N is not always an integer. In the case in which P/N is not an integer, (7) can be generalized as follows [35]:

$$L = \left\lfloor \frac{P}{N} \right\rfloor - 1, \tag{8}$$

where $\lfloor P/N \rfloor$ is the floor function of P/N, i.e., it means the maximum integer that does not exceed P/N. Similarly, a quasi-optimal ZCZ sequence set is defined as a ZCZ sequence set that satisfies the following condition:

$$L = \frac{P}{N} - 2. \tag{9}$$

(9) can be generalized in a similar manner to (8) as follows:

$$L = \left\lfloor \frac{P}{N} \right\rfloor - 2. \tag{10}$$

Next, we explain MO-ZCZ sequence sets and GMO-ZCZ sequence sets. Let M be a set with K ZCZ sequence sets containing N sequences of period P. Then, M can be represented as

$$\begin{aligned} &M = \left\{ M_k \mid 0 \le k \le K-1 \right\}, \\ &M_k = \left\{ M_n^{(k)} \mid 0 \le n \le N-1 \right\}, \\ &M_n^{(k)} = \left(m_0^{(k,n)}, \cdots, m_p^{(k,n)}, \cdots, m_{P-1}^{(k,n)} \right), \end{aligned} \tag{11}$$

where M_k, $M_n^{(k)}$, and $m_p^{(k,n)}$ represent a ZCZ sequence set, a sequence, and a sequence element, respectively. Suppose that the ZCZ length of the ZCZ sequence sets in M is L. If two arbitrary sequences that belong to different ZCZ sequence sets satisfy the following condition, M is referred to as an MO-ZCZ sequence set:

$$\begin{aligned} &\forall k_0 \ne k_1, \forall n_0, \forall n_1, \\ &\qquad R_{M_{n_0}^{(k_0)},M_{n_1}^{(k_1)}}(0) = 0. \end{aligned} \tag{12}$$

That is, an MO-ZCZ sequence set is a set of several small ZCZ sequence sets, and two arbitrary sequences that belong to different ZCZ sequence sets are orthogonal. In addition, if two arbitrary sequences that belong to different ZCZ sequence sets of an MO-ZCZ sequence set satisfy the following condition, M is referred to as a GMO-ZCZ sequence set:

$$\begin{aligned} &\forall k_0 \ne k_1, \forall n_0, \forall n_1, |\tau| \le \tilde{L}, \\ &\qquad R_{M_{n_0}^{(k_0)},M_{n_1}^{(k_1)}}(\tau) = 0. \end{aligned} \tag{13}$$

The integer $\tilde{L}$ is a ZCZ length between different ZCZ sequence sets, and $\tilde{L} < L$. That is, two arbitrary sequences that belong to different ZCZ sequence sets of a GMO-ZCZ sequence set have a zero-correlation zone instead of being orthogonal. From (6), it is obvious that the number of sequences is also restricted by the following mathematical upper bound:

$$N \le \frac{P}{L+1}. \tag{14}$$

MO-ZCZ sequence sets have been studied in order to increase the number of available sequences for AS-CDMA systems. Since two arbitrary sequences that belong to different ZCZ sequence sets of a GMO-ZCZ sequence set have a zero-correlation zone, GMO-ZCZ sequence sets are more suitable for reducing co-channel interference than MO-ZCZ sequence sets. Since a GMO-ZCZ sequence set has a zero-correlation zone between different ZCZ sequence sets, the GMO-ZCZ sequence sets can be regarded as a single ZCZ sequence set. If a GMO-ZCZ sequence set becomes a single optimal ZCZ sequence set, the GMO-ZCZ sequence set is referred to as an optimal GMO-ZCZ sequence set. Similarly, if a GMO-ZCZ sequence set becomes a single quasi-optimal ZCZ sequence set, the GMO-ZCZ sequence set is referred to as a quasi-optimal GMO-ZCZ sequence set.

III. New Quasi-Optimal GMO-ZCZ Sequence Sets

In this section, we propose a method for constructing quasi-optimal GMO-ZCZ sequence sets based on perfect sequences, orthogonal codes, and an interleaving technique. This method is a generalized version of our previously proposed method [34]. In addition, we prove the proposed method.

A. Sequence Generation

Let $S = (s_0, s_1, \cdots, s_p, \cdots, s_{P-1})$ be a perfect sequence of period P, i.e., S satisfies the following autocorrelation property:

$$\begin{aligned} &\text{For } \tau \neq 0, \\ &\quad R_{S,S}(\tau) = 0, \qquad (15) \\ &\text{For } \tau = 0, \\ &\quad R_{S,S}(\tau) = \sum_{p=0}^{P-1} |s_p|^2 = E_S, \qquad (16) \end{aligned}$$

where E_S is the sequence energy of S. Let N_0 be a positive integer, and assume that $N_0 \geq 2$. Let O be a set of N_0 orthogonal codes of length N_0, i.e., O can be represented as

$$\begin{aligned} &O = \{O_n \mid 0 \leq n \leq N_0 - 1\}, \\ &O_n = \left(o_0^{(n)}, o_1^{(n)}, \cdots, o_p^{(n)}, \cdots, o_{N_0-1}^{(n)}\right), \qquad (17) \\ &\forall n_0 \neq n_1, \\ &R_{O_{n_0}, O_{n_1}}(0) = \sum_{p=0}^{N_0-1} o_p^{(n_0)} o_p^{(n_1)*} = 0, \qquad (18) \end{aligned}$$

where O_n and $o_p^{(n)}$ represent a sequence and a sequence element, respectively. Two integers N_1 and N_2 are defined as follows:

$$\left\lfloor \frac{P}{N_0 N_2} \right\rfloor = N_1, \qquad (19)$$

$$N_1 \geq 2, N_2 \geq 1. \qquad (20)$$

Using S and O, a GMO-ZCZ sequence set G is obtained by the following formula:

$$\begin{aligned} &G = \{G_k \mid 0 \leq k \leq N_1 - 1\}, \\ &G_k = \left\{G_n^{(k)} \mid 0 \leq n \leq N_0 - 1\right\}, \\ &G_n^{(k)} = \left(g_0^{(k,n)}, g_1^{(k,n)}, \cdots, g_p^{(k,n)}, \cdots, g_{PN_0-1}^{(k,n)}\right), \\ &g_p^{(k,n)} = s_{((k+(p\%N_0)N_1)N_2 + \lfloor p/N_0 \rfloor)\%P} \cdot o_{p\%N_0}^{(n)}. \qquad (21) \end{aligned}$$

If $P = N_0 N_1 N_2$ is satisfied, the proposed method corresponds with our previous method [34].

Note that G includes N_1 ZCZ sequence sets, each ZCZ sequence set includes N_0 sequences, and the period of each sequence is PN_0. The ZCZ length of each small ZCZ sequence set, L, is $N_0 N_1 N_2 - 2$, and the ZCZ length between different ZCZ sequence sets, $\tilde{L}$, is $N_0 N_2 - 2$. Since G includes N_1 ZCZ sequence sets and each G_k includes N_0 sequences, the total number of sequences included in G is $N_0 N_1$. Therefore, G becomes a single ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_0 N_1, N_0 N_2 - 2)$. If $P = N_0 N_1 N_2$ is satisfied, G is a quasi-optimal GMO-ZCZ sequence set from the viewpoint of (9). On the other hand, if $\lfloor P/N_1 \rfloor = N_0 N_2$ is satisfied, G is a quasi-optimal GMO-ZCZ sequence set from the viewpoint of (10). Each G_k is a ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_0, N_0 N_1 N_2 - 2)$. If $P = N_0 N_1 N_2$ is satisfied, each small ZCZ sequence set becomes a quasi-optimal ZCZ sequence set from the viewpoint of (9).

B. Proof

Integers p, p_0, p_1, τ, τ_0, and τ_1 are defined as

$$\begin{aligned} &p = p_1 N_0 + p_0, \qquad (22) \\ &\tau = \tau_1 N_0 + \tau_0, \qquad (23) \\ &0 \leq p_0, \tau_0 \leq N_0 - 1, \qquad (24) \\ &0 \leq p_1, \tau_1 \leq P - 1, \qquad (25) \\ &0 \leq p, \tau \leq PN_0 - 1. \qquad (26) \end{aligned}$$

In addition, an integer ϵ is defined as follows:

$$\begin{aligned} &\text{For } p_0 + \tau_0 < N_0, \\ &\quad \epsilon = 0, \qquad (27) \\ &\text{For } p_0 + \tau_0 \geq N_0, \\ &\quad \epsilon = 1. \qquad (28) \end{aligned}$$

Then, we have

$$(p_0 + \tau_0)\%N_0 = p_0 + \tau_0 - \epsilon N_0, \qquad (29)$$

$$\lfloor (p_0 + \tau_0)/N_0 \rfloor = \epsilon. \qquad (30)$$

From (21), the periodic correlation function between $G_{n_0}^{(k_0)}$ and $G_{n_1}^{(k_1)}$ can be represented as

$$\begin{aligned} R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}}(\tau) &= \sum_{p=0}^{PN_0-1} g_p^{(k_0,n_0)} \cdot g_{(p+\tau)\%PN_0}^{(k_1,n_1)*} \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(n_1)*} \\ &\quad \cdot \sum_{p_1=0}^{P-1} s_{((k_0+p_0 N_1)N_2+p_1)\%P} \\ &\qquad \cdot s^*_{((k_1+(p_0+\tau_0-\epsilon N_0)N_1)N_2+p_1+\tau_1+\epsilon)\%P} \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(n_1)*} \\ &\quad \cdot R_{S,S}\left((k_1 - k_0)N_2 + (\tau_0 - \epsilon N_0)N_1 N_2 \right. \\ &\qquad \left. + \tau_1 + \epsilon\right). \qquad (31) \end{aligned}$$

For the sake of simplicity, we introduce the following notation:

$$\rho = ((k_1 - k_0) N_2 + (\tau_0 - \epsilon N_0) N_1 N_2 + \tau_1 + \epsilon) \% P, \quad (32)$$

$$\rho_0 = \tau_0 N_1 N_2 + \tau_1, \quad (33)$$

$$\rho_1 = \epsilon (N_0 N_1 N_2 - 1) + (k_0 - k_1) N_2, \quad (34)$$

$$\rho = (\rho_0 - \rho_1) \% P. \quad (35)$$

Since S is a perfect sequence, $R_{S,S}(\rho)$ becomes E_S when $\rho = 0$, and it becomes 0 when $\rho \neq 0$. Now, we consider the following two cases.

[A] Suppose that $k_0 = k_1$; then ρ_1 is represented as

$$\rho_1 = \epsilon (N_0 N_1 N_2 - 1). \quad (36)$$

In addition, suppose that $0 \leq \tau \leq N_0 N_1 N_2 - 1$; then from (23), (24), and (25),

$$0 \leq \tau_0 \leq N_0 - 1, \quad (37)$$

$$0 \leq \tau_1 \leq N_1 N_2 - 1. \quad (38)$$

Therefore, the range of ρ_0 becomes

$$0 \leq \rho_0 \leq N_0 N_1 N_2 - 1. \quad (39)$$

Suppose that $\epsilon = 0$; then $\rho_0 = \rho_1$ holds if and only if $\rho_0 = 0$, i.e.,

$$\tau_0 = \tau_1 = 0. \quad (40)$$

This means that $\tau = 0$. On the other hand, suppose that $\epsilon = 1$; then $\rho_0 = \rho_1$ holds if and only if $\rho_0 = N_0 N_1 N_2 - 1$, i.e.,

$$\tau_0 = N_0 - 1, \quad (41)$$

$$\tau_1 = N_1 N_2 - 1. \quad (42)$$

This means that $\tau = N_0 N_1 N_2 - 1$. Therefore, $R_{S,S}(\rho) = 0$ is satisfied, at least in the range of $1 \leq \tau \leq N_0 N_1 N_2 - 2$. Based on (31), this means that

$$\forall k_0, \forall n_0, \forall n_1, 1 \leq \tau \leq N_0 N_1 N_2 - 2, \quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_0)}}(\tau) = 0. \quad (43)$$

The periodic correlation function has the following symmetric property [36]:

$$R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}}(-\tau) = R^*_{G_{n_1}^{(k_1)}, G_{n_0}^{(k_0)}}(\tau). \quad (44)$$

Exchanging n_0 and n_1 in (43) and applying (44), we have

$$\forall k_0, \forall n_0, \forall n_1, -N_0 N_1 N_2 + 2 \leq \tau \leq -1, \quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_0)}}(\tau) = 0. \quad (45)$$

When $\tau = 0$, we have $\tau_0 = \tau_1 = \epsilon = 0$ from (23) and (27). Therefore, (31) becomes

$$R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_0)}}(\tau) = \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} o_{p_0}^{(n_1)*} R_{S,S}(0) = E_S \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} o_{p_0}^{(n_1)*} \quad (46)$$

Since O is a set of orthogonal codes, (46) means that

$$\forall k_0, \forall n_0 \neq n_1, \quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_0)}}(0) = 0. \quad (47)$$

From (43), (45), and (47), each G_k is a ZCZ sequence set that is represented as $Z_{cz}(PN_0, N_0, N_0 N_1 N_2 - 2)$.

[B] Suppose that $k_0 \neq k_1$. In addition, suppose that $0 \leq \tau \leq N_0 N_2 - 1$; then from (23), (24), and (25),

$$0 \leq \tau_0 \leq N_0 - 1, \quad (48)$$

$$0 \leq \tau_1 \leq N_2 - 1. \quad (49)$$

Therefore, the range of ρ_0 becomes

$$\tau_0 N_1 N_2 \leq \rho_0 \leq \tau_0 N_1 N_2 + N_2 - 1. \quad (50)$$

Note that the minimum value of ρ_0 is 0 and the maximum value of ρ_0 is $N_0 N_1 N_2 - N_1 N_2 + N_2 - 1 < P$. Since $k_0 \neq k_1$, the range of $k_0 - k_1$ becomes

$$-N_1 + 1 \leq k_0 - k_1 \leq -1, 1 \leq k_0 - k_1 \leq N_1 - 1. \quad (51)$$

Suppose that $\epsilon = 0$; then ρ_1 is represented as

$$\rho_1 = (k_0 - k_1) N_2. \quad (52)$$

The range of ρ_1 becomes

$$-N_1 N_2 + N_2 \leq \rho_1 \leq -N_2, N_2 \leq \rho_1 \leq N_1 N_2 - N_2. \quad (53)$$

Since the minimum value of $\rho_0 - \rho_1$ is $-N_1 N_2 + N_2 > -P$ and the maximum value of $\rho_0 - \rho_1$ is $N_0 N_1 N_2 - 1 < P$, $(\rho_0 - \rho_1) \% P = 0$ means $\rho_0 = \rho_1$. Since ρ_1 is a multiple of N_2, the following condition is necessary in order to satisfy $\rho_0 = \rho_1$:

$$\tau_0 N_1 N_2 = (k_0 - k_1) N_2. \quad (54)$$

However, $k_0 - k_1$ is not a multiple of N_1 from (51). Therefore, $\rho_0 \neq \rho_1$, i.e., $\rho \neq 0$ if $\epsilon = 0$. On the other hand, suppose that $\epsilon = 1$; then ρ_1 is represented as

$$\rho_1 = N_0 N_1 N_2 - 1 + (k_0 - k_1) N_2. \quad (55)$$

The range of ρ_1 becomes

$$N_0 N_1 N_2 - 1 - N_1 N_2 + N_2 \leq \rho_1 \leq N_0 N_1 N_2 - 1 - N_2, \quad (56)$$

$$N_0 N_1 N_2 - 1 + N_2 \leq \rho_1 \leq N_0 N_1 N_2 - 1 + N_1 N_2 - N_2. \quad (57)$$

From (28), if $\epsilon = 1$, $\tau_0 \neq 0$ is satisfied. Therefore, the minimum value of ρ_0 is $N_1 N_2$. Since the minimum value of $\rho_0 - \rho_1$ is $-N_0 N_1 N_2 + 1 + N_2 > -P$ and the maximum value of $\rho_0 - \rho_1$ is 0, $(\rho_0 - \rho_1) \% P = 0$ means $\rho_0 = \rho_1$. Since ρ_1 is one less than a multiple of N_2, the following condition is necessary in order to satisfy $\rho_0 = \rho_1$:

$$\tau_0 N_1 N_2 + N_2 - 1 = N_0 N_1 N_2 - 1 + (k_0 - k_1) N_2. \quad (58)$$

Note that $\tau_1 = N_2 - 1$ in this case. (58) can be represented as

$$(\tau_0 - N_0) N_1 = k_0 - k_1 - 1. \quad (59)$$

(59) holds if and only if $k_0 - k_1 = -N_1 + 1$ and $\tau_0 = N_0 - 1$. Since $\tau_0 = N_0 - 1$ and $\tau_1 = N_2 - 1$, we have $\tau = N_0 N_2 - 1$.

Therefore, $R_{S,S}(\rho) = 0$ is satisfied, at least in the range of $0 \le \tau \le N_0 N_2 - 2$. Based on (31), this means that

$$\begin{aligned} &\forall k_0 \ne k_1, \forall n_0, \forall n_1, 0 \le \tau \le N_0 N_2 - 2, \\ &\quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}}(\tau) = 0. \end{aligned} \tag{60}$$

Similar to [A], we have

$$\begin{aligned} &\forall k_0 \ne k_1, \forall n_0, \forall n_1, -N_0 N_2 + 2 \le \tau \le 0, \\ &\quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}}(\tau) = 0. \end{aligned} \tag{61}$$

From (60) and (61), when we regard G as a single sequence set, G is also a ZCZ sequence set that is represented as $Z_{cz}(PN_0, N_0 N_1, N_0 N_2 - 2)$.

Thus, the proposed method has been proven.

C. Example

Suppose that $P = 16$ and

$$S = (0000012302020321),$$

where 0, 1, 2, and 3 represent $+1$, $+\sqrt{-1}$, -1, and $-\sqrt{-1}$, respectively. Then, S is a quadriphase perfect sequence of period 16. In addition, suppose that $N_0 = 3$ and

$$\begin{aligned} O_0 &= (000), \\ O_1 &= (012), \\ O_2 &= (021), \end{aligned}$$

where each element represents a power of $\exp\left(2\pi\sqrt{-1}/3\right)$. Then, $O = \{O_n \mid 0 \le n \le 2\}$ is a set of three-phase orthogonal codes of length 3. Let $N_1 = 5$ and $N_2 = 1$; then a twelve-phase GMO-ZCZ sequence set with five ZCZ sequence sets containing three sequences of period 48 is obtained from (21). Each sequence of this GMO-ZCZ sequence set is given below:

$$\begin{aligned}
G_0^{(0)} &= (030066090000906630366090\\
&\qquad 0906600306030069096003 06), \\
G_1^{(0)} &= (0780A20180450A234B6A8948\\
&\qquad 0186A807864B042945648342), \\
G_2^{(0)} &= (0B402A05408102A387624984\\
&\qquad 0546240B468708A98168438A), \\
G_0^{(1)} &= (066090000906630366090009 0\\
&\qquad 66003060300690960030 6030), \\
G_1^{(1)} &= (0A20180450A234B6A8948018\\
&\qquad 6A807864B042945648342078), \\
G_2^{(1)} &= (02A05408102A387624984054\\
&\qquad 6240B468708A98168438A0B4), \\
G_0^{(2)} &= (090000906630366090009066 0\\
&\qquad 030603006909600306030066), \\
G_1^{(2)} &= (0180450A234B6A89480186A8\\
&\qquad 07864B0429456483420780A2), \\
G_2^{(2)} &= (05408102A387624984054624\\
&\qquad 0B468708A98168438A0B402A), \\
G_0^{(3)} &= (009066303660900009066003 0\\
&\qquad 603006909600306030066090), \\
G_1^{(3)} &= (0450A234B6A89480186A8078\\
&\qquad 64B0429456483420780A2018), \\
G_2^{(3)} &= (08102A38762498405462 40B4\\
&\qquad 68708A98168438A0B402A054), \\
G_0^{(4)} &= (066303660900009066003060 3\\
&\qquad 006909600306030066090009), \\
G_1^{(4)} &= (0A234B6A89480186A807864B\\
&\qquad 0429456483420780A2018045), \\
G_2^{(4)} &= (02A387624984054624 0B4687\\
&\qquad 08A98168438A0B402A054081),
\end{aligned}$$

where A and B represent 10 and 11, respectively, and each element represents a power of $\exp\left(2\pi\sqrt{-1}/12\right)$. The GMO-ZCZ sequence set $G = \{G_k \mid 0 \le k \le 4\}$ is a single quasi-optimal ZCZ sequence set that is represented as $Z_{cz}(48, 15, 1)$. In addition, each $G_k = \left\{G_n^{(k)} \mid 0 \le n \le 2\right\}$ is a ZCZ sequence set that is represented as $Z_{cz}(48, 3, 13)$.

IV. New Optimal GMO-ZCZ Sequence Sets

In this section, we propose a method for constructing optimal GMO-ZCZ sequence sets based on DFT matrices, orthogonal codes, and an interleaving technique. In addition, we prove the proposed method.

A. Sequence Generation

Let F be the P-dimensional DFT matrix, i.e., F can be represented as

$$\begin{aligned} F &= [f_{i,j}], \\ f_{i,j} &= \exp\left(\frac{-2\pi\sqrt{-1}}{P} ij\right), \end{aligned} \tag{62}$$

where $f_{i,j}$ represents the (i,j)-th element of F, and $0 \le i, j \le P - 1$. For the sake of simplicity, we introduce the following notation:

$$\exp\left(\frac{-2\pi\sqrt{-1}}{P} ij\right) = W_P^{ij}. \tag{63}$$

Let $R_{f_{i_0}, f_{i_1}}(\tau)$ be the periodic correlation function between the i_0-th row and the i_1-th row of the DFT matrix. The DFT matrix has the following correlation property:

$$\begin{aligned} &\forall i_0 \ne i_1, \forall \tau, \\ &\quad R_{f_{i_0}, f_{i_1}}(\tau) = 0. \end{aligned} \tag{64}$$

Let O be a set of N_0 orthogonal codes of length N_0 defined in Section III. In addition, suppose that all of the sequence elements in O has the same absolute value, i.e.,

$$\begin{aligned} &\forall n, \forall p, \\ &\quad \left|o_p^{(n)}\right| = o_{const.}. \end{aligned} \tag{65}$$

An integer N_1 is defined as follows:

$$\begin{aligned} P &= N_0 N_1, \\ N_1 &\ge 2. \end{aligned} \tag{66}$$

Using F and O, a GMO-ZCZ sequence set G is obtained by the following formula:

$$\begin{aligned} G &= \{G_k \mid 0 \le k \le N_0 - 1\}, \\ G_k &= \left\{G_n^{(k)} \mid 0 \le n \le N_1 - 1\right\}, \\ G_n^{(k)} &= \left(g_0^{(k,n)}, g_1^{(k,n)}, \cdots, g_p^{(k,n)}, \cdots, g_{PN_0-1}^{(k,n)}\right), \\ g_p^{(k,n)} &= f_{n+(p\%N_0)N_1, \lfloor p/N_0 \rfloor} \cdot o_{p\%N_0}^{(k)}. \end{aligned} \tag{67}$$

Note that G includes N_0 ZCZ sequence sets, each ZCZ sequence set includes N_1 sequences, and the period of each sequence is PN_0. The ZCZ length of each small ZCZ sequence set, L, is $N_0^2 - 1$, and the ZCZ length between different ZCZ sequence sets, $\tilde{L}$, is $N_0 - 1$. Since G includes N_0 ZCZ sequence sets and each G_k includes N_1 sequences, the total number of sequences included in G is $N_0 N_1$. Therefore, G becomes a single ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_0 N_1, N_0 - 1)$. Since the parameters of this ZCZ sequence set satisfy (7), G is an optimal GMO-ZCZ sequence set from the viewpoint of (7). Each G_k is a ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_1, N_0^2 - 1)$, and it becomes an optimal ZCZ sequence set the viewpoint of (7).

B. Proof

We define integers p, p_0, p_1, τ, τ_0, τ_1, and ϵ in the same manner as Section III. From (67), the periodic correlation function between $G_{n_0}^{(k_0)}$ and $G_{n_1}^{(k_1)}$ can be represented as

$$\begin{aligned} & R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}}(\tau) \\ &= \sum_{p=0}^{PN_0-1} g_p^{(k_0,n_0)} \cdot g_{(p+\tau)\%PN_0}^{(k_1,n_1)*} \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(k_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(k_1)*} \\ &\quad \cdot \sum_{p_1=0}^{P-1} f_{n_0+p_0N_1, p_1} \\ &\quad\quad \cdot f^*_{n_1+(p_0+\tau_0-\epsilon N_0)N_1, (p_1+\tau_1+\epsilon)\%P} \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(k_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(k_1)*} \\ &\quad \cdot R_{f_{n_0+p_0N_1}, f_{n_1+(p_0+\tau_0-\epsilon N_0)N_1}}(\tau_1 + \epsilon). \end{aligned} \tag{68}$$

Now, we consider the following three cases.
[A] Suppose that $k_0 = k_1$ and $n_0 = n_1$. In addition, suppose that $0 \le \tau \le N_0^2 - 1$. Then, from (23), (24), and (25),

$$0 \le \tau_0 \le N_0 - 1, \tag{69}$$
$$0 \le \tau_1 \le N_0 - 1. \tag{70}$$

If $\tau_0 \ne 0$, $\tau_0 \ne \epsilon N_0$ from (69). In this case, the following condition is satisfied:

$$n_0 + p_0 N_1 \ne n_0 + (p_0 + \tau_0 - \epsilon N_0) N_1. \tag{71}$$

From (64), we have

$$R_{f_{n_0+p_0N_1}, f_{n_0+(p_0+\tau_0-\epsilon N_0)N_1}}(\tau_1 + \epsilon) = 0. \tag{72}$$

Based on (68), this means that

$$R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_0)}}(\tau) = 0. \tag{73}$$

On the other hand, if $\tau_0 = 0$, $\epsilon = 0$ from (27). Therefore, the following condition holds if and only if $\tau_0 = 0$:

$$n_0 + p_0 N_1 = n_0 + (p_0 + \tau_0 - \epsilon N_0) N_1. \tag{74}$$

In this case, from (62) and (63),

$$\begin{aligned} & R_{f_{n_0+p_0N_1}, f_{n_0+p_0N_1}}(\tau_1) \\ &= \sum_{p_1=0}^{P-1} W_P^{(n_0+p_0N_1)p_1} \cdot W_P^{(n_0+p_0N_1)(p_1+\tau_1)*} \\ &= \sum_{p_1=0}^{P-1} W_P^{(n_0+p_0N_1)\tau_1 *} \\ &= P W_P^{(n_0+p_0N_1)\tau_1 *} \end{aligned} \tag{75}$$

Based on (65), (68), and (70),

$$\begin{aligned} & R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_0)}}(\tau) \\ &= P \sum_{p_0=0}^{N_0-1} o_{p_0}^{(k_0)} \cdot o_{p_0}^{(k_0)*} \cdot W_P^{(n_0+p_0N_1)\tau_1 *} \\ &= P W_P^{n_0\tau_1 *} \sum_{p_0=0}^{N_0-1} \left| o_{p_0}^{(k_0)} \right|^2 W_P^{p_0\tau_1N_1 *} \\ &= P W_P^{n_0\tau_1 *} o_{const.}^2 \sum_{p_0=0}^{N_0-1} \exp\left(\frac{2\pi\sqrt{-1}p_0\tau_1}{N_0}\right) \\ &= \begin{cases} 0 & (\tau_1 \ne 0), \\ PN_0 o_{const.}^2 & (\tau_1 = 0). \end{cases} \end{aligned} \tag{76}$$

Therefore, from (73) and (76), we have

$$\begin{aligned} & \forall k_0, \forall n_0, 1 \le \tau \le N_0^2 - 1, \\ & \quad R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_0)}}(\tau) = 0. \end{aligned} \tag{77}$$

[B] Suppose that $k_0 \ne k_1$ and $n_0 = n_1$. In addition, suppose that $0 \le \tau \le N_0 - 1$. Then, from (23), (24), and (25),

$$0 \le \tau_0 \le N_0 - 1, \tag{78}$$
$$\tau_1 = 0. \tag{79}$$

Similar to [A], if $\tau_0 \ne 0$,

$$R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_1)}}(\tau) = 0. \tag{80}$$

Moreover, similar to [A], if $\tau_0 = 0$,

$$\begin{aligned} R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_1)}}(\tau) &= P \sum_{p_0=0}^{N_0-1} o_{p_0}^{(k_0)} \cdot o_{p_0}^{(k_1)*} \\ &= 0. \end{aligned} \tag{81}$$

Note that $\tau_1 = 0$ and O is a set of orthogonal codes. Therefore, from (80) and (81), we have

$$\begin{aligned} & \forall k_0 \ne k_1, \forall n_0, 0 \le \tau \le N_0 - 1, \\ & \quad R_{G_{n_0}^{(k_0)}, G_{n_0}^{(k_1)}}(\tau) = 0. \end{aligned} \tag{82}$$

[C] Suppose that $n_0 \neq n_1$. The range of $n_0 - n_1$ becomes

$$\begin{aligned} -N_1 + 1 \leq n_0 - n_1 \leq -1, \\ 1 \leq n_0 - n_1 \leq N_1 - 1. \end{aligned} \tag{83}$$

Therefore, we have

$$n_0 - n_1 \neq (\tau_0 - \epsilon N_0) N_1. \tag{84}$$

Note that $n_0 - n_1$ is not a multiple of N_1. In this case, the following condition is satisfied:

$$n_0 + p_0 N_1 \neq n_1 + (p_0 + \tau_0 - \epsilon N_0) N_1. \tag{85}$$

From (64), we have

$$R_{f_{n_0+p_0N_1}, f_{n_1+(p_0+\tau_0-\epsilon N_0)N_1}} (\tau_1 + \epsilon) = 0. \tag{86}$$

Based on (68), this means that

$$\begin{aligned} &\forall k_0, \forall k_1, \forall n_0 \neq n_1, \forall \tau, \\ &\quad R_{G_{n_0}^{(k_0)}, G_{n_1}^{(k_1)}} (\tau) = 0. \end{aligned} \tag{87}$$

From (44), (77), (82), and (87), the proposed method has been proven.

C. Example

Suppose that $P = 8$; then we have the following 8-dimensional DFT matrix:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 0 & 6 & 4 & 2 & 0 & 6 & 4 & 2 \\ 0 & 5 & 2 & 7 & 4 & 1 & 6 & 3 \\ 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 \\ 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \\ 0 & 2 & 4 & 6 & 0 & 2 & 4 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix},$$

where each element represents a power of $\exp\left(2\pi\sqrt{-1}/8\right)$. In addition, suppose that $N_0 = 4$ and

$$\begin{aligned} O_0 &= (0000), \\ O_1 &= (0246), \\ O_2 &= (0404), \\ O_3 &= (0642). \end{aligned}$$

Then, $O = \{O_n \mid 0 \leq n \leq 3\}$ is a set of quadriphase orthogonal codes of length 4. Let $N_1 = 2$; then an eight-phase GMO-ZCZ sequence set with four ZCZ sequence sets containing two sequences of period 32 is obtained from (67). Each sequence of this GMO-ZCZ sequence set is given below:

$$\begin{aligned} G_0^{(0)} &= (00000642040402460000064204040246), \\ G_1^{(0)} &= (00007531626257134444317526261357), \\ G_0^{(1)} &= (02460000064204040246000006420404), \\ G_1^{(1)} &= (02467777642051514602333320641515), \\ G_0^{(2)} &= (04040246000006420404024600000642), \\ G_1^{(2)} &= (04047135666653174040357122221753), \\ G_0^{(3)} &= (06420404024600000642040402460000), \\ G_1^{(3)} &= (06427373602455554206373724601111). \end{aligned}$$

The GMO-ZCZ sequence set $G = \{G_k \mid 0 \leq k \leq 3\}$ is a single optimal ZCZ sequence set that is represented as $Z_{cz}(32, 8, 3)$. In addition, each $G_k = \left\{G_n^{(k)} \mid 0 \leq n \leq 1\right\}$ is an optimal ZCZ sequence set that is represented as $Z_{cz}(32, 2, 15)$.

V. Conclusion

In the present paper, we have proposed the two methods for constructing GMO-ZCZ sequence sets. One is a method for constructing quasi-optimal GMO-ZCZ sequence sets using perfect sequences and orthogonal codes, and this method is a generalized version of our previously proposed method [34]. The other is a method for constructing optimal GMO-ZCZ sequence sets using discrete Fourier transform (DFT) matrices and orthogonal codes. These proposed methods can generate new GMO-ZCZ sequence sets that cannot be obtained from known methods. The proposed GMO-ZCZ sequence sets are expected to be useful for designing spreading sequences for AS-CDMA systems.

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