# Frequency-Domain Equalization for SC-FDE in HF Channel

Xu He, Qingyun Zhu, and Shaoqian Li

***Abstract*—**HF channel is a common multipath propagation resulting in frequency selective fading, SC-FDE can better resist frequency selective fading, but commonly used frequency domain equalization algorithm needs to get the value of channel estimation. In this paper, the research is applicable to SC-FDE in HF channel. The algorithm is after the conventional channel estimation algorithm, used the interpolation algorithm to reduce the number of pilot in the frame structure to improve the transmission efficiency, use overlap FDE and MMSE-RISIC to improve the performance of communication system. Through the simulation prove that the algorithm has good use to estimate the channel and improve the spectrum utilization efficiency.

***Keywords*—**HF Channel. Overlap FDE. MMSE-RISIC, Interpolation Algorithm Channel Estimation.

## I. Introduction

Sky wave transmission of the HF communication is influenced by the ionosphere seriously which lend to more serious multipath effect. Multipath effect will lead to frequency selective fading, broaden the waveform of received signal and produce inter symbol interference (ISI) which limits the transmission performance and transmission efficiency of digital communication. The existing multicarrier orthogonal frequency division multiplexing (OFDM) and single carrier time-domain equalization technology (SC-TDE) in combating multipath fading channel has good performance, but OFDM suffers from peak-to-average power ratio (PAPR) are relatively high and also have the disadvantages of sensitive to carrier frequency offsets, and the computation complexity of TDEs is considerable high to handle severe ISI due to high speed data transmission in SC-TDE system. Single carrier frequency domain equalization (SC-FDE) technique with single carrier transmission overcomes
the disadvantage of PAPR are relatively higher in OFDM and the frequency domain equalization overcomes the disadvantage of time domain equalization with large amount of computation.

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HF channel is composed of many propagation paths with different time delays and has a narrow bandwidth lead to high rate transmission is so difficult and characterized by a severely frequency-selective channel. In this paper, based on the single carrier frequency domain equalization on the premise of guarantee the transmission performance and then proposed a single carrier frequency domain system which can improve transmission efficiency.

## II. CONVENTIONAL SC-FDE SYSTEM DESCRIPTION

SC - FDE baseband signal processing block diagram is shown in Figure 1. In the transmitter, after modulation the data blocks which length is N then insert Unique Word (UW) that length is $N_g$ and receiver is known. The $N_g$ must be greater than the value of multipath time delay symbol length is caused by the biggest path delay of channel. For example, generally in the HF channel considered the biggest path delay of channel is 5ms then we should use $N_g$ is greater than 5ms to fully estimate the channel impulse response (CIR). After the synchronization of receiver, extraction of each block UW sequence to estimate channel frequency response of the current time, then the transmit data blocks use FFT block transform to frequency domain to process frequency domain equalization. After that put the signal to process IFFT transform to get the time domain signal. At last, remove the UW sequences and demodulation the time domain signal is in order to restore bits. Different from the structure of time domain equalization, the coefficients of frequency domain equalization is multiple single filter is same to the frequency domain form of the time domain filter coefficient.

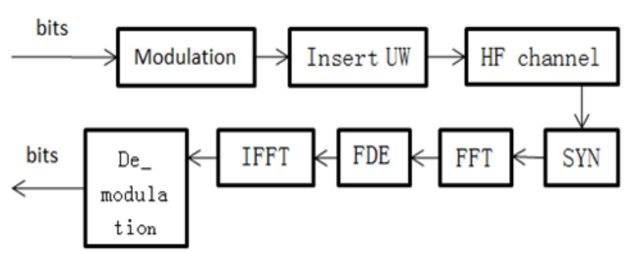


Fig.1. The signal processing flow of SC-FDE

In the SC - FDE system, UW sequence can be used as cyclic prefix (CP), which can used to pilot sequences to do channel estimation. As the UW sequence to be the CP, the length of data must be greater than the length of the response of the channel in order to eliminate ISI. Use the UW sequence as the pilot sequences, for UW sequences with good correlation and cyclical, smooth frequency response. According to the IEEE802.11 standard commonly use UW sequence such as: Chu, Frank - Zadaff, PN sequences, etc. In this paper, use the Chu sequence as UW sequence. The conventional frame structure is shown in Figure 2.

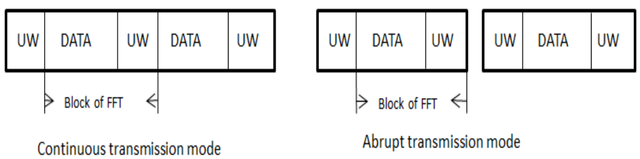


Fig.2. Conventional frame structure

The time domain impulse response (CIR) of Multipath fading channel can be represented as:

$$h(n)=\sum_{l=0}^{L-1} h_l \exp\left(\frac{j2\pi f_{Dl} T n}{N}\right)\delta(\tau-\tau_l) \qquad (2\text{-}1)$$

Where in (2-1) $L$ is the total number of paths, $f_{Dl}$ is Doppler frequency shift of $l_{th}$, $\tau_l$ is the time delay of $l_{th}$, $T$ is the cycle of a transmission block, $T=NT_s$, $T_s$ as the sampling period, $n$ as the sampling time.

The transmit data is additive white Gaussian noise (AWGN) in the channel. In the receiver, the received signal can be represented as:

$$y(n)=x(n)*h(n)+v(n) \qquad (2\text{-}2)$$

Where in (2-2) $*$ is convolution symbol, $x(n)$ represented as transmit signal, $h(n)$ represented as the channel impulse response, $v(n)$ is AWGN channel of mean to 0, variance for sigma is $\sigma^2$. In the ideal state, Because of the CP, the linear convolution could convert to cyclic convolution, namely:

$$y(n)=x(n)\otimes h(n)+v(n) \qquad (2\text{-}3)$$

In type (2-3) on both sides use Fourier transform which can get the expressions of the frequency domain can be represented as:

$$Y(k)=X(k)H(k)+V(k) \qquad (2\text{-}4)$$

In type (2-4) $H(k)$ is discrete Fourier transform of $h(n)$, that is channel frequency response. Discrete Fourier transform of $x(n)$、$y(n)$、$v(n)$ is $X(k)$、$Y(k)$、$V(k)$.

## III. FREQUENCY-DOMAIN EQUALIZATION

### A. The LS channel estimation algorithm

Line frequency domain equalization algorithm used MMSE, ZF commonly, which need the value $H_L$ to offset ISI. In order to get the $H_L$, must be on channel estimation. At present, channel estimation algorithm used least square (LS) channel estimation algorithm is proposed commonly which compare to minimum mean square error (LMMSE) channel estimation algorithm that has advantages of small amount of calculation and easy to realize. In this paper, the channel estimation algorithm use LS channel estimation.

LS channel estimation is according to the least squares criterion to get the channel estimation algorithm. In (2-4) can be represented by matrix as: $Y=XH+V$. According to the least squares criterion, defining the function as:

$$J_{LS}=(Y-XH)^H(Y-XH) \qquad (3\text{-}1)$$

Where $X$ is pilot sequences that is known by reception, $X=diag[X(0),X(1),\dots X(N_g-1)]$. $Y$ is pilot sequences signal via the channel to receive the column value, $Y=diag[Y(0),Y(1),\dots Y(N_g-1)]^T$. $H$ is the pilot position corresponding the channel frequency response column value, $H=diag[H(0),H(1),\dots H(N_g-1)]^T$. $N_g$ is the length of the pilot sequences. In order to get best channel estimate of $H$, so the $J_{LS}$ about $H$ the first-order and second-order derivatives:

$$\frac{\partial}{\partial H}J_{LS}=-2X^H(Y-XH) \qquad (3\text{-}2)$$

$$\frac{\partial}{\partial H}\left(\frac{\partial}{\partial H}J_{LS}\right)^H=2X^HX\geq 0 \qquad (3\text{-}3)$$

Could be concluded that the second derivative is bigger than zero, there exist a minimum value. Order first derivative is zero, you can get:

$$\hat{H}_{LS}=(X^HX)^{-1}X^HY=\frac{Y}{X} \qquad (3\text{-}4)$$

The LS channel estimation all operations in frequency domain, because of its simple structure, so it has been widely used.

### B. Pilot block interpolation channel estimation algorithm

Conventional LS channel estimation assumed channel to be a static channel within a block of data, that is in the duration of a block of data assumes that the channel impulse response remains the same. If want to get an accurate channel estimation value, must process channel estimation frequently as to need pilot sequences that will make system have lower transmission efficiency. Especially for HF channel which has narrow bandwidth, the resource of spectrum is extremely tense. Due to the HF channel under influence of multipath effect, Doppler frequency shift and the Doppler spread lead to higher-order modulation hardly, improved transmission rate was difficult. In conventional SC-FDE system used many UW sequences to improve the performance of the system but reduce the transmission efficiency of the system. Based on the situation, proposed a method of reduce pilot sequences and used the interpolation algorithm to obtain all data blocks corresponding channel estimates value. Figure 3 is based on the interpolation algorithm design of frame structure.

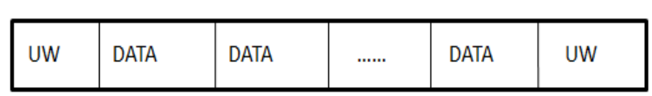


Fig.3. The improved frame structure of abrupt transmission mode

In the conventional system is a pilot sequences connect a block of data, but in the improved frame structure is a pilot sequences connect many blocks of data. For example, in figure 3 frame structure used fore and aft pilot blocks to get channel estimation $\hat{H}_a$, $\hat{H}_b$. Then the channel estimation of data $\hat{H}_1\dots\hat{H}_{i-1}$ ($i$ is the number of data blocks) were obtained through the interpolation algorithm.

If channel time varying is not fast and decline is not serious could use line interpolation algorithm to get the value of channel estimation. But in HF channel, the fading is frequently, the low order interpolation algorithm to get the accurate channel estimation value of data is hardly, could use high order interpolation algorithm to do that. The commonly used high order interpolation algorithm such as: spline, cubic, etc. Using high order interpolation algorithm need more reference point (pilot sequences) to improve the estimation accuracy. So if the transmission mode is continuous transmission, the frame structure as shown in Figure 4.


Fig.4. The improved frame structure of continuous transmission mode

In Figure 4 can get 4 values of channel estimation through 4 blocks of UW sequences according to the LS channel estimation algorithm respectively. $\hat{H}_{esti} = [\hat{H}_1, \hat{H}_6, \hat{H}_{11}, \hat{H}_{16}]$. Then used interpolation algorithm to get $\hat{H}$, inserted $N$ data between $\hat{H}_i$ and $\hat{H}_{i+1}$ in $\hat{H}_{esti}$. $N$ is the number of blocks of data that is between UW sequences. In figure 4 $N$ is 4. After that will get the value of channel estimation of all data blocks, $\hat{H} = [\hat{H}_2, \hat{H}_3, \hat{H}_4, \hat{H}_5, \hat{H}_7, \hat{H}_8, \hat{H}_9, \hat{H}_{10}, \hat{H}_{12}, \hat{H}_{13}, \hat{H}_{14}, \hat{H}_{15}]$, $\hat{H}$ is the value of channel estimation about the 12 blocks of data respectively. Figure 5 is a frequency spectrum curve line is used interpolation to estimate channel.

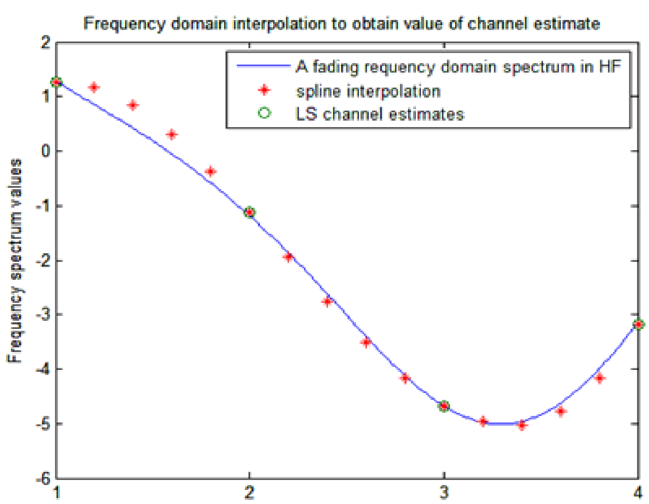


Fig.5. The sketch interpolation channel estimation algorithm

The number and length of data blocks should be adjusted according to different channel, the number and length of data blocks may be appropriately increased when the situation of channel is better. When channel condition is bad should decrease the number and length of data blocks to make data transmission correctly. This method use fewer pilot sequences to get the location of channel estimation, then use interpolation algorithm to get each channel estimation value of block data. Compared with conventional SC-FDE system, the interpolation algorithm saves many pilot blocks. The spectrum resource is very nervous in HF channel, used this method will decrease number of pilot block to improve the transmission efficiency.

### *C. Overlap FDE*

In SC-FDE system the frequency domain equalization is important, used line frequency domain equalization algorithm commonly, such as: MMSE, ZF. Both of them need the value of channel estimation $\hat{H}_z$. The value of channel estimation is more accurate then will get the more accurate result after equalization.

Theory suggests that, MMSE is better than ZF, because of stable and good performance, the equalizer coefficients are expressed as:

$$W = \frac{\hat{H}_z}{|\hat{H}_z|^2 + \sigma_W^2} \qquad (3\text{-}5)$$

Where $\sigma_W^2$ in (3-5) is the variance of noise. In this paper is based on MMSE frequency domain equalization.

In the improved frame structure reduce the UW sequence (without CP) would produce inter block Interference (IBI) between block of data. Overlap FDE requires no CP insertion and which could eliminate the IBI between the block of data. Assumes the channel impulse response (CIR) length is $L$, due to noise and multipath effect, the send block of data exist IBI. Then after a length of $M$ block of data reached the receiver, the length will spread to $M$+$L$, Interference between adjacent data block that length is $L$. Conventional SC-FDE system use CP to resist IBI. Overlap FDE specific process as shown in Figure 6.

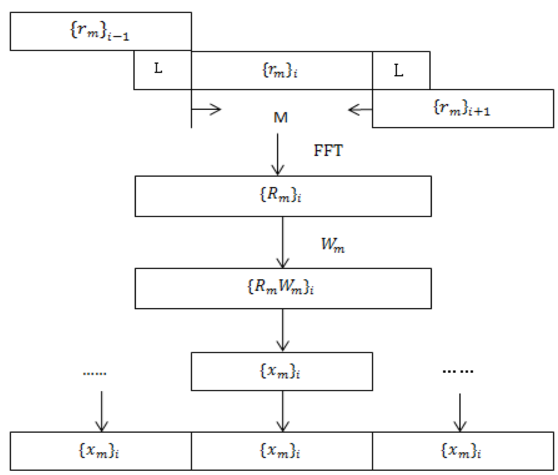


Fig.6. The signal processing flow of overlap FDE

The Overlap FDE need accurate value of channel estimation, otherwise Overlap FDE can resist the deep fading channel and dynamic channel performance badly. But pilot block interpolation channel estimation algorithm will get relative accurate channel estimation. If the channel conditions badly and IBI serious could increase overlap length $L$ to improve the system performance.

### *D. MMSE-RISIC FDE*

MMSE-RISIC is an algorithm based on MMSE, MMSE-RISIC is eliminating residual inter symbol interference (ISI) which is after MMSE equalization. Assume $\hat{S}_k$ is symbol after MMSE line equalization.

Via (2-4) and (3-5) could get:

$$\hat{X}_k = Y_k W_k = \frac{(X_k H_k + V_k)\ H_k^*}{|H_k|^2 + \sigma_w^2} =$$

$$X_k - \frac{x_k}{|H_k|^2/\sigma_w^2+1} + \frac{H_k^* V_k}{|H_k|^2/\sigma_w^2+1} = X_k + \Delta_k + \hat{V}_k \qquad (3\text{-}6)$$

In (3-6)

$$\Delta_k = -\frac{x_k}{|H_k|^2/\sigma_w^2+1} \qquad (3\text{-}7)$$

$$\hat{V}_k = \frac{H_k^* V_k}{|H_k|^2/\sigma_w^2+1} \qquad (3\text{-}8)$$

Where $\Delta_k = [\Delta_0, \dots, \Delta_{N-1}]^T$ , $\hat{V}_k = [\hat{V}_0, \dots, \hat{V}_{N-1}]^T$ , $\Delta$ is frequency domain form of residual inter symbol interference that is after frequency equalization. $\hat{V}_k$ is noise of frequency domain form. For (3-6) proceed IFFT transform could get:

$$\hat{x} = x + \delta + \hat{v} \qquad (3\text{-}9)$$

Where $\delta_k = [\delta_0, \dots, \delta_{N-1}]^T = \mathrm{IFFT}\{\Delta\}$ is residual ISI time domain form. $\hat{v}$ is time domain noise after equalization. Conventional process mode is direct decision after MMSE frequency equalization, because of $\delta$ exist, can not get best decision. If estimate $\delta$ and eliminate it will get more accurate decision. When calculat $\Delta_k$ use $\hat{x}$ instead of $x$, the estimate value compare with the real value will be difference to generate additional interference. In order to eliminate the additional interference can iterative use MMSE-RISIC.

The structure diagram of MMSE-RISIC as shown in Figure7.

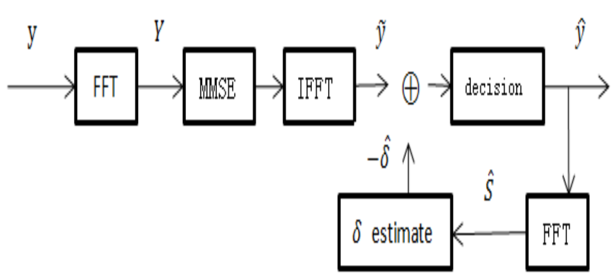


Fig.7.The signal processing flow of MMSE-RISIC

## IV. SIMULATION RESULTS

The bit error rate (BER) performance of improved SC-FDE is simulated with assumption that the synchronization is perfect and without channel coding. Used HF channel that bandwidth is 3KHz, Doppler frequency spread is 1.5Hz. The multipath is 2 and the biggest time of delay is 2ms.used BPSK modulation. The Monte -Carlo simulation times is 5000.

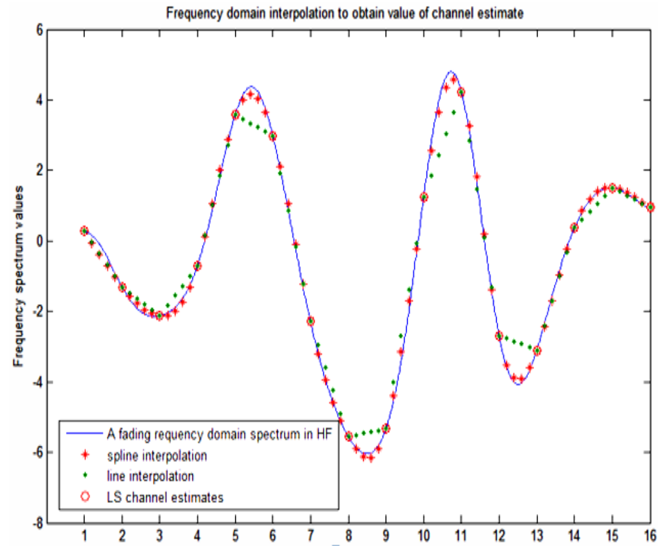


Fig.8. Compare different interpolation algorithm

As can be seen in Figure 8, consider LS value of channel estimation is perfect without consider Gaussian noise. The spline interpolation is better than line interpolation. The line used spline interpolation is more close to the real frequency fading line. The rest of the simulation use spline to interpolate.

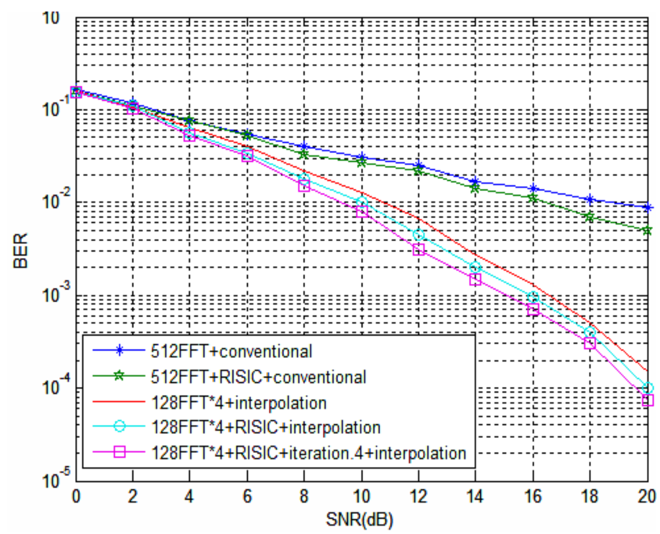


Fig.9. Analysis interpolation of HF channel estimation with CP

As can be seen in Figure 9, this simulation used abrupt frame structure. Conventional frame structure likes Figure 2.The length of FFT block is 512 and the length of CP is 16. Interpolation frame structure likes Figure 3, the number of DATA is 4, the length of FFT block is 128 and the length of CP is 16. In Figure 9, due to reduce the length of FFT by interpolation without increase times of channel estimation can have a larger performance improvement than conventional system. In the figure also can see MMSE-RISIC can have a better performance compare with MMSE and iteration 4 times is better than use RISIC 1 time.

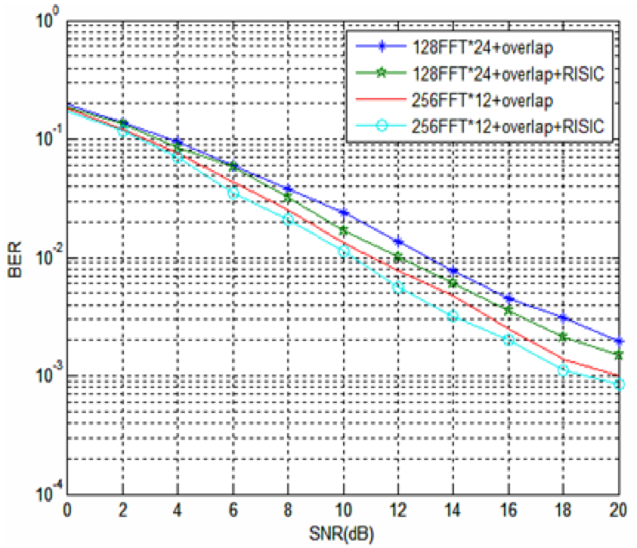


Fig.10. Analysis overlap FDE

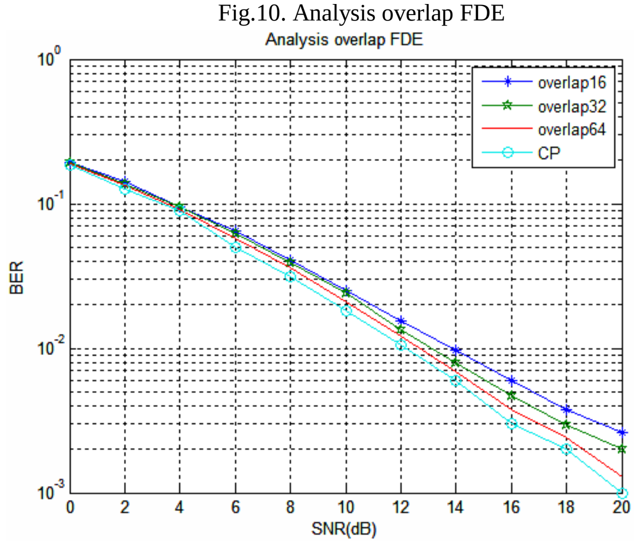


Fig.11. Analysis the length of overlap

As can be seen in Figure 10 used frame structure is Figure 4, As the FFT length is 256, the number of UW is 4, the number of DATA between UW is 4 and a total of DATA is 12.As the FFT length is 128, the number of UW is 4, the number of DATA between UW is 8 and a total of DATA is 24. In the simulation use overlap FDE without CP, used interpolation and overlap can improve the efficiency of system. In Figure 11 the FFT length is 256, increased the length of overlap can improve the performance of system and it is close to performance of system with CP.

## V. CONCLUSION

In this paper, used interpolation can decrease the length of FFT to improve the performance of system. Used interpolation and overlap can on the premise of ensuring performance of system to improve the transmission efficiency of system. Using MMSE-RISIC can have a better performance by eliminating ISI. In HF channel, conventional SC-FDE is wasted the frequency spectrum resources of HF channel, on the contrary, this paper provide methods can save the frequency spectrum resources and improve the transmission efficiency of system with good performance of system.

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