Design and Analysis of an Optimal Orbit Control for a Communication Satellite

Santosh Kumar Choudhary

***Abstract*—In this article, design and analysis of an optimal orbit control for a communication satellite is investigated. It is the challenging task and critical importance to control the orbit of a communication satellite especially the one used for worldwide communication. The main objective of this work is to evolve a design based on modelling and simulation of an optimal orbit controller for a satellite orbiting into a a geostationary orbit. This article first presents the mathematical model of satellite orbital dynamics and then illustrates the basic idea and technical formulation for controller design. The paper briefly explains the linear quadratic regulator (LQR) design method for optimal feedback control of the satellite orbital system. This approach has been considered in order to assure high control performance of the system. The simulation results show that if the satellite orbital system follow the control pattern obtained through the MATLAB simulation, the use of fuel for the thrusters can be optimized and the satellite orbit perturbation can be controlled within the specified design requirements. This can increase the efficiency of the thrusters and the lifetime of the satellite.

***Keywords*—Communication satellite, modelling, simulation, geostationary orbit, linear quadratic regulation, orbit control.

I. Introduction

THE orbit control of a communication satellite especially the one used for worldwide communication is having critical importance. The Fig. 1 denotes a schematic diagram of communication satellite that is in the geostationary orbit. Geostationary means that the satellite's orbital rate is equal to the rotational rate of the Earth. As a result, geostationary satellites appear to hover over a single point on the earth. Once a satellite is launched in a desired orbit, it never remains in the ideal orbit. Since satellite is a free floating body in space therefore it has a disturbance torque due to the external forces present in space. As the spacecraft orbits the earth, it is subject to solar pressure. This solar pressure generates a torque on the craft with a moment arm (shown in the Fig. 1). The torque varies in a sinusoidal manner as the vehicle orbits the Earth. Obviously, if the Earth shades the spacecraft from the Sun, then there is no solar pressure on the vehicle. We therefore expect the disturbance due to the solar pressure to vary in a sinusoidal manner over a 24 hour period. These external forces present in the the space cause perturbations to satellite's ideal orbit [1], [2]. To bring back the satellite into the desired orbit, on-board thrusters provides in-orbit propulsion. In this work, satellite orbital system is controlled by the thrust produced by the on-board thrusters installed in the radial and tangential directions.

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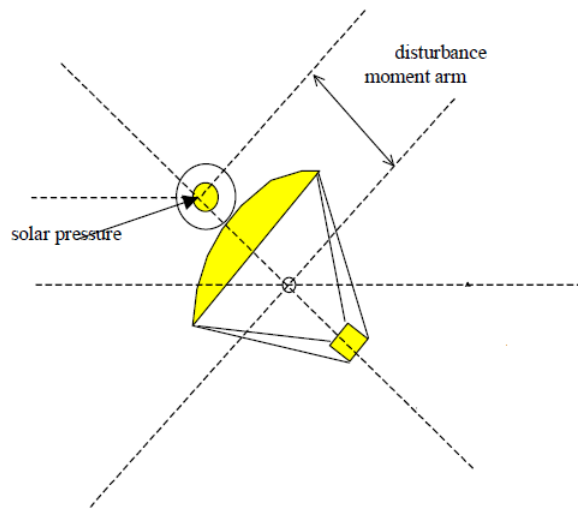

Fig. 1. Communication satellite

There are some other optimal control strategies also used and briefly discussed in [3], [4] but the aim of this work is to model and simulate the satellite orbit controller with the optimized use of the system inputs or thrusters to keep the satellite in the desired orbit. The satellite orbital control system simulation with help of MATLAB involves mathematical modelling, linearization of model and applying linear quadratic regulation (LQR) methodology to design an orbit controller to maintain the satellite into a desired orbit. If the satellite orbital system follow the pattern generated by the simulation, the use of fuel for the thrusters can be optimized and the satellite orbit perturbation can be controlled within the specified design requirements. This can increase the efficiency of the thrusters and the lifetime of the satellite.

The rest of the article is organized as follows: In section II, we brief mathematical modeling of satellite orbital dynamics. The section III describes the methodology for optimal orbit controller design and section IV deals with LQR based optimal orbit controller synthesis for the communication satellite. In section V, we present simulations and results analysis for the design and finally, we conclude this paper in section VI by some remarks and conclusion.

II. Mathematical modelling of satellite orbital dynamics

Understanding the system behavior has become essential to ensure control. Indeed, the ability to describe and explain the various phenomena involved and interacting in the satellite orbital dynamics has a large impact in practice. We can say the aim of modeling is to evaluate and control the system as much as possible.

Consider the planar motion of an orbiting satellite in the inverse-square gravitational field of the earth. For mathematical simplicity, the satellite is approximated as a particle mass M. The gravitational force exerted on the satellite is given by

$$F_g = Mg \tag{1}$$

The earth's varying gravitational field with the varying altitude can be written as [1], [5]

$$g = g\left(\frac{R}{R+h}\right)^2 \tag{2}$$

where R is the radius of earth and h is the altitude of satellite from the surface of the earth. If $r(t)$ is the distance from the center of the earth to the center of satellite,

$$R + h = r(t) \tag{3}$$

Using equation (2) in (1), we get

$$F_g = Mg\left(\frac{R}{R+h}\right)^2 \tag{4}$$

Now by substituting from (3) into (4), we get

$$F_g = Mg\left(\frac{R}{r}\right)^2 \tag{5}$$

This systems involves circular motion around a fixed center. Assume that the system is equipped with the ability to exert a thrust F_1 in the radial direction and F_2 in the tangential direction as shown in Fig. 2. The satellite motion is more conveniently described with polar coordinates. In this situation, it is often more convenient to represent all variables in vector form with complex numbers. Using Fig. 2, we can write satellite position vector, radial thrust vector, tangential thrust vector, gravity force vector and inertial force vector as

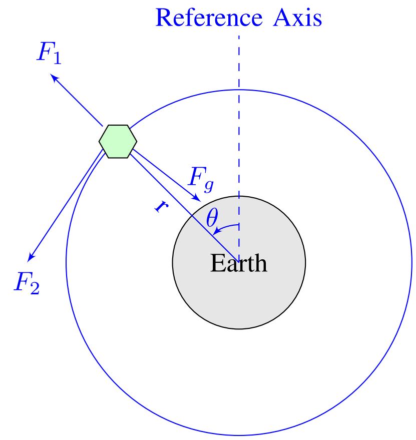

Fig. 2. Circular motion of communication satellite

$$r(t) = re^{i\theta} \tag{6}$$

$$F_1 = F_1 re^{i\theta} \tag{7}$$

$$F_2 = F_2 re^{i(\theta+90^\circ)} \tag{8}$$

$$F_g = -Mg\left(\frac{R}{r}\right)^2 re^{i\theta} \tag{9}$$

$$F = M\frac{d^2r(t)}{dt^2} re^{i\theta} \tag{10}$$

respectively. Now Newton's law can be applied to complex force vectors as follows [5]:

$$F_1 + F_2 + F_g = F \tag{11}$$

Using equations (7), (8), (9) and (10) in (11), we get

$$\begin{aligned} &F_1 re^{i\theta} + F_2 re^{i(\theta+90^\circ)} - Mg\left(\frac{R}{r}\right)^2 re^{i\theta} = M\frac{d^2r(t)}{dt^2} re^{i\theta} \\ &\Rightarrow F_1 e^{i\theta} + iF_2 e^{i\theta} - Mg\left(\frac{R}{r}\right)^2 e^{i\theta} = M\frac{d^2r(t)}{dt^2} e^{i\theta} \\ &\Rightarrow F_1 e^{i\theta} + iF_2 e^{i\theta} - Mg\left(\frac{R}{r}\right)^2 e^{i\theta} = M(\ddot{r}e^{i\theta} + i2\dot{r}\dot{\theta}e^{i\theta} \\ &\qquad + ir\ddot{\theta}e^{i\theta} - r\dot{\theta}^2 e^{i\theta}) \end{aligned} \tag{12}$$

By canceling the common vector $e^{i\theta}$ and equating the real and imaginary part respectively of (12), produces the two second order differential equations as follow:

$$\begin{cases} F_1 & = M\ddot{r} - Mr\dot{\theta}^2 + Mg\left(\frac{R}{r}\right)^2 \\ F_2 & = 2M\dot{r}\dot{\theta} + Mr\ddot{\theta} \end{cases} \tag{13}$$

To simplify the numerical problems, the time, distance and force variable are normalized into dimensionless quantities in the following way:

$$\begin{cases} \tau & = t/(R/g)^{1/2};\ \rho = r/R; \\ u_1 & = F_1/(Mg);\ u_2 = F_2/(Mg) \end{cases} \tag{14}$$

Hence, using (14), the equations of satellite motion (13) can be written as

$$\begin{cases} u_1 & = \rho^{''} - \rho\theta^{'2} + \frac{1}{\rho^2} \\ u_2 & = 2\rho^{'}\theta^{'} + \rho\theta^{''} \end{cases} \tag{15}$$

where the prime symbol $(')$ denotes $d/d\tau$.

A. Non-linear state space model

To obtain the non-linear model of satellite orbital dynamics, the state and input vectors of the model are chosen to describe all information about the system in following way:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},\ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where $x_1 = \rho,\ x_2 = \theta,\ x_3 = \dot{x_1} = \dot{\rho},\ x_4 = \dot{x_2} = \dot{\theta}$ and $\dot{x_3} = \ddot{\rho},\ \dot{x_4} = \ddot{\theta}$.
Hence a non-linear state space model $\dot{x} = f(x,u)$ of satellite orbital dynamics (15) is obtained as under

$$\begin{cases} \dot{x_1} = x_3 \\ \dot{x_2} = x_4 \\ \dot{x_3} = x_1x_4^2 - \frac{1}{x_1^2} + u_1 \\ \dot{x_4} = -\frac{2x_3x_4}{x_1} + \frac{1}{x_1}u_2 \end{cases} \tag{16}$$

B. Linear state space model

Suppose that satellite is to maintaine in circular geo-stationary orbit of the earth where angular velocity of the satellite orbiting is $\omega = 15.04$ degree per hour. We assume that steady state is maintained only by the gravitational force, so that the steady state components of two thrusts vectors are zero. To minimize energy consumption, thrusts are only applied to take transient corrective action to eliminate error. Thus at steady state,

$$u_1 = 0;\ \ u_2 = 0;\ \ x_3 = 0;\ \ x_4(\text{angular speed}) = \text{constant}$$

We now linearize the non-linear system (16) about the steady state solution (say operating point) to obtain the system in the form of the linear control system $\dot{x}(t) = Ax(t) + Bu(t)$. By linearizing the function $\dot{x} = f(x,u)$ about $x = [x_1\ \ 0\ \ 0\ \ x_4]^T$ and $u = [0\ \ 0]^T$, we have

$$\hat{f}(x,u) = f_x^{'}(x,u)x + f_u^{'}u \equiv Ax(t) + Bu(t) \tag{17}$$

where,

$$\begin{cases} A = f_x^{'}(x,u) & = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} \\ & = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ x_4^2 + \frac{2}{x_1^3} & 0 & 0 & 2x_1x_4 \\ 0 & 0 & -\frac{2x_4}{x_1} & 0 \end{bmatrix} \\ B = f_u^{'}(x,u) & = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \frac{1}{x_1} \end{bmatrix} \end{cases} \tag{18}$$

Here we find the fourth order linear model and apply to situation in which not only the angular speed $x_4(t)$ or $\omega(t)$ needs to be regulated but angular position $\theta(t)$ must also be accurately controlled for complete control of satellite. Since,

$$\omega = 15.0^\circ/h = \pi/43200 \text{ radian/sec.}$$

This corresponds to a normalized angular speed of

$$x_4 = (R/g)^{1/2}\omega = 0.0587$$

with R (radius of earth) equal to 6378 km.
Finally, on substituting the $x_1 = \rho = 6.6108$ and $x_4 = 0.0587$ in state equation given by (17) and (18) and considering angular position $x_2 = \theta(t)$ and angular speed $x_4 = \omega(t)$ as a output vector of the system, we can established the complete linear state space model of

satellite orbital dynamics as:

$$\begin{cases} \underbrace{\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.01036 & 0 & 0 & 0.7757 \\ 0 & 0 & -0.01775 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} \\ \qquad + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.1513 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u} \\ \underbrace{\begin{bmatrix} \theta \\ \omega \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u} \end{cases} \tag{19}$$

III. Design philosophy

In this section, a Linear Quadratic Regulator (LQR) design method is discussed, which is a part of optimal control strategy and provides a linear state space controller. The LQR design is stable and robust (except in the case where the system is not controllable) and it is based on the selection of feedback controller gains such that the system performance index or cost function can be minimized [6]–[9]. In this design method, system must be described by state space model [6]–[9]

$$\begin{cases} \dot{x} = Ax + Bu, & x \in \mathbb{R}^n,\ u \in \mathbb{R}^r \\ y = Cx + Du, & y \in \mathbb{R}^m \end{cases} \tag{20}$$

and the performance index J is defined as

$$\begin{aligned} J(x(t), u(t), t) &= \int_0^\infty \left[e(t)^T Q e(t) + u(t)^T R u(t) \right] dt \\ &= \int_0^\infty \left[(z(t) - x(t))^T Q (z(t) - x(t)) + u(t)^T R u(t) \right] dt \end{aligned} \tag{21}$$

where $x(t)$ is the n^{th} state vector, $y(t)$ is the m^{th} output vector, $z(t)$ is the n^{th} desired state vector, $u(t)$ is the r^{th} control vector and $e(t) = z(t) - x(t)$ is the error vector. If our objective is to keep the state $x(t)$ near zero i.e. $z(t)$=0, C=I, the error $e(t) = 0 - x(t)$ itself is the state. This situation often arises in satellite orbital system, where plant is subjected to unwanted disturbances that perturb original state. In this case, the performance index J can be written as [6]–[9]

$$J(x(t), u(t), t) = \int_0^\infty \left[x(t)^T Q x(t) + u(t)^T R u(t) \right] dt \tag{22}$$

The matrix Q and R are known as error weighted and control weighted matrix respectively. In order to keep the error $e(t)$ small and error squared non-negative, the integral of the expression $\frac{1}{2} e(t)^T Q e(t)$ should be non-negative and small. Thus, the matrix Q must be positive semi-definite. Due to the quadratic nature of the weightage, we have to pay more attention to large errors than small errors. On the other hand, the quadratic nature of the control cost expression $\frac{1}{2} u(t)^T R u(t)$ indicates that one has to pay higher cost for larger control effort. Since the cost of control has to be a positive quantity, the matrix R should be a positive definite.

The crucial and difficult task in LQR controller design is a choice of the weighting matrices. We generally select weighting matrices Q and R to satisfy expected performance criterion. The different Q and R values give different system response. The system will be more robust to disturbance and the settling time will be shorter if Q is larger (in certain range). But there is no straightforward way to select these weighting matrices and it is usually done through an iterative simulation process. In this work, we apply the Bryson's rule for a simple and reasonable choice of the matrices Q and R. According to the rule, we select Q and R diagonal [8], with

$$\begin{cases} Q_{ii} &= \frac{1}{\text{maximum acceptable value of } x_i^2}, \quad i \in \{1, 2, \dots, l\} \\ R_{jj} &= \rho \left[\frac{1}{\text{maximum acceptable value of } u_j^2} \right], \quad j \in \{1, 2, \dots, k\} \end{cases} \tag{23}$$

where ρ is a constant which establish the trade-off between controlled output and control input signal in the following way:

- When we chose ρ very large, the most effective way to minimize J is to employ a small control input, at the expense of a large controlled output.
- When we chose ρ very small, the most effective way to minimize J is to obtain a very small controlled output, even if this is achieved at the expense of employing a large control input.

Now the LQR design objective can be formulated as: find the control input $u(t), t \in [0 \quad \infty)$ to obtain feedback controller gain matrix K that gives optimal control vector

$$u^*(t) = -Kx^*(t) \tag{24}$$

to steer the system described by (20) from non-zero state to zero state such that performance index (22) is minimized. The control gain matrix K known as *Kalman gain* is given by

$$K = R^{-1}B^TP \tag{25}$$

where, P, the $n \times n$ constant, positive definite, symmetric matrix, is the solution of the nonlinear, matrix *algebraic Riccati equation* (ARE) [6]–[9]

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{26}$$

and the optimal trajectory x^* is the solution of

$$\dot{x}^*(t) = \left[A - BR^{-1}B^TP\right] x^*(t) \tag{27}$$

The implementation configuration of the closed loop optimal control is shown in Fig. 3.

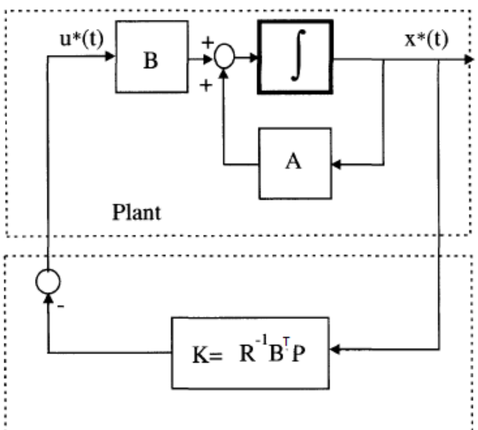

Fig. 3. Closed-loop optimal control

IV. Orbit Controller Design

In this section our objective is to evolve a LQR based orbit controller design for a satellite orbiting into a circular orbit. As we know that once a satellite is launched in a desired orbit, it never remains in that ideal orbit. The external forces present in space cause perturbations to this ideal orbit. The schematic path of satellite operational orbit and perturbed orbit for 24 hours are shown in Fig. 4. To bring back the satellite into the desired orbit, on-board thrusters provide in-orbit propulsion. These thrusters keep satellite in the desired orbit. In this work, the orbit of the satellite is controlled by the thrust produced by the on-board thrusters installed in the radial and tangential direction. However it is dictated by the controllability requirements.

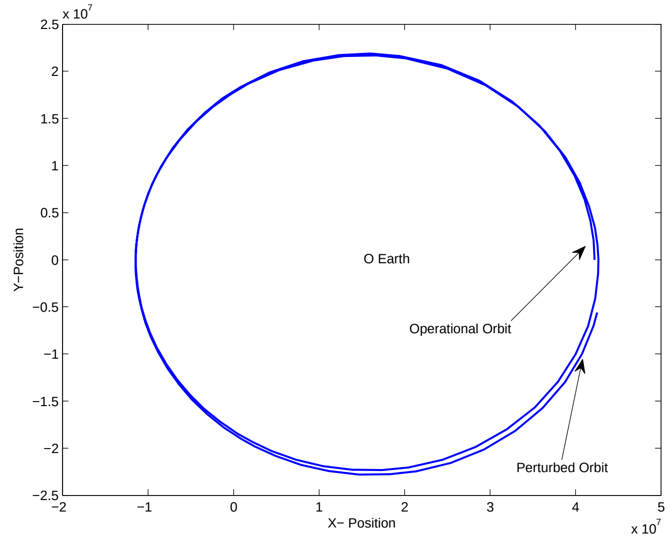

Fig. 4. Operational and drift path of satellite

A. Kalman's controllability test

State controllability involves the manner in which the input to the system is able to influence all state variables. Using Kalman's matrix model [9]

$$CO = \left[B \vdots AB \vdots A^2B \vdots \cdots \vdots A^{n-1}B\right] \tag{28}$$

of the system, it can be shown that the system is controllable if and only if the controllability matrix has rank n, where n is order of the system [8], [9]. For the linearized satellite orbital system (19), we can easily compute the controllability matrix (28) as

$$CO = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0.1173 & -0.0034 & 0 \\ 0 & 0 & 0 & 0.1513 & -0.0177 & 0 & 0 & -0.0021 \\ 1 & 0 & 0 & 0.1173 & -0.0034 & 0 & 0 & -0.0004 \\ 0 & 0.1513 & -0.0177 & 0 & 0 & -0.0021 & 0.0001 & 0 \end{bmatrix} \tag{29}$$

One can verify that rank of CO is 4 and hence the satellite orbital system is controllable. Now it is interesting to ask following question. What happens when one of the thrusters become in-operative? For this purpose set $u_2 = 0$ and hence B reduces to $B_1 = [0\ 0\ 1\ 0]^T$. So, the controllability matrix is given by

$$C_1O = \begin{bmatrix} B_1 & AB_1 & A^2B_1 & A^3B_1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 & 0 & -0.0034 \\ 0 & 0 & -0.0177 & 0 \\ 1 & 0 & -0.0034 & 0 \\ 0 & -0.0177 & 0 & 0.0001 \end{bmatrix} \tag{30}$$

C_1O has the rank 3 and hence the system is not controllable with the radial thruster alone. On the other hand if the radial thrusters fail i.e. $u_1 = 0$. In that case matrix B reduces to $B_2 = [0\ 0\ 0\ 0.1513]^T$ and this gives controllability matrix as

$$C_2O = \begin{bmatrix} B_2 & AB_2 & A^2B_2 & A^3B_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.1174 & 0 \\ 0 & 0.1513 & 0 & -0.0021 \\ 0 & 0.1174 & 0 & -0.0004 \\ 0.1513 & 0 & -0.0021 & 0 \end{bmatrix} \quad (31)$$

Note that C_2O has rank 4, so the system is controllable with input u_2 only. Since u_1 is radial thruster and u_2 is tangential thruster, we see that the loss of radial thruster do not destroy controllability where as loss of tangential thrusters do. Thus it is possible to stabilize the system with only thrust in the tangential direction.

B. Design specifications

After checking the controllability of the system it has been ascertained that there can be two different situations to stabilize the satellite. In first case the position of satellite will be controlled with both radial and tangential thrust. Second case would be dealt the control of angular rate with the tangential thrust only. In this way there will be two different possibilities and control objectives for maintaining the satellite into its desired or assigned orbit. In both cases it is required to have the following.

- Time required for steering the system completely from perturbed state to original state should be minimum.
- The overshoot of the system should not exceed more than 20%.
- The thrust magnitude of u_1 and u_2 are minimum and limited by the allowable energy consumption.

C. Orbit control with radial and tangential thrust

In the first case the orbit of satellite is being controlled with radial and tangential thrust. The linearized satellite orbital model equations are of fourth order and apply to the situation in which not only the angular speed $\omega(t)$ or $x_4(t)$ needs to be regulated but the angular position $\theta(t)$ must also be controlled.

At first, we select the weighting matrices Q and R, based on Bryson's rule as discussed in section III. The matrices Q and R for orbit control problem are selected as follow:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.7 \end{bmatrix} \quad (32)$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (33)$$

We now compute a controller gain matrix K which steers the satellite orbital system (19) from non-zero state to zero state i.e. from perturbed state to desired original state using the control law given by (24). The computation is carried out using `MATLAB` code `K=lqr(A,B,Q,R)` and obtained the following control matrix:

$$K = \begin{bmatrix} 1.0074 & -0.0781 & 1.5771 & 0.5651 \\ 0.0782 & 0.9969 & 0.0855 & 3.7998 \end{bmatrix} \quad (34)$$

D. Orbit control with tangential thrust

The controllability of system model (19) shows that the satellite can also be controlled with only single radial thrust by controlling the angular rate $\omega(t)$ of the satellite but with a constant non-zero offset error in angular position $\theta(t)$. Because of the reason the control of angular rate usually does not ensure exact alignment of angular position of the satellite. Moreover this controlling technique can be employed in this situation that the radial thruster has failed down and only the tangential thruster is operative. If this constant nonzero offset error in angular position $\theta(t)$ is tolerable then the second row of system model equation (19) should be removed. This reduced satellite orbit model can be written as

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0.01036 & 0 & 0.7757 \\ 0 & -0.01775 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.1513 \end{bmatrix}}_{B} \begin{bmatrix} u_2 \end{bmatrix} \quad (35)$$

To compute the control matrix K for this reduced satellite model, again we select the weighting matrices Q and R as

$$\begin{cases} Q &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \\ R &= 1 \end{cases} \quad (36)$$

The computation is carried out using `MATLAB` code `K=lqr(A,B,Q,R)` and obtained the following control matrix:

$$K = \begin{bmatrix} 1.0880 & 4.1533 & 6.5799 \end{bmatrix} \tag{37}$$

V. Simulation and Results Analysis

In this section, simulations are demonstrated with help of `MATLAB` to show the control of satellite orbit with both possible options i.e. orbit control with radial and tangential thrust and orbit control with tangential thrust only. The Fig. 5 shows that controlled states of

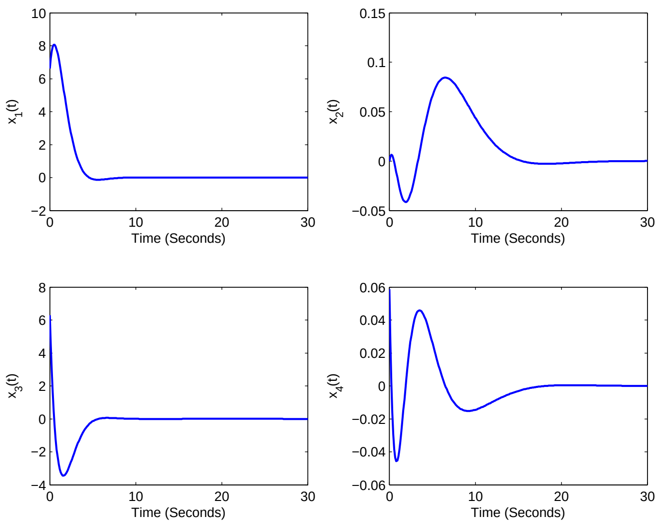

Fig. 5. Controlled states of satellite orbital system

satellite orbital system with radial and tangential thrust. We can see that our computed optimal control matrix (34) successfully steer the system from non-zero state to zero state. The Fig. 6 demonstrates the graphical representations of geo-stationary orbit with approximately 8.21 meters drift from nominal satellite orbit. The Fig. 6 also shows that the perturbed orbit reached to nominal orbit within 8 seconds only. The optimal steering control profile of satellite orbital system with two systems inputs are also shown in the Fig. 7.

On the other hand if the radial thrusters fails and tangential thruster works alone in the system. In this case the simulation of orbit control with tangential thrust is also performed and results are shown in Fig. 8 and Fig. 9. The Fig. 8 shows that controlled states of satellite orbital system with tangential thruster only and Fig. 9 illustrate drift of geostationary orbit with approximately 16.5 meters from nominal orbit and its steering profile from drift orbit to desired orbit in about 15 seconds.

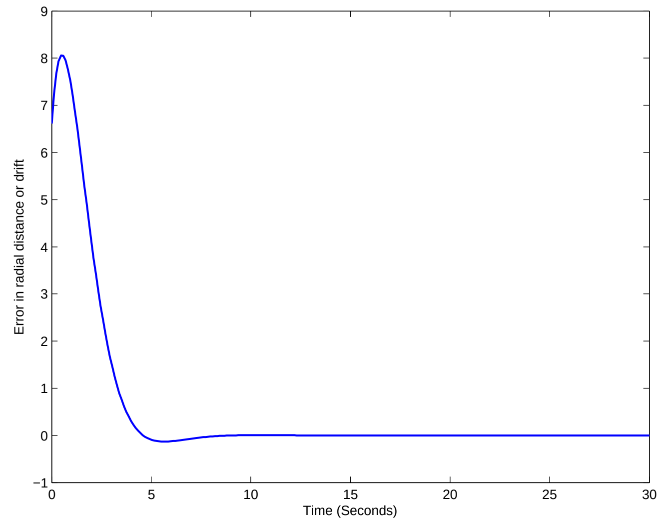

Fig. 6. Radial error or drift vs time

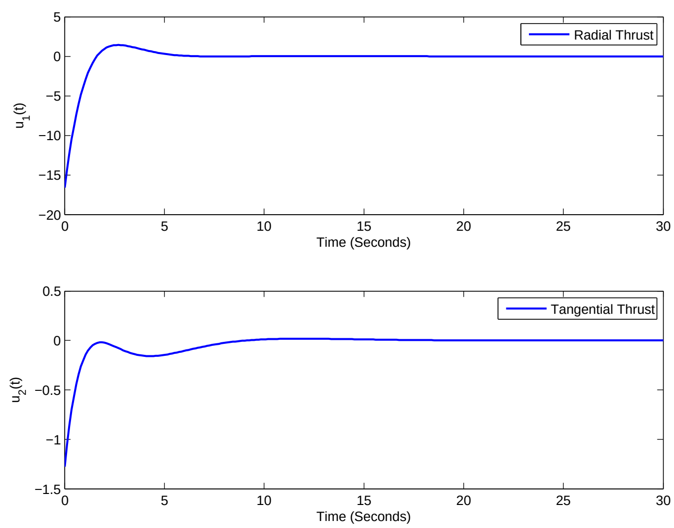

Fig. 7. Normalized inputs Vs Time

VI. Conclusion

In this article, LQR based optimal orbit control for the communication satellite is investigated. The satellite orbit dynamical equations along with it's state space model are first presented in the paper and than the basic ideas and technical formulations for designing an optimal orbit controller to maintain the satellite into a desired circular orbit are briefly illustrated. If the satellite orbital system follow the pattern generated by the `MATLAB` simulation, the use of fuel for the thrusters can be optimized and the satellite orbit perturbation can be controlled within the specified design requirements. This can increase the

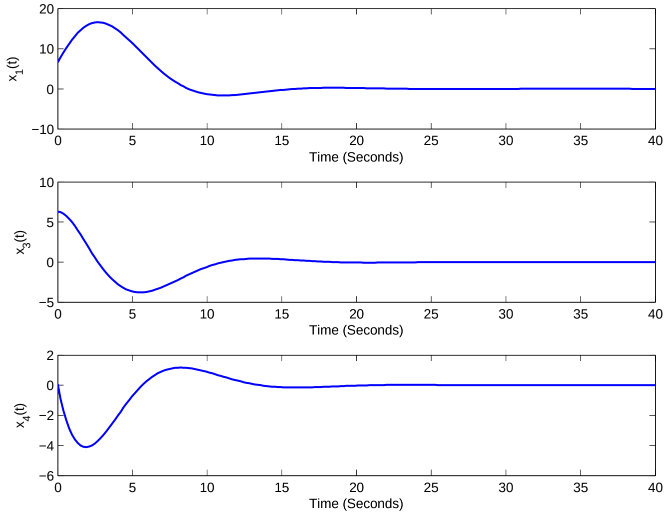

Fig. 8. Orbit controlled states with tangential thruster

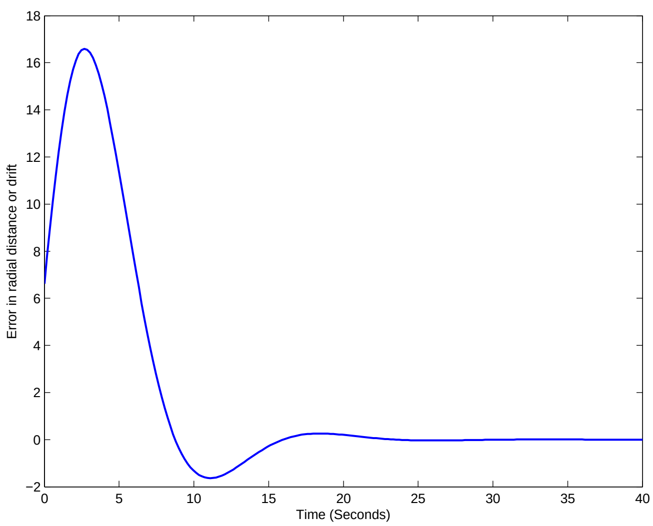

Fig. 9. Radial error or drift vs time with tangential thruster

efficiency of the thrusters and the lifetime of the satellite.

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