Uncertainty Test and Improvement in Quantum Information Transfer Employing Shannon Reverse Theorem

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***Abstract*—** just as data becomes important with regard to safety and security as well as effectiveness associated with information transfer, from a source to a destination node, the particular issue for how this data could possibly be compacted in a message for transfer can be another vital issue. In this paper we show up an experiment where arbitrary parameters could be chosen within a probabilistic manner of certainty and uncertainty. We explain results of the test base on the theory of Shannon entropy of classical information. This means a measured condition will have significantly less details in it because of the guarantee of pure states, the entropy of a pure state is zero. The higher the entropy the higher qubits can be enclosed in a message. We evaluate the way our test provide chances of retrieving more information out of a variable that has less probability to be selected, consequently recording a high number of entropy than the variable with a higher probability. We also show how the Shannon reverse theorem of both the classical and quantum channel could be simulated from noisy channel to noiseless one and vice versa.

***Keywords*—**Quantum information theory; quantum data compression; Shannon entropy.

I. Introduction

Traditionally a model of communication comprises sending and receiving nodes and a medium within which information is transferred (in our case quantum channel).

A particular channel transmitted some symbols as input which in connection also produces other symbols as output, the input and output symbols are different, and then there is a need for an encoding procedure, to ensure the effectiveness of the transmission[1, 2]. The encoding process involves the assignment of precise word to each input symbols, that this word has a built-in relation with output alphabet[3].

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Now the information senThis also applies to quantum information theory but with some changes as stated in[4]. A quantum state was assigned to the symbol generated by the source. Unlike traditional model the symbol of the input/output alphabet of the channel is 2D Hilbert space known as qubit [5]. The source encoding was also represented in qubit. There is a super operator representing the alteration of the transmitted information. But for our case we deliberately introduce noisy behavior in our channel due to the environmental effect on the system upon interaction. When the channels are free of error, each symbol is attached to a pure state. While a mixed state is associated to the symbol if the channel is noisy.

II. Literature Review

In this section the theoretical idea of information transfers in both classical and quantum information theories are discussed.

A. Shannon entropy as a concept of information transfer

This will be better understood if we start with a real life example.

Assuming we have a bag that contains 10 fruits and if nine of the fruits are apples and one is an orange, by extracting one fruit at random. Let's assume the random extraction as X. The result is measured by assigning the value 1 if the fruit is apple and value 0 if the fruit is orange.

$$P_{(x=1)} = \frac{9}{10} = 0.9$$
$$P_{(x=0)} = \frac{1}{10} = 0.1$$

The expected value of finding the variable X can be calculated as

$$E(X) = 1\left(P_{(x=1)}\right) + 0\left(P_{(x=0)}\right) \qquad (1.0)$$
$$E(X) = 1(0.9) + 0(0.1) = 0.9$$

The amount of information for the first event is defined as

$$I_i = -\log_2\left(P(x_i)\right) \qquad (1.1)$$

in our case the two values of information will be

$$I_1 = -\log_2(0.9) = 0.15$$
$$I_0 = -\log_2(0.1) = 3.32$$

Now there is little anomaly that the information associated with having orange is much higher than that of having apple despite having the highest probability [6]. Therefore the idea of random variable means the exact outcome is not known, and still the amount of information retrieved after the experiment is not known, due to the reason that every possible result has associated a different amount of information[3]. This is quite a brilliant idea that also gives a way in defining the amount of information acquired.

for every X,

$$X \rightarrow \{x_{1,}\, x_{2,} \ldots\ldots, x_n\}$$

Lies a probability $P \rightarrow \{p_{1,}p_{2,} \ldots \ldots, p_n\}$

That provides information $I \rightarrow \{I_{1,}I_{2,} \ldots\ldots I_n\}$

Now the amount of information associated with X is given as (X). Considering our example the $I(X)$ is 0.15 with probability 0.9 and 3.32 with probability 0.1.

This is an important new variable which is not dependent on the value of X. It relies on the probability of those values.

The expected value of

$$I(X) \text{ is } E(I(X)) \equiv H(X)$$
$$H(X) = 0.15 \times 0.9 + 3.32 \times 0.1 = 0.467$$

And that is the expected value of the amount of information obtained from the experiment by the random variable X.

In general the analytic expression of this value known as Shannon entropy can be written as [7]

$$H(X) = E\big(I(X)\big) = \sum_{i=1}^{n} P(x_i)\log_2(P(x_i) \qquad (1.2)$$

B. Von Neumann Entropy

We learn from Shannon entropy $H(X)$ that it is the number of incompressible bits of information performed for each letter[7]. It is known that the mutual information [7]

$$\begin{aligned} I(X;Y) &= H(X) - H(X \mid Y) \\ &= H(Y) - H(Y \mid X) \end{aligned} \qquad (1.3)$$

is the number of bits per character about X that we can acquire by reading Y (or the other way round). If the conditional probability characterized by a noisy channel, then $I(X;Y)$ is the amount of information for each character that can be transmitted through the channel[8]. Now assuming we have a source that generates data (messages) of length n, and each character is generated from a set of quantum states. These quantum states ρ_x are associated with probabilityP_x. As it is clearly discussed in[9] that the probability of any measured outcome of each character from the ρ_x where there is no prior knowledge of which character was transmitted can be written as [10]

$$\rho = \sum_x P_x \rho_x \qquad (1.4)$$

for the positive operator valued measure[11] (POVM) F_a

$$P_{(a)} = tr(F_a \rho)$$

We can now define the von Neumann entropy as

$$S(\rho) = -tr(\rho \log \rho) \qquad (1.5)$$

By choosing orthonormal basis $\{\mid a >\}$ that diagonalizes ρ

$$\rho = \sum_a \lambda_a \mid a >< a \mid, \qquad \lambda_a = eigenvalue$$

Then $S(\rho) = H(A)$

$$where\ H(A) is\ the\ shannon\ entorpy\ of$$

$$A = \{a, \lambda_a\}$$

In a situation where the pure state (mutual and orthogonal) have the signal alphabet then the equation source becomes a classical source;

$$S(\rho) = H(X) \qquad (1.6)$$

The more the signal state ρ becomes commuted mutually, the more interesting the quantum source becomes. Now our bigger argument is that von Neumann and Shannon entropies quantify the incompressible information content of the quantum source and classical source respectively [12]. For the quantum source we refer to the signal states that are pure. In this regard the von Neumann entropy is the interpretation of quantum information theory and classical for Shannon entropy.

C. Fidelity

Assuming a pure state is transmitted through a quantum channel and the pure state is the density operator[11, 13]

$$\hat{\rho} = \mid \varphi >< \varphi \mid \qquad (1.7)$$

Then the super operator that produces the alteration characterizes this

$$\hat{\rho}' = \$(\hat{\rho}) \qquad (1.8)$$

Now the fidelity function is defined by

$$F(\hat{\rho}, \hat{\rho}') = < \varphi \mid \hat{\rho}' \mid \varphi > \qquad (1.9)$$

If the fidelity is closer to 1 we expect better transmission and the application of fidelity is limited to pure states. And for the mixed states is even more complicated [7].

$$F(\hat{\rho}, \hat{\rho}') = tr^2 \left|(\sqrt{\hat{\rho}\hat{\rho}'}\sqrt{\hat{\rho}})^{1/2}\right| \quad (2.0)$$

where $\hat{\rho}$ and $\hat{\rho}'$ are the mixed states, now the fidelity equation can be can be re-written as

$$F(\hat{\rho}, \hat{\rho}') = \max |< \phi \mid \phi' >|^2 \quad (2.1)$$

where ϕ and ϕ' are purification of $\hat{\rho}$ and $\hat{\rho}'$ respectively.

Now the fidelity being an important factor the following are always true about it.

1. It ranges between 0 and 1, for fidelity to be 1 $\hat{\rho} = \hat{\rho}'$
2. $F(\hat{\rho}, \hat{\rho}') = F(\hat{\rho}', \hat{\rho})$
3. Given two real positive numbers a_1 and a_2 such that their summation is 1 then $F(\hat{\rho}P_1\hat{\rho}_1 + P_2\hat{\rho}_2) \geq P_1F(\hat{\rho}, \hat{\rho}_1) + P_2F(\hat{\rho},)\hat{\rho}_2$ also; $F(\hat{\rho}, \hat{\rho}') \geq tr(\hat{\rho}\hat{\rho}')$.
4. If $\hat{\rho}$ is pure state $\hat{\rho} = \mid \varphi\rangle\langle\varphi \mid$, then $$F(\hat{\rho}, \hat{\rho}') =< \varphi \mid \hat{\rho}' \mid \varphi >= tr(\hat{\rho}\hat{\rho}')$$
5. $F(\hat{\rho}_1 \otimes \hat{\rho}_2, \hat{\rho}_3 \otimes \hat{\rho}_4) = F(\hat{\rho}_1, \hat{\rho}_3)F(\hat{\rho}_2, \hat{\rho}_4)$
6. If $\hat{\rho}_1$ and $\hat{\rho}_2$ are transformed to $\hat{\rho}_1'$ $and \hat{\rho}_2'$ respectively it implies that $F(\hat{\rho}_1', \hat{\rho}_2') \geq F(\hat{\rho}_1, \hat{\rho}_2)$ which means the fidelity of the transformed state is greater or equal to the pure state depending on the value of $\hat{\rho}$ $and \hat{\rho}'$.

D. Schumacher's Coding Theorem

For a set of pure quantum state $\{\mid \varphi_1 >, \mid \varphi_2 >, \ldots\ldots, \mid \varphi_x >\}$ with probabilities $\{P_1, P_2, \ldots P_x\}$ the symbol from the source depends on the density operator

$$\hat{\rho} = \sum \lim_x P_x \mid \varphi_x > \mid \varphi_x > \quad (2.2)$$

Similarly, a series comprising N symbols is described using the tonsorial product formalism as

$$\hat{\rho}^N = \hat{\rho} \otimes \ldots\ldots \otimes \hat{\rho} \quad (2.3)$$

For a given two positive real numbers δ and ε and sequence of N symbols, each state is generated by the source with $S(\hat{\rho}) + \delta$ qubits keeping the fidelity as high as possible i.e $F > 1 - \delta$.

E. Quantum Channel without Noise

Now assume that we want to send conventional information triggered from a source with entropy $H(X)$ through an error free quantum channel. For a successful transmission of information it is obvious that a quantum source is needed. To proceed with the process an application is need to be prepared so as to take care of the characters of the original input alphabet together with their probabilities with a complete set of quantum state [7].

But the X becomes more uncertain when trying to obtain a good measurement for the transmitted quantum states. To overcome this, the concept of accessible information $A(I)$ is introduced, and it is defined as maximum information in bits that could be recovered, when the results of a measurement that give the highest information carried in a quantum state.

$$A(I) = max_{\varsigma} I(X, Y) \quad (2.4)$$

Where ς is an operator and also a variable that is obtained randomly by measuring the state and their corresponding probabilities, $I(X, Y)$ represents the classical mutual information between X and Y. The set of the quantum states are orthogonal.

F. Quantum Channel with Noise

The way of retrieving information in a noisy and un-noisy quantum channel is somehow identical but the only difference is that for the noisy quantum channel it is allowed that the input information of the quantum source is a static mixture[7]. Hence the pure states are transformed into mixed states due to the de coherence effect. We can model this channel with a super operator ₦ which has effect directly on the input alphabet source.

$$\mid \varphi_x > \mid \varphi_x > \rightarrow ₦(\mid \varphi_x > < \varphi_x = \hat{\rho}_x \quad (2.5)$$

Capacity of the Channels:

The quantum channel differs from classical one in terms of capacities. This resulted as a fact that the quantum channel has great dynamic abilities if we consider the transfer of information via the quantum channel, the variety of capacities comes up from numerous factors[14]:

- Type of information transfer in both quantum and classical channels
- Type of the input state either entangled or non-entangled states
- The measurement carried on the output , that is either single or collective
- Availability of subordinate resources i.e. initial state between the sender and receiver and if the communication between them is allowed classically.

Thus the capacity of a quantum channel is known as the ability of the channel to transfer information without the conventional communication and the ancillary resources[15]. Also quantum channel can establish an entanglement with the sender and the receiver which may lead as a resource for teleportation [16, 17], and this equal the capacity of the channel in transferring the quantum information.

These capacities were initially determined based on the asymptotic uses of the channel, with the assumption that the channel is memoryless, and also the noise on the input to the channel was assume to be not connected. Even though there is no justification of the made assumption and the asymptotic scenario.

The possibility of reaching a perfect or accurate information transmission or an entangled generated information over single or limited number of uses is in general focus and not feasible.

Hence the need for a probability error that is not zero. Thus made it possible to consider the capacities under the condition that the chance for this error is at most a value that is greater or equal to zero.

$$\varepsilon \geq 0$$

Shannon establishes a phenomenon in which a noisy quantum channel (memoryless) can be able to simulate a noiseless binary channel [16] by applying coding theorem of noisy channel [18] and also proves that the asymptotic effectiveness of the quantum channel can be shown by the following expression:

$$C(N) = \max I(X;Y)$$
$$= \max\{H(X) + H\big(Y - H(XY)\big)\}$$

Where H is the entropy, X is the random input variable, Y = N(X) which is the induced output variable, C is the capacity of the channel and N is the channel.

The classical reserved Shannon theorem: as discussed in [19] it says if in the transmission there is sharing of an infinite random bits between sender and receiver for N channels of capacity C, then we expect $C_n + 0(n)$ uses of noiseless binary channel to be sufficient to simulate n uses of the channel.

Let's consider a noisy quantum channel that is been simulated by a noiseless one, which is the reversed of the classical theorem where the noisy channels simulate the noiseless. The reversed case can take place having equal capacities if the random information is shared. This means the noisy channels are equivalent asymptotically.

This will greatly ease the complication in quantum channel capacity if the reversed theorem became true as: It says "the quantum channel can be simulated asymptotically provided the previous entangled state and amount of non-quantum communications equals to its assisted capacity" [16, 19]. Therefore in a situation where there are many entangled states, all quantum channels are categorized based on quality equivalence and quantitatively characterized by a single parameter.

C.H Bennett et-al where able to show this theorem as follows:

Let N be the memoryless classical channel, X be the random variable (input), Y be N(X) induced output
This implies that I(X; Y) will indicate mutual relationship between Y and X. and let's assume N_F represent the feedback copy of receiver's output. It is trivial that

$$N \leq N_F \ and \ (N{:}\,p) \leq \langle N_F{:}\,p\rangle$$

p is the input distribution for one use of a channel of input-output mutual information.

Quantum Reverse Shannon Theorem:

Let N be a quantum channel from A→B or equivalently an isometric from A→BE and N_F the feedback channel that results from given system E to the sender. If we are given an input density matrix ρ^A then entropy quantities such as I(R; B) or I(R; B)$_\rho$ refer to the state

$$\varphi^{RBE} = (I^R \otimes N^{A\rightarrow BE})(\Phi_\rho^{RA})$$

Where Φ_ρ is any state satisfying $\Phi_\rho = \rho$

G. *Mathematical Values of $S(\rho)$*

i. Purification:

For a pure ρ

$$\Rightarrow \rho = \mid \varphi_x >< \varphi_x \mid$$

Then $S(\rho)$ has to be zero

ii. Variation:

The unitary change of basis doesn't affect any change in entropy $S(\rho) = S(UPU^{-1})$ this is true because $S(\rho)$ depends on eigenvalues of ρ.

iii. Maximum:

For a value d as non-vanishing eigenvalues, then,

$$S(\rho) \leq \log D$$

This means the entropy is at highest when the non-zero eigenvalues are equal.

iv. Concavity:

For $\lambda_{1,}\lambda_{2,}\lambda_{3,} \ldots \lambda_n \ \geq 0 \ and \ \lambda_1 + \lambda_2 \ + \cdots + \lambda_n = 1$

$$S(\lambda_1\rho_1 + \cdots + \lambda_n \, \rho_n \geq \lambda_1 \, S(\rho_1) + \cdots \lambda_n \, S(\rho_n)$$

This shows the more we have less knowledge of how the state is generated the larger the von Neumann entropy becomes [20]. This is as the result of a concavity of the log function.

v. Entropy Measurement:

Suppose in a state ρ we measure an observable

$$A = \sum_y \mid a_y > a_y < a_y \mid \qquad (2.6)$$

Such that a_y has probability

$$P\big(a_y\big) =< a_y \mid a_y \mid a_y > \qquad (2.7)$$

Then Shannon entropy of the ensemble measurement outcomes [21]

$$Y = \{a_y, P\big(a_y\big)\} satisfies$$
$$H(Y) \geq \ S(\rho)$$

H. *Quantum Data Compression*

Let's assume we have a long message of n characters and each character in the ensemble pure state was chosen randomly.

$$\{|\varphi_x>, P_x\},$$

And we also assume that the states are not all mutually orthogonal
Now each character is described by

$$\rho = \sum_x P_x \,|\, \varphi_x >< \varphi_x \,| \quad (2.8)$$

And generally the whole message has

$$\rho^N = \rho \otimes \ldots\ldots \otimes \rho \quad (2.9)$$

The challenge here is to find a quantum code that will make us to compress the message and thereby reducing the Hilbert space to smaller, without compromising the message [22]. This compression of data was achieved optimally by ben Schumacher. The best compression that did not temper with the fidelity as $n \to \infty$ is this of Hilbert space with

$$\log(dim\mathcal{H}) = nS(\rho) \quad (3.0)$$

where von Neumann entropy here is the number of qubit of quantum information in each character of a message.

For a message to have n photons and needed to be compressed.

$$m = nS(\rho) \quad (3.1)$$

III. Results and Discussion

In this section analysis of the simple picking experiment was given and also the interpretation of information compressed in a single message for transfer. In the experiment we performed a random picking of the fruits one at a time, each time measuring the probability for the occurrence of each item. The amount of information after each picking is also measured and also the entropy is interpreted in each case.

TABLE I. RESULTS FOR THE EXPERIMENT

x	p(x) orange	I(orange)	p(x) app	I app	h(x) orange	h(x) app
1	0.9	0.04575749	0.1	1	0.041181742	0.1
2	0.888888889	0.05115252	1	0	0.045468909	0
3	0.875	0.05799195	1	0	0.050742954	0
4	0.857142857	0.06694679	1	0	0.057382963	0
5	0.833333333	0.07918125	1	0	0.065984372	0
6	0.8	0.09691001	1	0	0.07752801	0
7	0.75	0.12493874	1	0	0.093704052	0
8	0.666666667	0.17609126	1	0	0.117394173	0
9	0.5	0.30103	1	0	0.150514998	0
10	1	0	1	0	0	0

Table I shows the values of the probabilities p(x) orange which is the probability of picking orange and p(x) apple which is the probability of picking an apple, amount of information in orange I(orange) and I(apple) amount of information in apple and the entropies h(x) orange and h(x) apple for picking the fruits one at a time, this shows the certainty and the uncertainty of each event in trying to select either an orange or an apple. Therefore by the above interpretation the von Neumann entropy is the amount of information in picking each of the fruit.

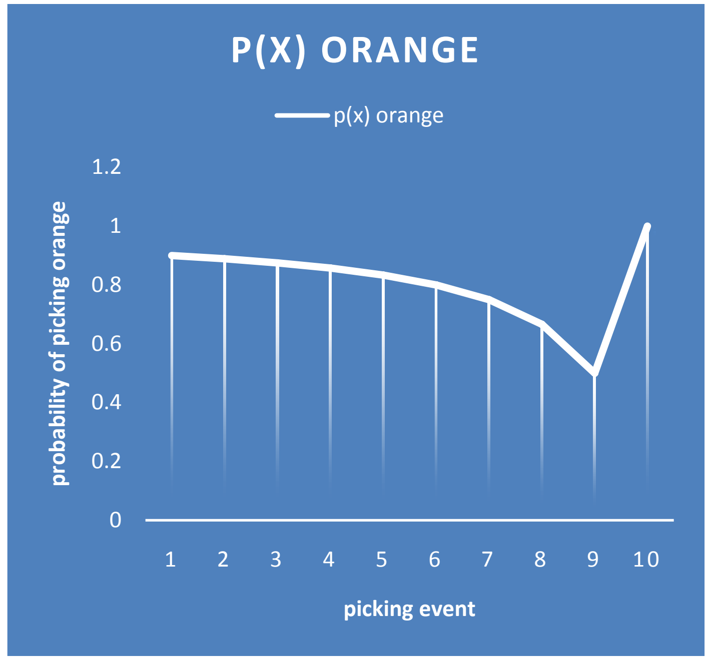

Fig 1. Probability of picking an orange

Fig. 1 shows how probability of picking an orange keeps changing after each event for ten times with the assumption that each time it is an orange that is picked. That is why we have uncertainty at each event, thus providing more information.

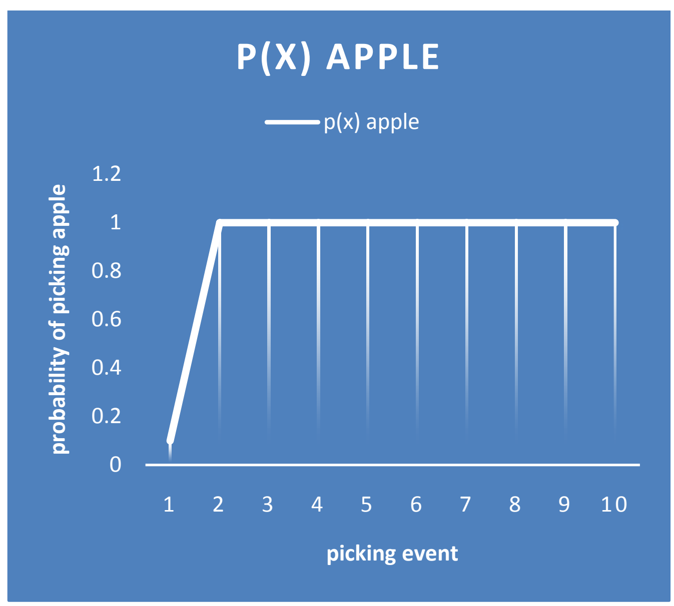

Fig 2. Probability of picking an apple

Fig 2 shows how probability of picking an apple remain constant after the first event, though providing less information due to the certainty of the outcomes. The assumption here is that the first picking was apple therefore all the remaining chances are certainly for orange therefore minimal amount of information is expected.

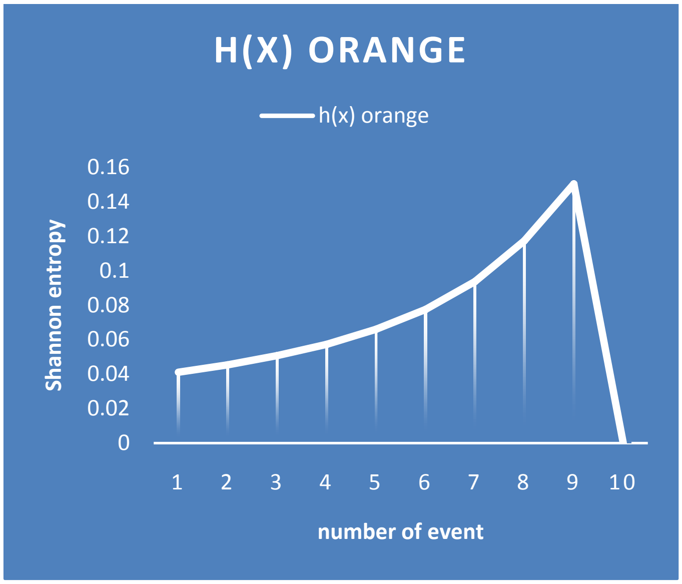

Fig 3. Entropy measure of picking orange

Fig 3 shows the entropy measure of uncertainty in picking the random variable orange, the measure keep changing while rising until it maximum, then it comes down to zero. Meaning that, it is only at the last event that certainty comes up. Therefore the message here could contain more information compressed as compared to Fig 4 below.

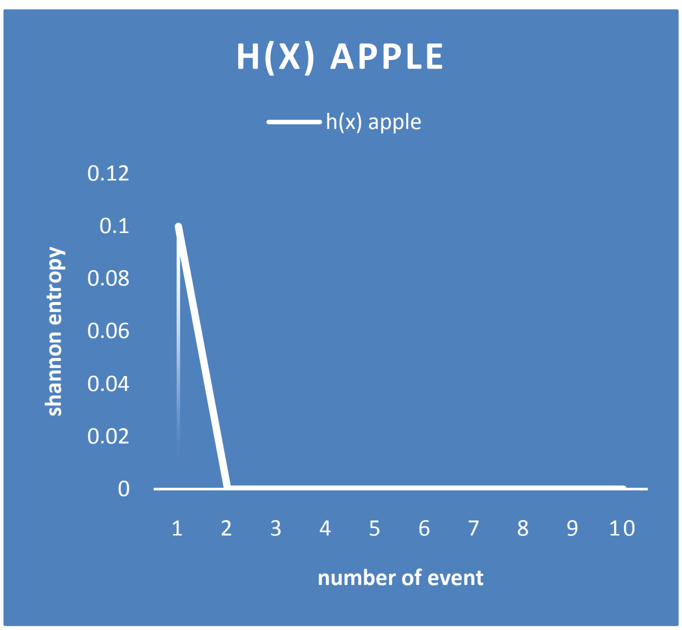

Fig 4. Entropy measure of picking an apple

Fig 4 shows that the amount of information that can be compressed in a message is very little because is at only the first prediction that it has a measure but all other events are certain therefore the entropy collapsed.

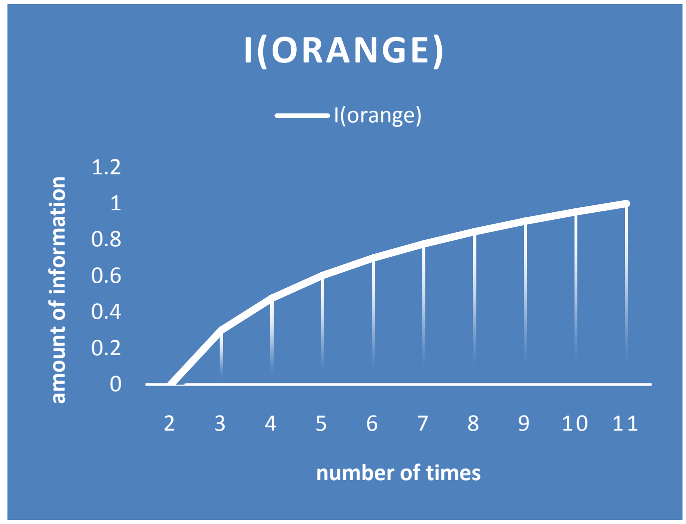

Fig 5. Measure of amount of information in orange

Fig 5 shows the measure of amount of information gathered considering the choice of orange, the channel will contain more information as the probability decreases.

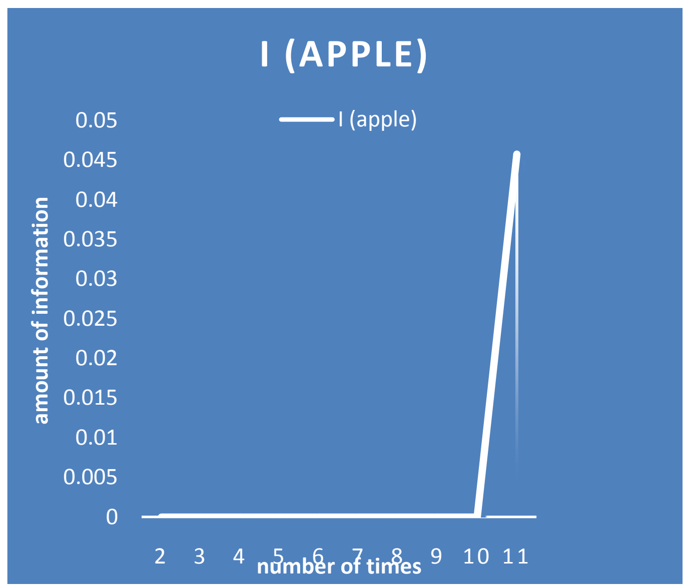

Fig 6. Measure of amount of information apple

Fig 6 shows the measure of amount of information that can be obtain in picking an apple. For the first nine events the information was zero because all this while the probability was at maximum thereby producing less information.

IV. Conclusion and Future Work

We were able to show how information can be maximum or minimum in an event. Such events were assumed to be the messages and the informations obtained in picking the fruits are the qubits contained in those messages. The qubits are ready for transmission using the probability experiment and applying the theory of Shannon entropy of classical information. Also we were able to show that the noisy classical or quantum channels can be used to simulate their noiseless counterpart channels and vice versa by applying the

Shannon reverse theorem of both classical and quantum channels. This is an opener to start for the quantum information transfer and message compression. We will focus in our next work the number of qubit of quantum information contains in each character of a message, which is the von Neumann entropy. The known results of the Shannon entropy will give us a lead.

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