Performance Analysis of CDMA in Optical Transmission Using PPM Signaling

N. Alsowaidi[1], Tawfig Eltaif[1*], M. R. Mokhtar[2], K. Alaghbari[1]

Abstract**—** In this article we aim to theoretically investigate the performance of optical code-division multiple-access (OCDMA) system with *M*-ary pulse-position modulation (PPM) signaling. The system is implemented on both direct sequence (DS) and spectral amplitude coding (SAC) using different spreading sequence codes, and considering the effects of the loss produced from the splitting during encoding and decoding, the channel loss, and multiple access interference (MAI). It is noticed that, PPM levels increases system improves significantly. In addition, SAC-Optical PPM-CDMA system has better performance than DS-Optical PPM-CDMA system.

Keywords**—** pulse-position modulation (PPM), multiple access interference (MAI), optical code-division multiple-access (OCDMA)

I. INTRODUCTION

IN the last three decades, optical code-division multiple-access (CDMA) schemes have attracted much attention and become the next candidate for next generation passive optical network (NG-PON) because this technique allows multiple users to access the network asynchronously or simultaneously [1]. Unfortunately, it was identified early that the performance of OCDMA was limited by multiple access interference (MAI). In optical CDMA systems, OOK and PPM are mostly used as modulation schemes. It is believed that PPM is more power-efficient than OOK because M-ary PPM can transmit log2M bits with only one pulse. In PPM, a laser pulse with few picoseconds width is shifted into one of a set of possible pulse locations (i.e, M) for data transmission. Each pulse position corresponds to a different PPM symbol [2-5]. In addition, the random locations of the transmitted sequences due to the PPM encoding can reduce the probability of concentrated buildup of pulse overlaps is any given one slot time. Therefore, one of the main advantages of using pulse position modulation in optical CDMA system is the improvement in system performance (e.g. BER) by increasing number of the slots M as compared with on-off keying modulation. Many researchers have introduced different techniques for further enhance in the OCDMA performance such as, the synchronous spectral-amplitude-coding (SAC)-OCDMA system, which used a family of codes with fixed weight and fixed cross-correlation for both lower and upper bounds of the bit error rate was investigated in [6]. While Shalaby analyzed the BER of the synchronous SAC-OCDMA system based on Gaussian approximation using codes with fixed cross-correlation [7]. While Sedaghat, et al. have found that the asynchronous SAC-OCDMA system has a better performance than the synchronous SAC-OCDMA when phase-induced intensity noise (PIIN) effect exists [8].

In this paper, the performance of an optical PPM-CDMA system for both direct sequence and spectral amplitude coding (SAC) with modified quadratic congruence (MQC) code, modified prime code (MPC) and double padded modified prime code (DPMPC) is theoretically analyzed. The performance is evaluated in terms of error free transmission.

II. SYSTEM PERFORMANCE

In PPM signaling format, each symbol is represented by a single pulse of duration τ , positioned in one of M possible time slots. Each slot has a width $\tau=T_s/M$. The achievable bit rates of an OCDMA channel depends on PPM parameters and chosen code sequence. If there is $M = 2^k$ slots per PPM frame, where $k > 0$ bit resolution is an integer, then the operating bit rate $R_b = kR_s = \log M/T_s = \log M/M\ \tau$. Using MQC codes [6], where P^2 Codes can be generated; each code has a weight equal to $(P+1)$and a length (P^2+P). Let T_c denotes the chip time duration of a CDMA signal. Hence, the relation between the optical pulse width T_c, τ, and T_s, can be expressed as $T_c = \frac{\tau}{P^2+P} = \frac{T_s}{M(P^2+P)}$ [4]. The users signals must be send with sufficient power to overcome the network losses. The delay line splitting during the encoding process produces a splitting and combining loss $1/(P+1)^2$. The same loss will be produced at the receiver side. Hence, the total loss from the laser source to the corresponding photodetector is $L/N(P+1)^4$, where$1/N$ is the network splitting loss and L fiber transmission losses. Let us assume that, P_a is the average laser power, then, it's peak pulse power after *M-ary* PPM framing is $P_p = M(P^2+P)P_a$ then the peak received signal pulse power is [4];

$$P_r = P_P\left(\frac{L}{N(P+1)^4}\right) = P_a\frac{PML}{N(P+1)^3} \tag{1}$$

Then, the intensity of the n^{th} user signal is $X_n(t) = P_p b_n(t) c_n(t)$ where $c_{k,n} \in \{0,1\}$ refers to sequence code, and $b_{i,n} \in (0,1,....,M-1)$ is the *PPM signal* of the n^{th} user. Hence, the received signal can be expressed as:

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1 Faculty of Engineering & Technology, Multimedia University, 75450, Bukit Beruang, Melaka, Malaysia

2 Faculty of Engineering, Multimedia University, 63100, Cyberjaya, Selangor, Malaysia

*Corresponding author e-mail: tefosat@ieee.org

$$r(t)=P_a\frac{PML}{N(P+1)^3}\sum_{n=1}^{N}\sum_{k=0}^{P^2-1}b_n(t-\tau_n)c_{k,n}(t-\tau_n)+n(t) \tag{2}$$

Where $n(t)$ represents the noise signal, and τ_n is the relative delay. To have more insight on the problem under consideration, we assume a synchronous scenario with $\tau_n = 0$.

A. DS-Optical PPM-CDMA System

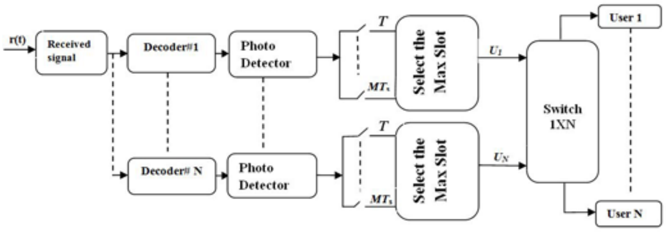

Fig1. Block diagram of DS-Optical PPM-CDMA receiver

The receiver block diagram of the DS optical PPM-CDMA is shown in figure 1, and it illustrates that the received signal goes through the bank correlator which consists of decoder and photo-decoder after that it goes through PPM demodulation then finally goes to the destination.

To measure the probability of bit error of the system, we need to use probability of detecting a correct symbol [5]. Assuming that, without loss of generality the sample $y_1=x_1+n_o$ among M slots was transmitted, where x_1 the symbol = $M\ P^2 P_a b_n$ and n_o is the thermal noise. Then,

$$P[correct\ symbol]= P[x_1+n_1>x_2+n_2, x_1+n_1>x_3+n_3, ...x_1+n_1>x_M+n_M] \tag{3}$$

The probability of each sample in Eq(3) can be expressed in terms of Q function as follows;

$$P[correct\ symbol]=E\left\{\begin{array}{l}\left[1-Q\left(\frac{x_1-x_2+n_1}{\sqrt{N_0}}\right)\right].\left[1-Q\left(\frac{x_1-x_3+n_1}{\sqrt{N_0}}\right)\right].\\ ..\left[1-Q\left(\frac{x_1-x_M+n_1}{\sqrt{N_0}}\right)\right]\end{array}\right\} \tag{4}$$

$$P[correct\ symbol]=E\left[\begin{array}{l}1-Q\left(\frac{x_1-x_2+n_1}{\sqrt{N_0}}\right)-\\ Q\left(\frac{x_1-x_3+n_1}{\sqrt{N_0}}\right)-Q\left(\frac{x_1-x_M+n_1}{\sqrt{N_0}}\right)\end{array}\right] \tag{5}$$

Since the mean and variance of both random variables x and n are (μ, σ^2), then $E(Q(X))=Q\left(\frac{\mu}{\sqrt{\sigma^2}}\right)$. Then, the probability of symbol error can be calculated as [5]

$$\begin{aligned}P[symbol\ \ error]&=1-P[correct\ symbol]\\ &=Q\left(\frac{x_1-x_2+n_1}{\sqrt{N_0}}\right)-Q\left(\frac{x_1-x_3+n_1}{\sqrt{N_0}}\right)-Q\left(\frac{x_1-x_M+n_1}{\sqrt{N_0}}\right)\\ &=\prod_{m=2}^{M}Q\left(\frac{S_n^1(t)-S_n^m(t)}{\sqrt{2N_0}}\right)\end{aligned} \tag{6}$$

Then, the probability of bit error;

$$P_e=\frac{M}{2(M-1)}\left[\prod_{m=2}^{M}Q\left(\frac{S_n^1(t)-S_n^m(t)}{\sqrt{2N_0}}\right)\right] \tag{7}$$

B. SAC-Optical PPM-CDMA System

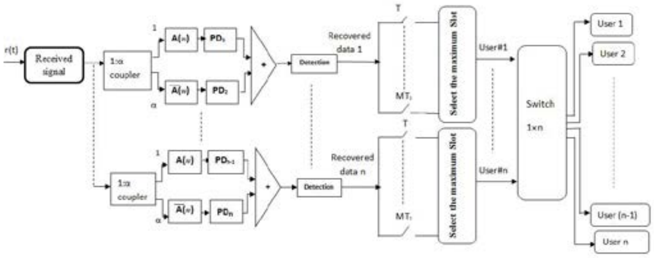

Fig 2. Block diagram of SAC-Optical PPM-CDMA receiver

The receiver block diagram of the SAC optical PPM-CDMA is shown in figure 2, SAC scheme become more attractive because it is capable to mitigate noises caused by the overlapping between the signals, especially when the code used to encode the data has fixed cross correlation. A significant degradation in the system performance is introduced due to the PIIN, shot noise and thermal noise. Using Modified congruence code (MQC) with p^2 codes can be generated where p is the prime number; each code has a weight equal to $p+1$ and a length p^2+p. Let T_c denotes the chip time duration of a CDMA signal. Hence, chip time duration T_c can be expressed as

$$T_c=\frac{T_s}{p^2+p}=\frac{T}{M(p^2+p)}.$$

$$\sum_{i=1}^{p^2+p}c_k(i)c_l(i)=\begin{cases}p+1, k=l\\ 1,\ k\neq l\end{cases}$$
$$\sum_{i=1}^{p^2+p}c_k(i)\overline{c}_l(i)=\begin{cases}0, k=l\\ p,\ k\neq l\end{cases} \tag{8}$$

The received signal of SAC Optical PPM-CDMA is expressed as:

$$G^i(v)=\frac{P_{er}}{\Delta v}\sum_{n=1}^{N=p^2} b_n \sum_{i=1}^{p^2+p} c_n(i).$$
$$\left(\begin{array}{l} u\left[v-v_o-\frac{\Delta v}{2P^2}(-p^2+2i-2)\right]- \\ u\left[v-v_o-\frac{\Delta v}{2P^2}(-P^2+2i)\right] \end{array}\right) \tag{9}$$

where; b_n is the PPM signal $b_n(t)=\sum_{i=-\infty}^{\infty} P_{T_s}(t-b_{i,n}T_s-iT)$, where $P_{T_s}(t)$, $u(v)$, P_{er} are a rectangular pulse of *M-ary* symbols with duration T_s.

The system would consider shot noise, incoherent intensity noise and thermal noise. The variance of the photocurrent consists of PIIN noise, thermal noise and shot noise, which can be stated as follows [9].

$$\left\langle i^2\right\rangle = 2eIB + I^2 B\tau_c + 4K_b T_n B / R_L \tag{10}$$

where; *B, K_b, e, I, T_n, R_l are* the noise-equivalent electrical bandwidth of the receiver; the Boltzmann's constant; the electron's charge; the average current; the absolute receiver noise temperature and the receiver load resistor respectively.

The light source coherent time τ_c can be expressed as [9]

$$\tau_c=\frac{\int_{v=0}^{\infty} G^2(v)dv}{\left[\int_{v=0}^{\infty} G(v)dv\right]^2} \tag{11}$$

The power spectral density output of upper and lower arms of SAC scheme of the first user can be expressed as

$$G_1(v)=\frac{P_{er}}{(p-1)\Delta v}\sum_{n=1}^{p^2} b_n \sum_{i=1}^{p^2+p} c_n(i)\bar{c}_1(i).$$
$$\left(\begin{array}{l} u\left[v-v_o-\frac{\Delta v}{2p^2}(-p^2+2i-2)\right]- \\ u\left[v-v_o-\frac{\Delta v}{2p^2}(-p^2+2i)\right] \end{array}\right) \tag{12a}$$

$$G_2(v)=\frac{P_{er}}{\Delta v}\sum_{n=1}^{p^2} b_n \sum_{i=1}^{p^2+p} c_n(i)c_1(i).$$
$$\left(\begin{array}{l} u\left[v-v_o-\frac{\Delta v}{2p^2}(-p^2+2i-2)\right]- \\ u\left[v-v_o-\frac{\Delta v}{2p^2}(-p^2+2i)\right] \end{array}\right) \tag{12b}$$

Then, the photocurrents at SAC photo detectors for the first user are given by (12(a, b))

$$I_1=\int_0^{\infty} G_1(v)dv=\frac{P_{er}}{p^2}\sum_{n=2}^{N} b_n \tag{13a}$$

$$I_2=\int_0^{\infty} G_2(v)dv=\frac{P_{er}}{p}b_1+\frac{P_{er}}{p^2}\sum_{n=2}^{N} b_n \tag{13b}$$

The subtraction of the photocurrent I_1 and I_2 will results to the first desired user data.

$$I=I_1-I_2=\Re\frac{P_{er}}{p}b_1 \tag{14}$$

The responsivity of the PD's is given by $\Re=\eta e/hv_c$. Here, η, e , *h*, v_c are the quantum efficiency, the electron's charge, the Plank's constant, and the central frequency of the original broadband optical pulse respectively.

Then the desired user data along with the total noise can be express as follows:

$$S=\Re\frac{P_a ML}{Np^3}T_c w P_i + n_0 \tag{15}$$

where *w* is the weight of the signature code; P_{1i} is an indicator of the event that the desired user is ending its information in i^{th} slot.

Then, the probability of bit error for the SAC optical PPM-CDMA system as follows;

$$P_e=\frac{M}{2(M-1)}\left[\prod_{m=2}^{M} Q\left(\frac{S_n^1(t)-S_n^m(t)}{\sqrt{2N_0}}\right)\right] \tag{16}$$

Where S_n^1 and S_n^m are the first and m^{th} components of the n^{th} user.

III. NUMERICAL RESULTS AND DISCUSSION

In this section we presented numerical results of both direct-sequence and spectral amplitude coding CDMA in optical transmission using PPM signaling. According to Eq (1) the laser diode placed after PPM modulation, and clearly as M increases the effective power increases, then the system performance improves significantly. The bit error rate (BER) performance is evaluated for 121/132 chips per bit encoding using analytical expression.

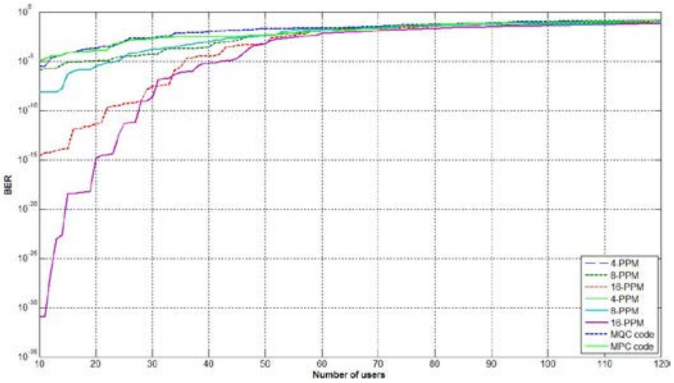

Fig 3. BER performance of DS Optical PPM-CDMA

Figure 3 and Figure 4 show the performance of DS-Optical PPM-CDMA system, while figure 5 shows the performance of SAC-Optical PPM-CDMA system.

Figure 3 shows that more users can be accommodated as M increases for both modified quadratic congruence (MQC) code and the modified prime code (MPC), channel length is 50 km, fiber transmission losses is 0.25dB/km. The optimum receiver sensitivity as shown in figure 4 at large *M*=16 is -12dBm as the transmit power is 0dBm. This improvement because of random location of the sequences due to the PPM encoding, which can reduce the probability of concentrated buildup of the pulse overlap in any one slot time, which means reducing MAI effect.

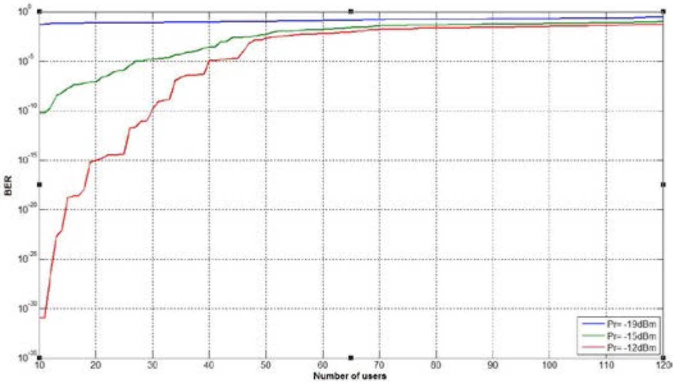

Fig 4. BER versus number of active users under different effective power

Results presented considering SAC Optical PPM-CDMA system under some parameters tabulated in table 1.

Table1 System Parameters

Parameter	**Value**
Prime number	$p = 11$
Channel length	50km
Effective power	12dBm
Fiber transmission losses	0.25dB/k
Operation Wavelength	193.1 THz
Quantum efficiency	0.6
Receiver noise temperature	300 k
Receiver load resistor	1030 Ω
Electrical equivalent bandwidth	80 MHz
Line-width of the thermal noise	Δv=3.75 THz

Different optical codes were applied to the proposed system, such as modified quadratic congruence codes (MQC), modified prime codes (MPC) and double padded modified prime codes (DPMPC) as signature codes. The effects of shot noise, PIIN and thermal noise have been taken into account. In Figure 5 we have compared the results of the system without cancellation using three different sequence codes. The BER from the obtained result proven that the system with conventional scheme using MPC has better performance than the system using MQC or double DPMPC and this due to code length and weight which considers very serious parameter. It is desirable to have smaller code length in case of conventional scheme is used.

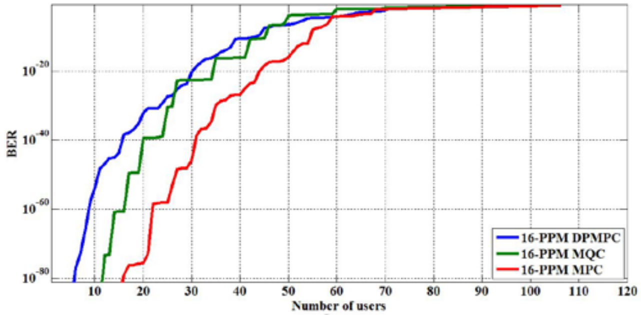

Fig 5. BER performance of SAC Optical PPM-CDMA

IV. CONCLUSION

A different theoretical analysis has been presented for both direct detection and SAC optical PPM-CDMA system using three family prime codes as signature sequence codes. Results show that the system performance improves significantly as PPM levels increases, so the receiver sensitivity since it depends on M slots and number of accommodated users. In addition, DS-Optical PPM-CDMA system with MPC as signature code achieves significantly BER improvement over 4, 8, 16-PPM-CDMA system as compared to the one MQC code. On the other hand SAC-Optical PPM-CDMA system has better performance than DS-Optical PPM-CDMA system where the latter system can support up to 37 users while the former can support up 55 users. The system capacity increases because the average number of interfering optical pulses equals (N-1)/M.

REFERENCES

[1] Ohtsuki, T., "Performance analysis of atmospheric optical PPM CDMA systems," Journal of Lightwave Technology, vol.21, no.2, pp.406-411, Feb. 2003.

[2] Shalaby, H.M.H., "Performance analysis of optical synchronous CDMA communication systems with PPM signaling," IEEE Transactions on Communications, vol.43, no.2/3/4, pp.624-634, Feb./March/April 1995.

[3] Jin Young Kim; Poor, H.V., "Turbo-coded optical direct-detection CDMA system with PPM modulation," Journal of Lightwave Technology vol.19, no.3, pp.312-323, Mar 2001.

[4] Gagliardi, R.M.; Mendez, AJ.; Dale, M.R.; Park, E., "Fiber-optic digital video multiplexing using optical CDMA," Journal of Lightwave Technology , vol.11, no.1, pp.20,26, Jan 1993.

[5] M. D. Audeh, and J. M. Kahn,"Performance evaluation of L-pulse-position modulation on non-directed indoor infrared channels," Proceedings of ICC/SUPERCOMM'94 - 1994 International Conference on Communications, vol., no., pp.660-664 vol.2, 1994.

[6] Noshad, M., Jamshidi, K.: “Bounds for the BER of codes with fixed cross correlation in SAC OCDMA systems”, J. Lightwave Technol., pp. 1944–1950, 2011,

[7] Shalaby, H.M.H.: “Closed-form expression for the bit-error rate of spectral-amplitude-coding optical CDMA systems’, IEEE Photonics Technol. Lett., pp. 1285–1287, 2012

[8] Sedaghat, M. A., Müller, R. R., Marvasti, F. “Performance analysis of asynchronous optical code division multiple access with spectral-amplitude coding”, IET Communications, IET Commun., 2014, Vol. 8, Iss. 6, pp. 956–963, 2013

[9] Zou Wei; Shalaby, H.M.H.; Ghafouri-Shiraz, H., "Modified quadratic congruence codes for fiber Bragg-grating-based spectral-amplitude-coding optical CDMA systems," Journal of Lightwave Technology vol.19, no.9, pp.1274-1281, Sep 2001.