Observer-based synchronization for coupled non identical chaotic systems

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***Abstract*—In this work, the proposed framework for the synchronization of two non identical continuous-time chaotic systems, based on the use of observer concept and aggregation technique, is applied, with success. Sufficient conditions for synchronization are obtained by the use of Borne and Gentina practical criterion for stabilization study associated to the Benrejeb arrow form matrix for system description. Application of the proposed approach to two master-slave Lorenz and Rôssler systems shows the efficiency of the proposed approach.**

***Keywords*—Continuous-time chaotic systems, Nonlinear observer, Synchronisation, Stability, Benrejeb arrow form matrix, Aggregation techniques.**

I. INTRODUCTION

The synchronization phenomenon is an interesting and well-known property of chaotic systems. Since its introduction by Pecora and Carrol in 1990, [1], chaos synchronization has attracted increasing interest in both theory and applications, [2] to [4], as far as several fields are concerned. As a matter of fact, the synchronization of chaotic systems has been successfully applied in secure communication and image encryption, information processing, life science, [5] to [10], and so on.

Recently, chaos synchronization has been studied from various angles and a variety of different synchronization phenomena have been discovered, such as generalized synchronization, [11] and [12], phase synchronization [13], lag synchronization [14], anti-synchronization, [15] and [16], observer-based synchronization [17] to [20], etc.

The purpose of this work is to determine a necessary and sufficient conditions for the convergence to zero of the states synchronization error between two non identical chaotic continuous-time processes, [21].

The proposed stabilizing conditions for nonlinear continuous-time hierarchical systems are based on the Borne and Gentina practical criterion for stability study, [23] and [24], associated to the Benrejeb arrow form matrix for system description, [26] to [28]. It constitutes an extension of previous results on synchronization studies of continuous two identical chaotic systems, [4] and [25].

The paper is organized as following. In Section II, is proposed an approach to design an nonlinear observer, effective and systematic in achieving synchronization of continuous-time chaotic systems, guarantying the asymptotic stability for the synchronization errors, characterized, in the state space, by an arrow form matrix. The implementation of the proposed synchronization approach to two non identical chaotic systems using two master-slave continuous-time chaotic Lorenz and Rôssler systems is performed in Section III.

II. OBSERVER-BASED CONTINUOUS-TIME SYNCHRONIZATION - BASIC IDEA

A proposed synchronization approach for a class of continuous time chaotic systems is applied in this section for two non identical chaotic systems.

Consider the n-dimensional chaotic continuous-time master system.

It is modeled as follows

$$\left\{\begin{array}{l} \dot{x}_m(t) = A_m x_m(t) + f_m(x_m(t)) + V_m \\ y_m(t) = C x_m(t) \end{array}\right. \tag{1}$$

x_m is the state vector, $x_m \in \mathbf{R}^n$, y_m the output $y_m \in \mathbf{R}$ of the master system and C an $(1 \times n)$ constant matrix.

We associate to this system a not identical slave system, defined by

$$\left\{\begin{array}{l} \dot{x}_s(t) = A_s x_s(t) + f_s(x_s(t)) + V_s + Bu(t) \\ y_s(t) = C x_s(t) \end{array}\right. \tag{2}$$

$x_s \in \mathbf{R}^n$ and $y_s \in \mathbf{R}$ are respectively the state vector and the output of the slave system.

$A_m = \{a_{m_{ij}}\}$ and $A_s = \{a_{s_{ij}}\}$ are constant matrices, and $f_m(x_m(t))$ and $f_s(x_s(t))$ nonlinear vectors.

$V_m \in \mathbf{R}^n$ et $V_s \in \mathbf{R}^n$ are constant vectors introduced in (1) and (2).

Lot the synchronization error vector defined by

$$e(t) = x_s(t) - x_m(t) \tag{3}$$

Using (1) and (2), the error system can be described by

$$\dot{e}(t) = \left\{\begin{array}{l} A_s x_s(t) - A_m x_m(t) + f_s\left(x_s\left(t\right)\right) - f_m\left(x_m\left(t\right)\right) \\ +V_s - V_m + Bu(t) \end{array}\right. \tag{4}$$

with $B = I_{n \times n}$ and the structure of the control law $u(t)$, retained in this case, on one hand by the output feedback control law and, on the other hand, by compensation of the expression of the nonlinear observer of Luenberger represented as follows

$$u(t) = \begin{cases} (A_s - A_m)\, x_s(t) + f_m\,(x_s\,(t)) - f_s\,(x_s\,(t)) \\ -V_s + V_m + L(.)\,(y_m(t) - y_s(t)) \end{cases} \tag{5}$$

The vector $L(.) = \{l_i(.)\} \in \mathbf{R^n}$ of the following Luenberger continuous-time observer parameters gain [29]

$$L(.) = \left[\begin{array}{ccc} l_1(.) & \cdots & l_n(.) \end{array}\right]^T, l_i \in \mathbf{R}, i = 1, 2, ..., n \tag{6}$$

has to be choosen to satisfy master-slave synchronization [1, 2] i.e.,

$$\lim_{t \to +\infty} e_i(t) = \lim_{t \to +\infty} (x_{s_i}(t) - x_{m_i}(t)) = 0, \forall i = 1, ..., n \tag{7}$$

The substitution of the new control law (5) in the system (4) leads to the representation of the error system as follows

$$\dot{e}(t) = A_m e(t) - BL(.)Ce(t) + f_m(x_s(t)) - f_m(x_m(t)) \tag{8}$$

We can note that the choice of the output feedback control law $u(t)$ (5) returns the study of the synchronization of two non identical chaotic systems to two identical ones.
For several chaotic systems, the expression $f_m(x_s(t)) - f_m(x_m(t))$ can be factorized as follows, [30] and [31],

$$f_m(x_s(t)) - f_m(x_m(t)) = Q(x_m(t),\ x_s(t))e(t) \tag{9}$$

$Q(x_m(t),\ x_s(t))$ is a borned matrix with nonlinear elements.

Then, the error system can be rewritten as

$$\dot{e}(t) = A(x_m(t),\ x_s(t))e(t) \tag{10}$$

with

$$A(x_m(t),\ x_s(t)) = A_m + Q(x_m(t),\ x_s(t)) - BL(.)C \tag{11}$$

The following theorem 1, based on the use of Borne and Gentina criterion, [22] and [23], associated to the specific canonical Benrejeb arrow form matrix $A(.) = \{a_{ij}(.)\}$, [26] to [28], gives sufficient conditions of complete synchronization of slave (2) with master (1) systems.

Theorem 1: The process described by (10) converges towards zero, if the matrix $A(.)$, such that the matrix given by (11) is in the arrow form matrix, if the following conditions are satisfied

- the nonlinear elements are located only in either one row or one column of the matrix $A(.)$
- the diagonal elements, $a_{ii}(.)$ of the matrix $A(.)$ are chosen such that

$$a_{ii}(.) < 0\ \forall\ i = 1, 2,..., n - 1 \tag{12}$$

- there exist $\varepsilon > 0$, such that

$$a_{nn}(.) - \sum_{i=1}^{n-1} |a_{ni}(.)a_{in}(.)|\, a_{ii}^{-1}(.) \leq -\varepsilon \tag{13}$$

Proof: The comparison system (14) of error system (10), associated to the vectorial norm $p(z) = \left[\begin{array}{cccc} |z_1| & |z_2| & \cdots & |z_n| \end{array}\right]^T, z = \left[\begin{array}{cccc} z_1 & z_2 & \cdots & z_n \end{array}\right]^T$ [28]

$$\dot{z}(t) = M(A(.))z(t) \tag{14}$$

is such that the elements $m_{ij}(.)$ of overvaluing system matrix characteristic $M(A(.))$ are deduced from the matrix $A(.)$ ones, by substituting the off-diagonal elements by their absolute values

$$\begin{cases} m_{ii}(.) = a_{ii}(.) & \forall\ i = 1, 2, ..., n \\ m_{ij}(.) = |a_{ij}(.)| & \forall\ i, j = 1, 2, ..., n,\ \forall i \neq j \end{cases} \tag{15}$$

The error system (10) is then stabilized by the output feedback law (5), if the matrix $M(A(.))$ is the opposite of an M-matrix [28]. By the application of Borne and Gentina stability criterion, the sufficient stability conditions, for $\varepsilon > 0$, are the followings

$$\begin{cases} a_{ii}(.) < 0\ \forall\ i = 1, 2, ..., n - 1 \\ (-1)^n \det(M(A(.))) \geq \varepsilon \end{cases} \tag{16}$$

when nonlinear elements are isolated in one row of $M(A(.))$ matrix.
The development of the first member of the last condition of (16)

$$(-1)^n \det(M(A(.))) = \left(\begin{array}{l} (-1)a_{nn}(.) \\ -\sum\limits_{i=1}^{n-1} (|a_{ni}(.)a_{in}(.)|)a_{ii}^{-1}(.)(-1)^{n-1} \prod\limits_{j=1}^{n-1} a_{jj}(.) \end{array} \right) \tag{17}$$

achieves easily the proof of the theorem 1.

This theorem 1 is used, in the next section, to synchronize two coupled master-slave non identical chaotic systems.
The goal now is to synchronize (1) and (2) and, at the same time, to design a parameter update law for nonlinear observer gain L such that the error system is asymptotically stable using (12) and (13) conditions of the theorem listed below.

III. Application to non identical master-slave chaotic systems

A. Proposed observer-based synchronization for coupled Lorenz Rôssler chaotic systems

In this section, the performance of the proposed synchronization approach is illustrate for coupled the Lorenz system with the Rôssler system returning the study to two identical chaotic Lorenz systems.
Consider the master Lorenz system given by [32]

$$\begin{cases} \dot{x}_{m_1}(t) = -ax_{m_1}(t) + ax_{m_2}(t) \\ \dot{x}_{m_2}(t) = rx_{m_1}(t) - x_{m_2}(t) - x_{m_1}(t)x_{m_3}(t) \\ \dot{x}_{m_3}(t) = -bx_{m_3}(t) + x_{m_1}(t)x_{m_2}(t) \end{cases} \tag{18}$$

a, b and r are three positive parameters of the Lorenz chaotic system such that $a = 10, b = 8/3$ and $r = 28$. For this parameters, the figure 1 shows the Lorenz system as a chaotic attractor.
The description of such system can be reformulated, as previously, in the state space as

$$\begin{cases} \dot{x}_m(t) = A_m(.)x_m(t) + f_m(x_m(t)) \\ y_m(t) = Cx_m(t) \end{cases} \tag{19}$$

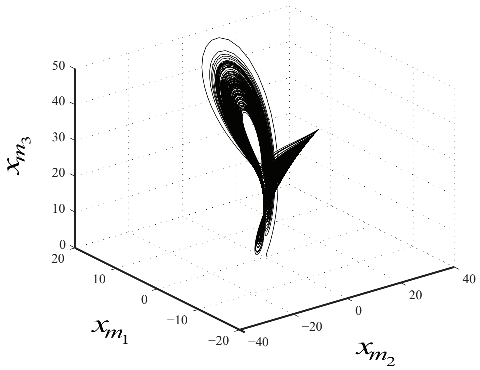

Fig. 1. Dynamic attractor of the chaotic Lorenz system

with

$$A_m(.) = \begin{bmatrix} -a & a & 0 \\ r - x_{m_3}(t) & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \tag{20}$$

and

$$f_m(x_m(t)) = \begin{bmatrix} 0 & 0 & x_{m_1}(t)x_{m_2}(t) \end{bmatrix}^T \tag{21}$$

Besides, let consider the slave Rôssler system described by [33]

$$\begin{cases} \dot{x}_{s_1}(t) = -x_{s_2}(t) - x_{s_3}(t) + u_1(t) \\ \dot{x}_{s_2}(t) = x_{s_1}(t) + \mu x_{s_2}(t) + u_2(t) \\ \dot{x}_{s_3}(t) = \rho + x_{s_3}(t)x_{s_1}(t) - \eta x_{s_3}(t) + u_3(t) \end{cases} \tag{22}$$

μ, ρ and η are three positive parameters such that $\mu = 0.398, \rho = 2$ and $\eta = 4$. For this parameters, the figure 2 shows the Rôssler system as a chaotic attractor.

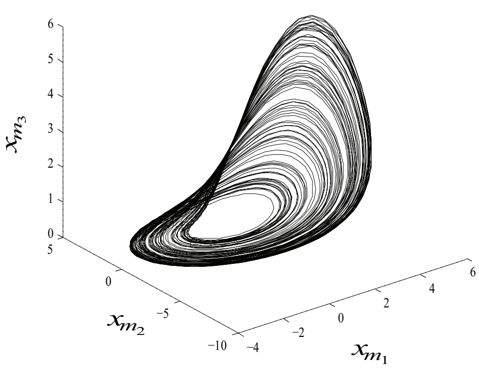

Fig. 2. Dynamic attractor of the chaotic Rôssler system

The description of such system (2) can be reformulated, also, in the state space as

$$\begin{cases} \dot{x}_s(t) = A_s x_s(t) + f_s(x_s(t)) + V_s + u(t) \\ y_s(t) = C x_s(t) \end{cases} \tag{23}$$

with

$$A_s = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \mu & 0 \\ 0 & 0 & -\eta \end{bmatrix}$$
$$f_s(x_s(t)) = \begin{bmatrix} 0 & 0 & x_{s_1}(t)x_{s_3}(t) \end{bmatrix}^T \tag{24}$$
$$V_s = \begin{bmatrix} 0 & 0 & \rho \end{bmatrix}^T$$

$x_m(t) = [\; x_{m_1}(t) \quad x_{m_2}(t) \quad x_{m_3}(t) \;]^T$ is the state vector of the master system and $x_s(t) = [\; x_{s_1}(t) \quad x_{s_2}(t) \quad x_{s_3}(t) \;]^T$ the state vector of a slave system.

The control law proposed for this slave Rôssler system such that

$$u(t) = \begin{cases} (A_m(.) - A_s)\, x_s(t) + f_m(x_s(t)) - f_s(x_s(t)) \\ -V_s + L(.)\,(y_m(t) - y_s(t)) \end{cases} \tag{25}$$

has to synchronize the non identical Lorenz and Rôssler systems.

Consider the Luenberger continuous-time observer gain

$$L(.) = \{l_i(.)\}, \forall i = 1, 2, 3 \tag{26}$$

which will return the study of the synchronization of two non identical chaotic systems to two identical chaotic Lorenz systems.

Let consider the synchronization error composant $e_i(t)$, between systems (19) and (23),

$$e_i(t) = x_{s_i}(t) - x_{m_i}(t), \;\forall i = 1,\; 2,\; 3 \tag{27}$$

leads to the error system description in the state space by

$$\dot{e}(t) = A(.)e(t) \tag{28}$$

where $A(.)$ is defined by (11).

and in the form

$$A(.) = \begin{bmatrix} -a - l_1 c_1 & a - l_1 c_2 & -l_1 c_3 \\ r - x_{m_3}(t) - l_2 c_1 & -1 - l_2 c_2 & -l_2 c_3 \\ x_{m_2}(t) - l_3 c_1 & x_{s_1}(t) - l_3 c_2 & -b - l_3 c_3 \end{bmatrix} \tag{29}$$

for $Q(x_m(t), x_s(t))$ defined by

$$Q(x_m(t), x_s(t)) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x_{m_2}(t) & x_{s_1}(t) & 0 \end{bmatrix} \tag{30}$$

It comes the instantaneous characteristic matrix $M(A(.))$ of the comparaison system

$$M(A(.)) = \begin{bmatrix} -a - l_1 c_1 & |a - l_1 c_2| & |l_1 c_3| \\ |r - x_{m_3}(t) - l_2 c_1| & -1 - l_2 c_2 & |l_2 c_3| \\ |x_{m_2}(t) - l_3 c_1| & |x_{s_1}(t) - l_3 c_2| & -b - l_3 c_3 \end{bmatrix} \tag{31}$$

The choice of correction parameters l_2, l_3, c_2 et c_3 as following

$$\begin{cases} l_2 c_3 = 0 \\ x_{s_1}(t) - l_3(.)c_2 = 0 \end{cases} \; i.e. \begin{cases} c_3 = 0 \\ l_3(.) = \dfrac{x_{s_1}(t)}{c_2} \end{cases} \tag{32}$$

isolates all the nonlinearities of $A(.)$ in it last column.

Synchronization is achieved when stabilization conditions are satisfied using the conditions *(ii)* of the theorem 1, namely

$$\begin{cases} -1 - l_2 c_2 < 0 \\ -b - l_3 c_3 < 0 \end{cases} \tag{33}$$

For this purpose, possible choices of l_2 and c_2 are the followings

$$\begin{cases} l_2 = 28 \\ c_2 = 1 \end{cases} \tag{34}$$

Finally, inequalities (12) are satisfed and (13) one becomes

$$\begin{pmatrix} (-a - l_1 c_1) - (|l_1 c_3|\,|x_{m_2}(t) - l_3 c_1|)\,(-b - l_3 c_3)^{-1} \\ -\,(|a - l_1 c_2|\,|r - x_{m_3}(t) - l_2 c_1|)\,(-1 - l_2 c_2)^{-1} \end{pmatrix} < 0 \tag{35}$$

Then, instantaneous gains l_1 and c_1, have to satisfy inequalities (35). The appropriate solution, corresponding to the matrices gain $L(.)$ et C such as

$$L(.) = \begin{bmatrix} 1 \\ 28 \\ x_{s_1}(t) \end{bmatrix} \tag{36}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \tag{37}$$

guaranties the synchronization between state vectors of the master and slave coupled non identical Lorenz and Rôssler systems.

B. Numerical simulation results

In the next, simulation results are outlined which will depict the synchronization in continuous-time system and its application to two non identical Lorenz and Rôssler chaotic systems.
The simulation results for the two coupled non identical Lorenz and Rôssler systems, without observer gains, show, in figure 3, the evolutions of states variables with different amplitudes, and figure 4 shows chaotic error dynamics when observer is desactived.

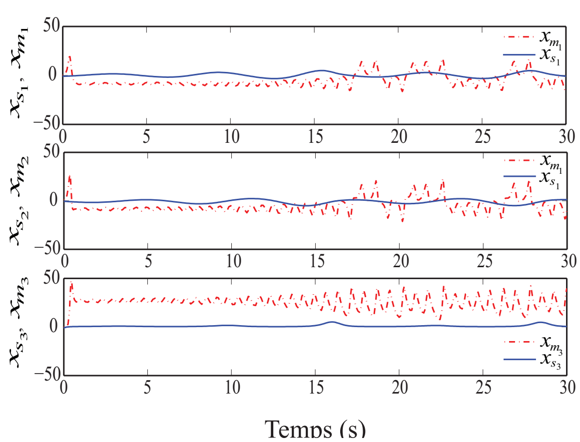

Fig. 3. Synchronization dynamics between the coupled master-slave Lorenz and Rôssler systems when the controller is desactived

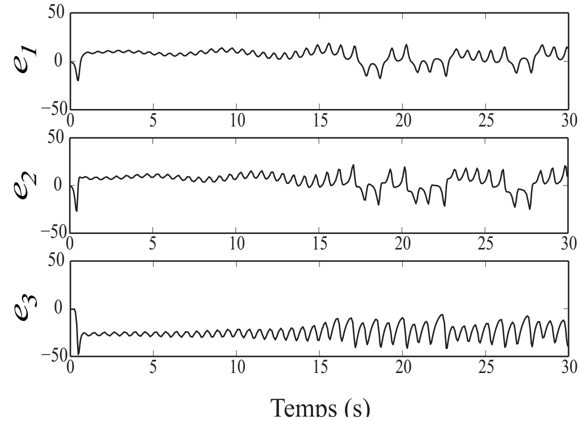

Fig. 4. Evolutions of error dynamics of the studied coupled master-slave Lorenz and Rôssler systems, when the controller is desactived

The figure 5 shows the state vector of the slave Rôssler system achieving synchronism with state vectors of the master Lorenz system, and the figure 6 the time response of the synchronization errors; one can observe that $e_1(t)$, $e_2(t)$ and $e_3(t)$ converges to zero after the activation of the proposed nonlinear observer.

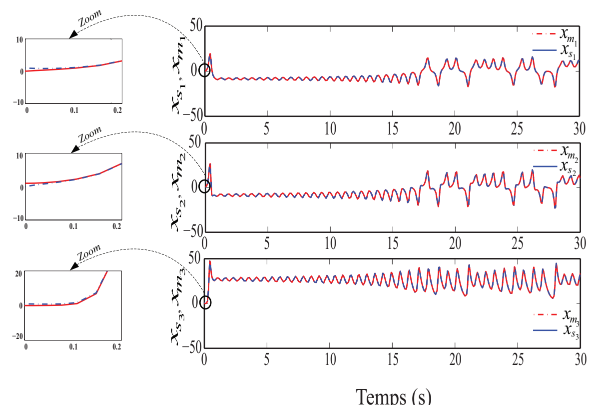

Fig. 5. Evolutions of state variables of the coupled master-slave Lorenz and Rôssler systems when proposed observer is activated

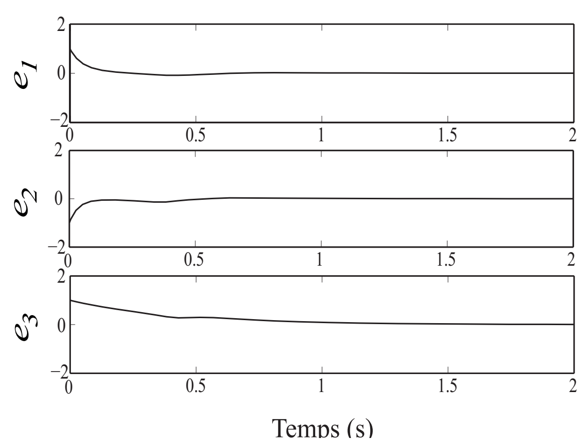

Fig. 6. Evolutions of the error dynamics of the studied coupled master-slave Lorenz and Rôssler systems when proposed observer is activated

IV. Conclusion

In this paper, suitable stabilization conditions are proposed for observer-based synchronization of chaotic continuous-time systems. Two main results have been obtained. The first one guaranties synchronization between master and slave systems based on the use of a Borne and Gentina technique associated to the Benrejeb arrow form matrix. The second ones, it is worth noting that the proposed nonlinear observer is easy to apply to two non identical master-slave chaotic systems. Numerical simulations illustrate the influence of the control gains parameters on the dynamic performances then the efficiency of the proposed approach.

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