Enhancing Performance of multi-rate WLANs: Hopfield Neural Network Approach

Qiang Ma, Abdullah Al-Dhelaan, Mznah Al-Rodhaan

Abstract—In a multi-rate 802.11 WLAN environment, the trade-off between users’ fairness and network throughput might be unacceptable. In this paper, we will design a new intuitive mathematical model called simplified coefficient of variation (SCV) model that would closely reflect our topic. SCV could optimize and enhance the trade-off problem through controlling the power of Access Points. Since our topic is a NP-hard problem, we use Hopfield Neural Network solution to solve our SCV model in a practical scenario. The simulation gives excellent results indicating our model is efficient and superior to an existing method through a comparison analysis. In addition, we use software SAS to further reveal the relationships among the three indicators to illustrate the essence of our algorithm and an existing algorithm.

Keywords—coefficient of variation, dynamic function, energy function, Hopfield neural network, optimization, power control

I. INTRODUCTION

The rapid development of the Internet and the progress of wireless technology are making wireless networks play an increasingly important role in many areas. This is particularly true for the IEEE 802.11 wireless local area network (WLAN) technology. With its development, the increasing demands of service quality and a sharp rise in the number of user groups, the problem has become heavily concentrated in some places such as offices, meeting rooms and other crowded places. In this case, many access points may be allocated, but without an overall channel or power planning and this will result in a large amount of co-channel interference, load imbalance, and network throughput decline, which will degrade the user experience. As it is one of the hot spots in the wireless area, research institutions, academic institutions, and commercial companies, have developed many valuable solutions to solve the problems, but these solutions cannot be applied easily.

Currently, most research on WLAN technology is mainly focused on the following two aspects:

(a) Wireless channel planning. Through different methods, the limited channel resources will be reasonably assigned to all access points (APs) to make it possible to reduce co-channel interference and network overhead in order to improve overall network throughput.
(b) Power control to achieve load balancing. Power control mainly uses the proportional relationship of AP signal strength and the power of the AP selected by the user accessing the wireless network, increasing or decreasing the power to adjust the signal strength of the AP. It thus changes the access topology of the user-AP in the network in order to reduce the scheduling overhead and improve load balance etc.

This article involves both aspects above. The rest of the paper is organized as follows: related work is discussed in Section II, and Section III shows the motivation. After that, a brief introduction of Hopfield Neural Network as background will be provided in Section IV. Then we start to explain the new model SCV and apply the Hopfield Neural Network, the simulation using Matlab is explained in Section V. After this, we give comparison and SAS analysis in section VI and then draw the conclusions in Section VII.

II. RELATED WORK

According to IEEE 802.11, a high-density WLAN deployment environment offers a short distance between APs and users. In this case, each user will connect with the AP by the strongest received signal strength indicator (RSSI) by default. We know that the users are not uniformly distributed in an area, which makes some APs connect more users than the other APs. This will produce the load imbalance problem, as some APs are hungry while some APs are overloaded. This situation results in unfair use of resources.

As a part of our research, the basic solution has been introduced in [1].

In order to improve the Quality of Service, the authors in [2] provided an enhanced method called DCF which providing weighted fairness among multiple priority classes in 802.11-based WLAN to properly control the transmission probability of nodes. The method was expected to achieve not only the weighted fairness but also maximize the system throughput and minimize the frame delay at the same time.

The authors in [3] proposed an Improved Power Control MAC (IPCM) protocol which improves the throughput and yields energy saving. The protocol adopted optimal transmission power to send all kinds of packets in order to save the energy, which also made spatial reuse of the wireless
channels, and achieved the maximum throughput compared to the other schemes.

The authors in [4] proposed a new protocol based on integration of WLAN and CDMA2000 networks. The protocol allowed mobile nodes to send request and get reply using two different networks simultaneously to improve the efficiency and throughput. The simulation results showed that the mobile nodes had higher data rates and efficiently utilized network resources compared to single network.

The popular 802.11 MAC protocol provides equal transmission chances to all users, which may achieve throughput-based fairness if all users have the same frame size during a cycle [5]-[8]. Recent studies have shown that time-based fairness is much better than throughput-based fairness in multi-rate WLANs [9].

So far, we have two fairness criteria factors that are widely used in network management: proportional fairness [7] that allocates bandwidth to users in proportion to their bit rates to maximize the sum of the bandwidth utilities of the users, and max-min fairness [10], which allocates throughput as equally as possible through maximizing the minimum throughput. Proportional fairness and time-based fairness are equivalent in multi-rate WLANs when all users have the same weight [11]. The equivalence of max-min fairness and throughput-based fairness under the same condition (integral association) was proved in [12].

The authors in [13] proposed a new algorithm called Power Control for AP (PCAP) to optimize the network utility by maximizing the average and minimizing the variance of the AP utility, the result directly maximizing the “throughput” as its target, and then the author started to calculate the “J” (Jain’s fairness index [14]). The author did not mention the “J” at the beginning, though the result showed significant improving of trade-off. We will analyze the relationship between these two variables.

According to IEEE802.11, AP transmission powers can be changed in an allowable range, this technique is called power control. Some previous studies, such as [15]-[16], have assumed that the user-AP associated topology will not change when adjusting the power of APs, so this assumption is not the reality. On the contrary, some papers have noticed this phenomenon and developed techniques called cell breathing [17].

In [18], a variable polyhedron genetic algorithm (GA) was proposed. To tackle the challenges of access points (AP) service cheating and AP service loophole, which not only provides an AP service availability guarantee but also yields a near-optimal beacon range for each AP when the number of evolutions is large enough. Their simulation study indicates that the algorithm is superior over the default 802.11 AP association model in terms of load-balancing and network throughput enhancement.

The authors in [19] proposed an algorithm that transformed the problem into a monotonic optimization problem. It is solved with geometric programming [20], but it is not suitable for the low Signal to Interference Ratio (SIR) case.

The authors in [21]-[23] provided inspiration and guidance for us to compare our problem with TSP problem.

In [24], the authors proposed a centralized algorithm called Non-Linear Approximation Optimization for Proportional Fairness to derive the user-AP association via relaxation, and proposed a distributed heuristic algorithm called Best Performance First, which provides an AP selection criterion for new comers.

In [25], the authors jointly considered the channel allocation and AP association, aims to maximize the system performance in terms of throughput and fairness. They introduced two penalty functions to relax the constraints, and a discrete particle swarm optimization algorithm to solve the problem.

In this paper, the contributions are modeling and analysis. The contributions are listed as follows: 1). we describe the “trade-off” using “J of user” and “J of AP”, which refer to the fairness of users and fairness of APs respectively. Then we design our target function using many skills to deal with those complex formulas, after that we use our simplified coefficient of variation (SCV) model, which is a clear mathematical function to solve such trade-off problem. This is the key contribution of our paper. 2). we define the problem as an informed search NP-hard problem and apply Hopfield Neural Network algorithm to solve the SCV model. 3). we use multi-channel allocation to improve the transmission rate. 4). we use Statistical Analysis System (SAS) for analysis to reveal the relationships of three indicators and the essence of algorithms. 5). SCV opens a door for many AI algorithms; it is a bridge between Network & AI.

III. MOTIVATION

A. The Essence of PCAP: Throughput

From our SAS analysis in Fig.6, three indicators (Juser: J of user; Jap: J of AP; Tpt: relative Throughput) show that J of AP can represent Throughput (value>0.8, so it is highly linear related).

Through our Statistics calculation, PCAP focus on J of AP only, which means it only focus on Throughput. This is a deficiency of Target Function design, which is not well reflecting our topic.

B. The Essence of SCV

The problem is defined as an Informed Search problem from AI perspective. It is a NP-hard problem since we apply a practical scenario that includes 20 APs, each AP has 10 levels of power, so the state space of the problem will be 10^{20}, making it neither solvable nor verifiable in polynomial time, which makes it as a NP-hard problem.

From the computation theory, we know that we cannot get an accurate solution. Compared with other NP-hard problems such as TSP (Traveling Salesman Problem), we get some heuristic methods. Since existing models are complicated by using a definition of utility and disturbed by many parameters such as channel gain, those models are not clear enough to apply informed search techniques, so first we need to build a clear, simplified model SCV, and then apply the Hopfield Neural Network to solve the model.
Since our topic is: “J of user (fairness of users) & Throughput”, which means to make balance between these two parameters. Obviously the two parameters have different units, then we have to convert the “Throughput” to “J of AP” (already explained, it can represent Throughput, with high linear relation).

Then our SCV gives a new designed target function: 
\[ F = \frac{1}{J_{\text{users}} - 1} + \alpha \frac{1}{J_{\text{APs}} - 1} \], which reflects the balance of two parameters (J of user & Throughput), and we will rewrite to get its final form \( f \).

### IV. HOPFIELD NEURAL NETWORK

In 1982, Hopfield artificial neural network model was proposed. The author introduced the concept of the energy function in an artificial neural network and gave a stability criterion to develop a new method of associative memory and calculation optimization of an artificial neural network. Fig. 1 shows a model of the Hopfield neural network.

Hopfield network is divided into two network models: Discrete Hopfield Neural Network (DHNN) and Continuous Hopfield Neural Network (CHNN). As shown in Fig.1, each neuron can be represented using a nonlinear dynamic equation; \( n \) neural nodes constitute a set of simultaneous nonlinear differential equations (continuous) or difference equations (discrete). Depending on the selection of network parameters, the system can converge to a stable state (attractor network) or an oscillation state, or it can enter a chaotic state. The network weights according to the optimization problem are given beforehand or by the Hebb rule set.

In a dynamic system, the learning procedure is similar to a feed forward neural network, after forming a network, prediction becomes an easy job. Some network attractors have been formed through learning. When faced with a new input vector, we can get a corresponding steady state, the attractor likes an output vector. The Hopfield network has an associative memory function. In addition, for certain types of problems, such as TSP, we have to design an energy function and then get a differential or difference equation to decide the architecture of the network and then initialize the weight matrix. The network begins its iteration through a learning procedure until it reaches a maximum number of iterations according to its configuration. The focus of such networks is of a human-designed energy function in which a neural network spans the learning process. Thus, associative memory and optimization calculations have different algorithms based on the same Hopfield neural network structure.

The optimization application is to design the minimum value of the target function, then derive energy function and dynamic function. When the energy function converges to the minimum value, we can get an optimal solution. The algorithm is as follows:

#### Hopfield Neural Network Combinatorial Optimization Procedure:

1. Initialize NN model: fix the number of neurons and the set of states (the states of neuron permutation matrix) to initialize the neural network, the parameter values \( A, B, C \), etc., the sampling time, and set the maximum number of iterations “L” and the counter count=0.
2. Design energy function: constraint items + optimization items (target function).
3. Derive dynamic equations with the weight matrix of the neural network.
4. Compute the updated value for all the nodes.
5. If count <L then go to step (4); otherwise output Vector

#### V. MODEL DESIGN AND SIMULATION

Now we are going to explain our SCV model and apply it in Hopfield neural network.

**A. The way APs attract users**

The user will select the strongest received signal strength indicator (RSSI) as default. In the model [26], 
\[ RSSI = \alpha \frac{P}{X^3} \]

where “\( \alpha \)” is a constant factor, “\( P \)” is received power, “\( X \)” is distance between user and selected AP, while “\( \alpha \)” has different value in different scenarios, generally between 1.6 and 6.5 [27].

The formula only determines the association matrix of User-AP. In practice, the general power range of the AP is 10dBm ~ 30dBm, i.e. 1mw ~ 1w, here we adopt \( \alpha = 3 \) for indoor case. From the formula, the value of “\( \alpha \)” does not affect the association results, to simplify the mathematical form, we take \( \alpha = 1 \), so our model adopts a simplified form:

\[ RSSI = \frac{P}{X^3} \]  \hspace{1cm} (1)

**B. Study the SINR\( [r_{ij}] \) of the user[i]**

Assuming the user[i] connects to AP[j], the power of AP[j] is \( P_j \). Where “\( g \)” are channel gains, \( A_i \) is a set of all APs within the same channel of AP[j], \( N_{0j} \) is an additive white Gaussian noise generated by AP[j].

\[ r_{ij} = \sum_{k \in A_i \kappa \neq j} \frac{g_{ij} P_j}{g_{ik} P_k + N_{0j}} \]  \hspace{1cm} (2)

It is worth noting that \( N_{0j} \) can be adjusted to an exact value [28]-[29]. So we can set a constant \( \mu > 0 \),

\[ \mu = \sum_{k \in A_i \kappa \neq j} \frac{g_{ij} P_j}{g_{ik} P_k + N_{0j}} \]  \hspace{1cm} (3)
then the value of $N_{ij}$ should be adjusted as:

$$N_{ij} = \frac{g_{ij}}{\mu} - \sum_{k=\delta_i(\delta_j)} g_{ik} p_k.$$  

### C. Study the relationship between user[i]'s transmission rate $v_i$ and its SINR $[r_{ij}]$

Table 1. $v_i$ - $r_{ij}$ relationship

<table>
<thead>
<tr>
<th>$r_{ij}$ (dB)</th>
<th>2.4</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
<th>5.5</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_i$ (Mbps)</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
</tr>
</tbody>
</table>

From Table 1 in [13], we see the monotonically increasing relationship between the two variables. Here we might assume that two variables meet the linear relationship as an approximation, $v_i = \beta r_{ij}$, $\beta > 0$ is a constant of proportionality.

Then connect this to (2) and (3) we have:

$$v_i = \beta r_{ij} \mu = \lambda p_j$$  \hspace{1cm} (4)

So $\lambda$ is a constant: $\lambda = \beta \mu$  \hspace{1cm} (5)

### D. Study the effective speed of $v_i$

Since many users connect with AP[j], let $N[j]$ denote the total number of users that connect with AP[j]. Because the users are time-based share the chance of AP[j], so the effective speed of $v_i$ is:

$$v_i = \frac{\lambda p_j}{N[j]}$$  \hspace{1cm} (6)

From this formula we know that it is better to decrease the $N[j]$, and increase the $p_j$ and $\lambda$.

### E. Study the AP power

According to the simulation result in [13], we know that usually 10 levels of AP power will be enough to achieve a good result. Therefore, in our model, the $p_{max}$ and $p_{min}$ have relationship as following: $p_{max} p_{min}$=10, $p_{max}$ will be the basis of calculation, since we need to increase the $p_j$, so the 10 power levels are in Table 2.

Table 2. level-value relationship

<table>
<thead>
<tr>
<th>level</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>$p_{min}$</td>
<td>$2p_{min}$</td>
<td>...</td>
<td>$10p_{min}$</td>
</tr>
</tbody>
</table>

Note here the unit of power is “mw”, not “dBm”. Since $p_j$=$p_{min}$+$l_j$ ($l_j$=1,2,...10), note that $l_j$ denotes the level of AP power, so the formula (6) can be rewritten as follows:

$$v_i = \frac{\lambda p_{min} l_j}{N[j]} = \lambda p_{min} \frac{l_j}{N[j]}$$  \hspace{1cm} (7)

Let $M$ be the total number of users and $N$ be the total number of APs. From statistics, we know that the expectation of $\bar{V}_i$ for all users is denoted as $E(\bar{V}_i)$, and variance of $\bar{V}_i$ for all users is denoted as $S^2(\bar{V}_i)$ . We have the following (i=1,2,...M; $j=1,2,...,N)$:

$$E(\bar{V}_i) = \lambda p_{min} E(\frac{l_j}{N[j]})$$  \hspace{1cm} (8)

$$S^2(\bar{V}_i) = (\lambda p_{min})^2 S^2(\frac{l_j}{N[j]})$$  \hspace{1cm} (9)

Let $b[i]$ denote the average transmission speed from user[i] to AP[j], we have $b[i]$=$\bar{V}_i$. Moreover, let $U[j]$ denote the transmission speed from the AP[j] to backbone. The Expectation of $b[i]$ is denoted as: $E(b[i])$, and Variance of $b[i]$ is denoted as: $S^2(b[i])$, and Expectation of $U[j]$ is denoted as: $E(U[j])$, and Variance of $U[j]$ is denoted as: $S^2(U[j])$, so continue we have formulas as following:

$$E(b[i])=E(v_i)=\lambda p_{min} E(\frac{l_j}{N[j]})$$  \hspace{1cm} (10)

$$S^2(b[i])=S^2(\bar{V}_i)=(\lambda p_{min})^2 S^2(\frac{l_j}{N[j]})$$  \hspace{1cm} (11)

$$E(U[j])=\lambda p_{min} E(l_j)$$  \hspace{1cm} (12)

$$S^2(U[j])=S^2(l_j)=(\lambda p_{min})^2 S^2(l_j)$$  \hspace{1cm} (13)

Let cvusers denote the coefficient of variation of transmission speed of all users and cvAPs denote the coefficient of variation of transmission speed of all APs, we have:

$$cvusers^2 = S^2(l_j) = \frac{(\lambda p_{min})^2 S^2(l_j)}{E(l_j)^2} = \frac{S^2(l_j)}{E(l_j)^2} = \frac{\sum_{j=1}^{M} \frac{l_j^2}{N[j]^2} - 1}{\sum_{j=1}^{M} \frac{l_j^2}{N[j]^2}} - 1$$  \hspace{1cm} (14)

$$cvAPs^2 = S^2(l_j) = \frac{(\lambda p_{min})^2 S^2(l_j)}{E(l_j)^2} = \frac{S^2(l_j)}{E(l_j)^2} = \frac{\sum_{j=1}^{N} \frac{l_j^2}{N[j]^2} - 1}{\sum_{j=1}^{N} \frac{l_j^2}{N[j]^2}} - 1$$  \hspace{1cm} (15)

Note, here we adopt the definition of J in [13], where we have $J = \sum_{i=1}^{n} \frac{S^2(x_i)}{n} - \frac{S^2(l_j)}{n}$. This is the relationship between J and the square of coefficient of variation.

### F. Cost function $f$ construction

In our topic, we need a function that can describe the tradeoff between fairness of users and throughput of network. In [13], the author’s algorithm is divided into two steps: increase throughput of network. They are equal to decreasing cvusers or cvAPs. So increasing J of users is equal to decreasing cvusers$^2$.
Let $F$ denote a target function as follows: $F = cv\text{users} + o(cv\text{APs})$, $\omega$ is weight proportion factor, it is very important reflecting our requirement how to make the balance between fairness and throughput, it gives us a quantifiable indicator.

Here we do some mathematical derivation to illustrate how we get a reasonable value of $\omega$. Considering the static grouping problem: $m$ numbers are average divided by $n$ groups, therefore each group has $m/n$ numbers. Given that the expectation of total numbers is $\sigma$, and their variance is $s^2$, so for group[i] we have:

$$E(\text{group}[i]) = E\left(\sum_{j=1}^{N} \text{number}[j]\right) = \sum_{j=1}^{N} E(\text{number}[j]) = \frac{m}{n} \sigma$$

$$S'(\text{group}[i]) = S'\left(\sum_{j=1}^{N} \text{number}[j]\right) = \sum_{j=1}^{N} S'(\text{number}[j]) = \frac{m}{n} s^2$$

$$cv\text{numbers}^2 = \frac{s^2}{\sigma}$$

$$cv\text{groups}^2 = \frac{S'^2(\text{group}[i])}{E(\text{group}[i])} = \frac{\frac{m}{n} s^2}{\frac{m}{n} \sigma} = \frac{1}{\sigma} s^2 = \frac{1}{\sigma} cv\text{numbers}^2$$

(16)

So it means $cv\text{groups}^2$ is much smaller than $cv\text{numbers}^2$, comparing this example to our function $F$, in function $F$ we should amplify the small part since two parts have relationship. So we decide to give value to $\omega$, let $\omega = M/N$.

$$F = cv\text{users} + o(cv\text{APs}) = \frac{M \sum_{j=1}^{N} l_j^2}{(\sum_{j=1}^{N} l_j)^2} - 1 + \frac{M \sum_{j=1}^{N} l_j^2}{N \left(\sum_{j=1}^{N} l_j\right)^2} - 1$$

$$= M \left(\sum_{j=1}^{N} l_j^2\right) - (1 + \frac{M}{N}) \left(1 + \frac{M}{N}\right)$$

(17)

Wherein:

$$f = \frac{\left(\sum_{j=1}^{N} l_j^2 \sum_{j=1}^{N} I_j^2\right)}{\left(\sum_{j=1}^{N} I_j\right)^2} = \frac{\sum_{j=1}^{N} (I_j (1 + \frac{1}{N}))}{\sum_{j=1}^{N} l_j^2}$$

(18)

Note that $M$ and $N$ are constants as defined before. $M$ is total number of users; $N$ is total number of APs. When “$F$” goes to minimum, it is equal to “$f$” goes to minimum. Therefore, (18) will be our simplified target function, to achieve the purpose of the tradeoff between Fairness (users) and Throughput (network).

G. Throughput

From formula (12) we know that:

$$Throughput_{real} = \sum_{j=1}^{N} U[j] = \lambda p_{\text{min}} \sum_{j=1}^{N} l_j = \lambda p_{\text{min}} Throughput_{relative}$$

(19)

$$Throughput_{relative} = \sum_{j=1}^{N} l_j$$

(20) Since the $\lambda p_{\text{min}}$ is constant, we use $Throughput_{relative}$ to represent $Throughput_{real}$.

H. Hopfield neural network design & simulation

In this part, we are going to place a total number of $N=20$ APs on a 4 by 5 grid, with each AP on a grid point. The coverage area of each AP can cross the whole area. The distance between two adjacent APs is set to 100 meters. The maximum transmission power of each AP is set to 20dBm (100mw), and so according to our model, the minimum transmission power of each AP is set to $100/10=10$mw=10dBm.

We arrange $M=200$ users randomly distributed in the whole area. According to [30], a separation of four channels can be used without reducing the performance, so the possibilities could be opened to channels 1, 5, 9 and 13. In this paper we decide to use these channels in order to get a bigger $Throughput_{real}$

Let $AP_j \rightarrow Ci$ denote $AP_j$ using channel $i$, we use 1, 5, 9, 13 these channels to configure the network as in Table 3.

<table>
<thead>
<tr>
<th>$AP_1 \rightarrow C1$</th>
<th>$AP_2 \rightarrow C9$</th>
<th>$AP_3 \rightarrow C1$</th>
<th>$AP_4 \rightarrow C9$</th>
<th>$AP_5 \rightarrow C1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AP_6 \rightarrow C5$</td>
<td>$AP_7 \rightarrow C13$</td>
<td>$AP_8 \rightarrow C5$</td>
<td>$AP_9 \rightarrow C13$</td>
<td>$AP_{10} \rightarrow C5$</td>
</tr>
<tr>
<td>$AP_{11} \rightarrow C9$</td>
<td>$AP_{12} \rightarrow C1$</td>
<td>$AP_{13} \rightarrow C9$</td>
<td>$AP_{14} \rightarrow C1$</td>
<td>$AP_{15} \rightarrow C9$</td>
</tr>
<tr>
<td>$AP_{16} \rightarrow C13$</td>
<td>$AP_{17} \rightarrow C5$</td>
<td>$AP_{18} \rightarrow C13$</td>
<td>$AP_{19} \rightarrow C5$</td>
<td>$AP_{20} \rightarrow C13$</td>
</tr>
</tbody>
</table>

Similar with the TSP problem, we have to design an energy function as our target function, and then derive dynamic function. When the energy function converges to the minimum value, we can get an optimal solution. Neural network structure is in Table 4.

<table>
<thead>
<tr>
<th>$AP_1$</th>
<th>$v_{i1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AP_j$</td>
<td>$v_{ji}$</td>
</tr>
<tr>
<td>...</td>
<td>$v_{Nj}$</td>
</tr>
</tbody>
</table>

N=total number of APs. $v_{ji}=0$ or 1. Total number of neurons: $N=10=20\times10=200$.

Constraint Item: each row is only allowed to have one “1”, the others should be “0”.

Optimization Item: related to $f$ in (18).

Energy Function:
\[
E = \frac{A}{2} \sum_{j=1}^{N} \left( \sum_{i=1}^{10} v_{ji} - 1 \right)^2 + \frac{B}{2} \left( \sum_{j=1}^{N} \left( \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \right)
\]

\[
\frac{\partial E}{\partial v_{ji}} = A \left( \sum_{i=1}^{10} v_{ji} - 1 \right) + B \left( 1 + \frac{1}{N[j]} \left( \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \right)
\]

(21)

Derived Dynamic Function:

\[
\frac{\partial E}{\partial v_{ji}} = A \left( \sum_{i=1}^{10} v_{ji} - 1 \right) + B \left( 1 + \frac{1}{N[j]} \left( \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \right)
\]

\[
- \frac{B}{2} \left( \sum_{j=1}^{N} \left( \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \right)
\]

\[
\left( \sum_{i=1}^{10} \left( \sum_{j=1}^{N} \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \right)
\]

(22)

Two points in programming need to be mentioned, let

\[
t_1 = \left( \sum_{i=1}^{10} v_{ji} - 1 \right) \\
t_2 = (1 + \frac{1}{N[j]}) \\
t_3 = 2 \left( \sum_{i=1}^{10} i \cdot v_{ji} \right)
\]

\[
t_4 = \left( \sum_{i=1}^{10} \sum_{j=1}^{N} i \cdot v_{ji} \right)^2 \\
t_5 = \sum_{i=1}^{10} \left( \sum_{j=1}^{N} \sum_{i=1}^{10} i \cdot v_{ji} \right)^2
\]

\[
t_6 = 2 \left( \sum_{j=1}^{N} \sum_{i=1}^{10} i \cdot v_{ji} \right)^2 \\
t_7 = \left( \sum_{j=1}^{N} \sum_{i=1}^{10} i \cdot v_{ji} \right)^4
\]

(1). \( t_1 \) is a matrix(20,10). \( t_2 \) is a column vector(20,1), repeating it to 10 columns and then we get a matrix \( V_2(20,10) \). \( t_3 \) is a column vector(20,1), repeating 10 times and then we get a matrix \( V_3(20,10) \). The element in matrix \( V_2(20,10) \) should multiply the element at the same position in matrix \( V_3(20,10) \) and then we get a matrix \( V_23(20,10) \). \( t_4, t_5 \) and \( t_7 \) are real numbers. \( t_6 \) is a row vector(1,10), repeating it to 20 rows and then we get a matrix \( V_6(20,10) \). So \( \frac{\partial E}{\partial v_{ji}} \) is a matrix (20,10).

(2). When there is no user attracted by AP, the \( N[j] \) and \( L_j \) will be “0”, namely whatever the power of AP is, if it doesn’t attract any user, its throughput will be “0”.

VI. RESULT ANALYSIS

In this section, we will explain the simulation results.

A. Explanation of parameters

We assume constraint Item parameter \( A=2 \) and optimization Item parameter \( B=2 \). We can increase \( A \) or \( B \) when an item is not satisfying our needs. If the hit ratio is low, then we can increase \( A \), and if we want to enhance \( J \) and Throughput, etc., we can increase \( B, u_0=0.01 \) and \( \text{step}=0.001 \); these 2 values are better to be set small, otherwise the neural network will converge quickly without a legal solution, because they control the speed and quality of convergence. The number of iterations \( K=1000 \). See the results in Fig. 2 and Fig.3.

Here we keep the parameters’ value, only change \( \text{step}=0.005 \), run again, see results in Fig. 4 and Fig. 5.
B. Simulation analysis

(1). We select two groups of results to compare the difference when changing the parameter values, we can see only change one parameter “step” from 0.001 to 0.005, the results will be different. Obviously, the second group (Fig. 4&5) is better.

(2). We need to run the same data set multiple times using the same parameters and select the average to represent the results. In addition, we have to search for the most suitable parameters. We know that the maximum of \( \text{Throughput}_{\text{relative}} \) is \( 10 \times 20 = 200 \), but it will never be achieved at least because of users’ distribution.

(3). All parameter values can affect the result: parameter A controls the number of legal solutions, namely the hit ratio. If the hit ratio is low, then we can increase A. We can also increase B if we want to enhance J, Throughput, etc. In our experiment, it is suitable to set \( A=B=2 \). When the hit ratio almost reaches 80% or higher, it is suitable to set \( u_0=0.01 \) and step=0.005. The number of cycles \( K \) can be set 500 or higher.

From the two groups of results, we can see that when we increase the step from 0.001 to 0.005, the J of AP and Throughput grow significantly and the energy decreases significantly. In each group of results, the energy plot monotonically decreases, which is consistent with the energy theory of the Hopfield neural network and our target cost function \( f \) in (18). This proves that the model is logical and effective.

(4). Calculation Matrix and Explanation Matrix: A Calculation Matrix is actually involved in the calculation of the neural network, and the value of each neuron can be a non-integer because of the transfer function. When converted by transfer function, the value of each neuron should be 0 or 1, such matrix is called Explanation Matrix. Therefore, in many cases the Calculation Matrix changes while the Explanation Matrix does not. This is why sometimes the plot is shown as non-changing because the plot is derived from the Explanation Matrix.

(5). From the Figures, administrators can select a satisfying configuration according to their requirement of fairness and throughput.

C. SAS analysis

We use the samples from experimental data to study the correlation coefficients among these indicators. Wherein Juser denotes J of user, Jap denotes J of AP, Tpt denotes \( \text{Throughput}_{\text{relative}} \), cost denotes the \( f \) in (18).

Fig.6 shows that at alpha=0.05 significance level, all the p-values are less than 0.05, we reject the \( H_0 \) and accept \( H_1 \) that these variables are linearly related, wherein the Tpt-(Jap, Juser) have highly significant linear correlations, while correlations of Jap-Juser is weak. We compared the degree of concentration of those data points in Fig. 7&8. It is clear that data points are more concentrated in Fig. 8. This means the linear correlation of Tpt-Jap is much higher than the linear correlation of Tpt-Juser, which also proves the effectiveness of SCV model (coefficient of Tpt-Jap>0.8, Tpt-Juser=0.17, so it is more effective to use J of AP whereas not J of user to represent the throughput).
D. Comparison analysis

We select average case in Fig.4, at the 600th time, the J of User is almost equal to 0.67, and corresponding J of AP is almost equal to 0.9, the \( \text{Throughput}_\text{relative} \) is almost equal to 138, since its maximum value is 200 as mentioned before, then the throughput of the network is equal to 138/200=69% of the network bandwidth. Moreover, the corresponding cost of \( f \) is almost equal to 0.063.

Table 5. The Statistics of the Results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Average AP Utility (( U^P ))</th>
<th>AP Utility Variance (( \sigma^2 ))</th>
<th>Total Network Utility</th>
<th>Jain’s Fairness Index</th>
<th>Average Power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCAP</td>
<td>3.82 x 10^9</td>
<td>2.98 x 10^9</td>
<td>117.42</td>
<td>0.90</td>
<td>17.15</td>
</tr>
<tr>
<td>MARL</td>
<td>2.67 x 10^9</td>
<td>5.00 x 10^9</td>
<td>102.30</td>
<td>0.65</td>
<td>17.28</td>
</tr>
<tr>
<td>SR</td>
<td>1.46 x 10^9</td>
<td>12.80 x 10^9</td>
<td>96.66</td>
<td>0.75</td>
<td>18.06</td>
</tr>
<tr>
<td>SSF</td>
<td>1.11 x 10^9</td>
<td>27.31 x 10^9</td>
<td>79.59</td>
<td>0.37</td>
<td>20.00</td>
</tr>
</tbody>
</table>

Here we want to compare our solution with PCAP in [13], we can see the above Table 5 from [13], since we use different definitions to denote throughput of AP and throughput of network, we have to use an indirect method to illustrate some issues.

According to [13], we can transfer and calculate their J of AP and their throughput percentage of network bandwidth:

\[
\text{cvAP}^2 = \frac{S^2(U(j))}{E(U(j))} = \frac{\log(2.98 \times 10^9)}{(\log(3.82 \times 10^9))^2} = 0.1 = \frac{1}{J_{\text{up}}}, \quad 1
\]

so their \( J_{\text{up}} = 0.9 = J_{\text{users}} \)

(24)

And we have:

\[
U \leq n \log(U^P) = 20 \log(3.82 \times 10^9) = 191.64 = U_{\text{max}}
\]

(25)

then their throughput percentage of network bandwidth is:

\[
\frac{U_{\text{network utility}}}{U_{\text{max}}} = \frac{117.42}{191.64} \approx 61.3\%
\]

(26)

In Fig. 4, our J of AP is equal to theirs in (24), from the throughput point of view, our throughput percentage of network bandwidth is 69%>61.3% in (26), so our method is better than PCAP. However, from the fairness of users (J of user) point of view, PCAP is better than ours since 0.67<0.9 in (24).

According to (17), we convert (24) into our function \( F \), we have:

\[
F_{\text{PCAP}} = \left(1 - \frac{1}{J_{\text{max}}}ight) + \left(\frac{M}{N}\right)\left(1 - \frac{1}{J_{\text{up}}}ight)
\]

(27)

\[
F_{\text{SCV}} = \left(1 - \frac{1}{J_{\text{max}}}ight) + \left(\frac{M}{N}\right)\left(1 - 0.9\right) = 2.17
\]

(28)

So the overall performance depends on the requirement of administrators, what indicator they most concern. If we define the value of “\( F \)" as the overall performance criteria of algorithm, note smaller “\( F \)" is better, then from (27) and (28) we know that our SCV model is much better than PCAP. The above comparison analysis result is in Table 6.

Table 6. Comparison Result

<table>
<thead>
<tr>
<th></th>
<th>PCAP</th>
<th>SCV-hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>J of user (↑win)</td>
<td>0.9</td>
<td>0.67</td>
</tr>
<tr>
<td>J of AP (↑win)</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Throughput % (↑win)</td>
<td>61.3%</td>
<td>69%</td>
</tr>
<tr>
<td>Function “( F )” value (↓win)</td>
<td>2.17</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Theoretically, our design of target function “\( F \)” in (17) is more simple and rational than PCAP algorithm, since we joint consider the J of user and Throughput (represented by J of AP), we regard them as two variables to reflect our topic, which is a balance problem. While the target of PCAP is the Throughput, the author used two sub-algorithms to achieve J of AP only, and then got their by-product: J of user.

Technically, our SCV math model is a door that leads this problem to AI algorithms. The clear target function “\( F \)” is easy to be applied to other AI algorithms, while PCAP cannot.

VII. CONCLUSIONS

The objective of this paper is to improve the trade-off between user fairness (J of user) and network throughput (represented by J of AP) via power control in multi-rate WLANs.

In this article, we first construct a new simplified model called SCV. The goal of the model is to derive a target function “\( F \)” in (17) and its simplified form “\( f \)” (18) as our key foundation. Then we use Hopfield neural network to solve this model, we conduct a simulation in Matlab. After that we give analysis of our SCV model and simulation results which confirm that our model is efficient and superior to PCAP in some aspects and overall performance under a new criteria designed for such specific problem. In addition, based on the data samples from the state space, we use SAS to conduct correlation analysis mainly among three indicators, and reveal their relationships.

SCV (Target function F) opens a door for many AI algorithms to apply in this problem; it is a bridge between Network & AI.

Our future work is to derive a more accurate target function, and adjust the values of parameters to find more suitable combination so that to improve the results. In addition, we are working on other AI solutions based on SCV model.

REFERENCES


Qiang Ma received the M.S. degree in Computer Science from Lanzhou University, Lanzhou, China, in 2009. Now he is pursuing the Ph.D. degree in Computer Science at King Saud University, Riyadh, Saudi. His current research interests include network, artificial intelligence and data mining.

Abdullah Al-Dhelaan received the B.S. degree in from King Saud and the MS and Ph.D. in from Oregon State and 1989 respectively. He Professor of Computer Computer and Sciences, King Saud Saudi Arabia. He has special issues for the Journal (Springer), and

Mznah Al-Rodhaan has received her BS in Computer Applications (Hon) and MS in Computer Science both from King Saud University on 1999 and 2003 respectively. In 2009, she received her Ph.D. in Computer Science from the University of Glasgow in Scotland. She is currently working as the Vice Chair of the Computer Science Department in College of Computer & Information Sciences, King Saud University, Riyadh, Saudi Arabia. Moreover, she has served in the editorial boards for some journals such as the Ad Hoc journal (Elsevier) and has participated in several international conferences. Her current research interest includes: Mobile Ad Hoc Networks, Sensor Networks, Cognitive Networks, Network Security, and High Performance Computing.

Mznah Al-Rodhaan has received her BS in Computer Applications (Hon) and MS in Computer Science both from King Saud University on 1999 and 2003 respectively. In 2009, she received her Ph.D. in Computer Science from the University of Glasgow in Scotland. She is currently working as the Vice Chair of the Computer Science Department in College of Computer & Information Sciences, King Saud University, Riyadh, Saudi Arabia. Moreover, she has served in the editorial boards for some journals such as the Ad Hoc journal (Elsevier) and has participated in several international conferences. Her current research interest includes: Mobile Ad Hoc Networks, Sensor Networks, Cognitive Networks, Network Security, and High Performance Computing.

Mznah Al-Rodhaan has received her BS in Computer Applications (Hon) and MS in Computer Science both from King Saud University on 1999 and 2003 respectively. In 2009, she received her Ph.D. in Computer Science from the University of Glasgow in Scotland. She is currently working as the Vice Chair of the Computer Science Department in College of Computer & Information Sciences, King Saud University, Riyadh, Saudi Arabia. Moreover, she has served in the editorial boards for some journals such as the Ad Hoc journal (Elsevier) and has participated in several international conferences. Her current research interest includes: Mobile Ad Hoc Networks, Sensor Networks, Cognitive Networks, Network Security, and High Performance Computing.

Mznah Al-Rodhaan has received her BS in Computer Applications (Hon) and MS in Computer Science both from King Saud University on 1999 and 2003 respectively. In 2009, she received her Ph.D. in Computer Science from the University of Glasgow in Scotland. She is currently working as the Vice Chair of the Computer Science Department in College of Computer & Information Sciences, King Saud University, Riyadh, Saudi Arabia. Moreover, she has served in the editorial boards for some journals such as the Ad Hoc journal (Elsevier) and has participated in several international conferences. Her current research interest includes: Mobile Ad Hoc Networks, Sensor Networks, Cognitive Networks, Network Security, and High Performance Computing.

Mznah Al-Rodhaan has received her BS in Computer Applications (Hon) and MS in Computer Science both from King Saud University on 1999 and 2003 respectively. In 2009, she received her Ph.D. in Computer Science from the University of Glasgow in Scotland. She is currently working as the Vice Chair of the Computer Science Department in College of Computer & Information Sciences, King Saud University, Riyadh, Saudi Arabia. Moreover, she has served in the editorial boards for some journals such as the Ad Hoc journal (Elsevier) and has participated in several international conferences. Her current research interest includes: Mobile Ad Hoc Networks, Sensor Networks, Cognitive Networks, Network Security, and High Performance Computing.