# Level Crossing Rate of Macrodiversity in the Presence of Mixed Short Term Fading, Gamma Long Term Fading and Co-channel Interference 

Dragana Krstic, Srdjan Milosavljevic, Bojana Milosavljević, Suad Suljovic, and Mihajlo Stefanovic


#### Abstract

In this paper, macrodiversity with macrodiversity selection reception and two microdiversity selection receptions operating over short term fading channel in the presence of long term fading and co-channel interference is considered. Desired signal in both, the first and the second microdiversity receivers is subjected to Rayleigh short term fading and Gamma long term fading. Co-channel interference in the first microdiversity receiver experiences Nakagami-m short term fading and Gamma long term fading and cochannel interference in the second microdiversity receiver experiences Rician short term fading and Gamma long term fading. Level crossing rate of the ratio of Rayleigh random variable and Nakagami- $m$ random variable and the ratio of Rayleigh random variable and Rician random variable is evaluated. These formulae are used for calculation the level crossing rate of macrodiversity selection combining (SC) receiver output signal to interference ratio. The influence of Nakagami- $m$ short term fading severity parameter and Rician $\kappa$ factor on level crossing rate is analyzed and discussed.


Keywords- Gamma fading, macrodiversity reception, microdiversity reception, level crossing rate, Nakagami-m short term fading, Rayleigh fading, Rician short term fading.

## I. Introduction

NAKAGAMI- $m$ short term fading, Rayleigh short term fading, Rician short term fading, Gamma long term fading and co-channel interference degrade level crossing rate, average fade duration and outage probability of wireless mobile communication radio systems. By using macrodiversity reception, the influence of short term fading, long term fading and co-channel interference on system performance can be reduced. Macrodiversity with macrodiversity selection combining (SC) receiver and two microdiversity SC receivers is used in this paper to mitigate short term fading effects, long term fading effects and co-channel interference on level

[^0]crossing rate, average fade duration, outage probability and bit error probability, simultaneously [1] - [3].

Desired signal channel and co-channel interference channel in the first microdiversity have not dominant component and co-channel interference channel in the second microdiversity has dominant component. Nakagami-m and Rayleigh distributions describes signal envelope in non line of sight multipath fading channels and Rician distribution describes signal envelope in line of sight multipath fading channel [4][8].

There are many papers in the literature considering performance analysis in macrodiversity reception in the presence of short term fading, long term fading and co-channel interference. In paper [9], macrodiversity reception in the presence of Weibull short term fading, long term fading and co-channel interference is considered and level crossing rate (LCR) of macrodiversity SC receiver output signal to interference ratio process is evaluated. Outage probability of macrodiversity with two microdiversity SC receiver in the presence of Nakagami-m short term fading, Gamma long term fading and co-channel interference is efficiently evaluated as expression in the closed form in [10].

In this paper, macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers is analyzed. Desired signal experiences Rayleigh small scale fading and correlated Gamma large scale fading. Co-channel interference in the first microdiversity receiver suffer Nakagami-m small scale fading and Gamma large scale fading and co-channel interference in the second microdiversity receiver is affected by Rician small scale fading and Gamma large scale fading. Level crossing rate of the ratio of Rayleigh and Nakagami-m random process is calculated and this expression is used for evaluation the level crossing rate of the first microdiversity SC receiver output signal to interference ratio process. Level crossing rate of the ratio of Rayleigh and Rician ratio random process is calculated and this expression is used for evaluation the level crossing rate of the second microdiversity SC receiver output signal to interference ratio prcess. These expressions are used for calculation the level crossing rate of macrodiversity SC receiver output signal to interference ratio process. The macrodiversity SC receiver reduces Gamma long term fading effects on level crossing rate, the first microdiversity SC receiver reduces Nakagami-m short term fading effects on level crossing rate and the second
microdiversity SC receiver reduces Rician short term fading effects on level crossing rate. Macrodiversity receiver and microdiversity SC receivers reduce co-channel interference effects on level crossing rate.

## II. RATIO OF RAYLEIGH RANDOM VARIABLE AND

 NAKAGAMI-M RANDOM VARIABLEThe ratio of Rayleigh and Nakagami- $m$ random variable is:

$$
\begin{equation*}
z=\frac{x}{y}, x=z \cdot y, \tag{1}
\end{equation*}
$$

where $x$ follows Rayleigh distribution:

$$
\begin{equation*}
p_{x}(x)=\frac{2 x}{\Omega} \cdot e^{-\frac{x^{2}}{\Omega}}, x \geq 0 \tag{2}
\end{equation*}
$$

where $\Omega$ is average square value of $x ; y$ follows Nakagami-m distribution:

$$
\begin{equation*}
p_{y}(y)=\frac{2}{\Gamma(m)}\left(\frac{m}{s}\right)^{m} y^{2 m-1} e^{-\frac{m}{s} y^{2}} \tag{3}
\end{equation*}
$$

$m$ is Nakagami- $m$ severity parameter and $s$ is average square value of $y$.

The probability density function (PDF) of $z$ is:

$$
\begin{gather*}
p_{z}(z)=\int_{0}^{\infty} d y \cdot y p_{x}(z y) p_{y}(y)=\frac{2}{\Omega} z \cdot \frac{2}{\Gamma(m)}\left(\frac{m}{s}\right)^{m} \\
\cdot \frac{1}{2}(\Omega s)^{m+1} \frac{1}{\left(z^{2} s+m \Omega\right)^{m+1}} \Gamma(m+1)= \\
=m^{m+1} \Omega^{m} \frac{2 s z}{\left(z^{2} s+m \Omega\right)^{m+1}} \tag{4}
\end{gather*}
$$

Cumulative distribution function (CDF) of $z$ is:

$$
\begin{gather*}
F_{z}(z)=\int_{0}^{z} d t p_{z}(t)= \\
=m^{m+1} \Omega^{m} \frac{1}{m}\left(\frac{1}{(m \Omega)^{m}}-\frac{1}{\left(m \Omega+s z^{2}\right)^{m}}\right)= \\
=1-\frac{1}{\left(1+\frac{s z^{2}}{m \Omega}\right)^{m}} \tag{5}
\end{gather*}
$$

Level crossing rate of $z$ is:

$$
\begin{equation*}
N_{z}=\int_{0}^{\infty} d y y p_{x}(y z) p_{y}(y) \frac{\sigma_{\dot{z}}}{\sqrt{2 \pi}} \tag{6}
\end{equation*}
$$

where variance of $z$ is:

$$
\begin{equation*}
\sigma_{\dot{z}}=\frac{\pi f_{m}}{\sqrt{2 \pi} y m^{1 / 2}} \sqrt{\Omega m+z^{2} s} \tag{7}
\end{equation*}
$$

After substituting, the expression for level crossing rate becomes:

$$
\begin{align*}
& N_{z}= \frac{\pi f_{m}}{\sqrt{2 \pi} m^{1 / 2}} \frac{2}{\Omega}\left(\frac{m}{s}\right)^{m} z \sqrt{\Omega m+z^{2} s} . \\
& \cdot \frac{1}{2}(\Omega s)^{m+1 / 2} \frac{1}{\left(\Omega m+s z^{2}\right)^{m+1 / 2}}= \\
&=\frac{\sqrt{2 \pi} f_{m} m^{m-1 / 2} \Omega^{m-1 / 2} s^{1 / 2} z}{\left(\Omega m+z^{2} s\right)^{1 / 2}} \tag{8}
\end{align*}
$$

## III. Ratio of Rayleigh Random Variable and Rician Random Variable

The ratio of Rayleigh random variable and Rician random variable is:

$$
z=\frac{x}{y}, x=z \cdot y
$$

where $p_{\chi}(x)$ is given by (2):

$$
p_{x}(x)=\frac{2 x}{\Omega} \cdot e^{-\frac{x^{2}}{\Omega}}, x \geq 0
$$

and $p_{y}(y)$ is

$$
\begin{gather*}
p_{y}(y)=\frac{2(\kappa+1)}{e^{\kappa} s} y \cdot I_{0}\left(2 \sqrt{\frac{\kappa(\kappa+1)}{s}}\right) e^{-\frac{(\kappa+1)}{s} y^{2}}= \\
=\frac{2(\kappa+1)}{e^{\kappa} s} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} y^{2 i_{1}+2-1} e^{-\frac{(\kappa+1)}{s} y^{2}}, y \geq 0 \tag{9}
\end{gather*}
$$

The probability density function of $z$ is:

$$
\begin{gathered}
p_{z}(z)=\int_{0}^{\infty} d y \cdot y p_{x}(z y) p_{y}(y)= \\
=\frac{2}{\Omega} z \cdot \frac{2(\kappa+1)}{e^{\kappa} s} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} . \\
\cdot \frac{1}{2}(\Omega s)^{i_{1}+2} \frac{2 s z}{\left((\kappa+1) \Omega+z^{2} s\right)^{i_{1}+2}} \Gamma\left(i_{1}+2\right)= \\
=\frac{(\kappa+1)}{\Omega e^{\kappa}} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} \Omega^{i_{1}+2} \Gamma\left(i_{1}+2\right)
\end{gathered}
$$

$$
\begin{equation*}
\cdot \frac{2 s z}{\left((\kappa+1) \Omega+z^{2} s\right)^{i_{1}+2}} \tag{10}
\end{equation*}
$$

Cumulative distribution function of $z$ is:

$$
\begin{gather*}
F_{z}(z)=\int_{0}^{z} d t p_{z}(t)= \\
=\frac{(\kappa+1)}{\Omega e^{\kappa}} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} \Omega^{i_{1}+2} \Gamma\left(i_{1}+2\right) \frac{1}{i_{1}+1} \\
\left(\frac{1}{((\kappa+1) \Omega)^{i_{1}+1}}-\frac{1}{\left((\kappa+1) \Omega+z^{2} s\right)^{i_{1}+1}}\right) \tag{11}
\end{gather*}
$$

Level crossing rate of $z$ is given in (6):

$$
N_{z}=\int_{0}^{\infty} d y y p_{x}(y z) p_{y}(y) \frac{\sigma_{\dot{z}}}{\sqrt{2 \pi}}
$$

with variance of $z$ being:

$$
\begin{gather*}
\sigma_{\dot{z}}=\frac{\sigma_{\dot{x}}}{y^{2}}+\frac{x^{2}}{y^{4}} \sigma_{\dot{y}}= \\
=\frac{1}{y^{2}}\left(\pi^{2} f_{m}^{2} \Omega+z^{2} \pi^{2} f_{m}^{2} \frac{\Omega}{\kappa+1}\right)= \\
=\frac{\pi^{2} f_{m}^{2}}{y^{2}(\kappa+1)}\left(\Omega(\kappa+1)+z^{2} s\right) . \tag{12}
\end{gather*}
$$

After substituting, the expression for level crossing rate becomes:

$$
\begin{gather*}
N_{z}=\frac{\pi f_{m}}{(\kappa+1)^{1 / 2}} z \frac{2}{\Omega}\left(\Omega(\kappa+1)+z^{2} s\right)^{1 / 2} \\
=\frac{2(\kappa+1)}{e^{\kappa} s} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}}(\Omega s)^{i_{1}+3 / 2} \\
\frac{1}{\left((\kappa+1) \Omega+z^{2} s\right)^{i_{1}+3 / 2}} \Gamma\left(i_{1}+3 / 2\right) . \tag{13}
\end{gather*}
$$

IV. Level Crossing Rate of Macrodiversity SC

Receiver Output Signal to Interference Ratio
Model of macrodiversity SC receiver considered in this paper is shown in Fig. 1.

Random variables $x_{11}, x_{12}, x_{21}$ and $x_{22}$ follow Rayleigh distribution:


Fig.1. System model

$$
\begin{equation*}
p_{x_{i j}}\left(x_{i j}\right)=\frac{2 x_{i j}}{\Omega_{i}} \cdot e^{-\frac{x_{i j}^{2}}{\Omega_{i}}}, \quad x_{i j} \geq 0, i=1,2, j=1,2 \tag{14}
\end{equation*}
$$

Random variables $y_{11}$ and $y_{12}$ follow Nakagami-m distribution:

$$
\begin{equation*}
p_{y_{1 j}}\left(y_{1 j}\right)=\frac{2}{\Gamma(m)}\left(\frac{m}{s_{j}}\right)^{m} y_{1 j}^{2 m-1} e^{-\frac{m}{s_{j}} y_{1 j}^{2}}, y_{1 j} \geq 0, j=1,2 \tag{15}
\end{equation*}
$$

Random variables $y_{21}$ and $y_{22}$ follow Rician distribution:

$$
\begin{gather*}
p_{y_{2 j}}\left(y_{2 j}\right)=\frac{2(\kappa+1)}{e^{\kappa} s_{j}} . \\
\sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s_{j}}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} y_{2 j}^{2 i_{i}+1} e^{-\frac{(\kappa+1)}{s_{j}} y_{2 j}^{2}}, y_{2 j} \geq 0, j=1,2 \tag{16}
\end{gather*}
$$

Joint probability density function (JPDF) of $\Omega_{1}$ and $\Omega_{2}$ is:

$$
\begin{gather*}
p_{\Omega_{1} \Omega_{2}}\left(\Omega_{1} \Omega_{2}\right)=\frac{2}{\Gamma(c)\left(1-\rho^{2}\right) \rho^{c-1} \Omega_{0}^{c+1}} \cdot \\
\cdot \sum_{i_{2}=0}^{\infty}\left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2 i_{2}+c-1} \cdot \frac{1}{i_{2}!\Gamma\left(i_{2}+c\right)} \cdot \\
\Omega_{1}^{i_{2}+c-1} \Omega_{2}^{i_{2}+c-1} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}}, \Omega_{1} \geq 0, \Omega_{2} \geq 0 \tag{17}
\end{gather*}
$$

where $\rho$ is Gamma long term fading correlation coefficient, $c$ is Gamma long term fading severity parameter of desired signal, and $\Omega_{0}$ is average value of $\Omega_{1}$ and $\Omega_{2}$.

Joint probability density function of $s_{1}$ and $s_{2}$ is:

$$
p_{s_{1} s_{2}}\left(s_{1} s_{2}\right)=\frac{1}{\Gamma\left(c_{1}\right) \beta^{c_{1}}} \cdot s_{1}^{c_{1}-1} e^{-\frac{1}{\beta} s_{1}}
$$

$$
\begin{equation*}
\cdot \frac{1}{\Gamma\left(c_{1}\right) \beta^{c_{1}}} \cdot s_{2}^{c_{1}-1} e^{-\frac{1}{\beta} s_{2}}, s_{1} \geq 0, s_{2} \geq 0 \tag{18}
\end{equation*}
$$

$c_{1}$ is Gamma long term fading severity parameter of interference, $\beta$ is average value of $s_{1}$ and $s_{2}$.

The macrodiversity $S C$ receiver selects the first microdiversity SC receiver to enable service to user when the power at inputs of the first microdiversity SC receiver is higher than the power at inputs of the second microdiversity SC receiver, and macrodiversity SC receiver selects the second microdiversity SC receiver to provide service to user when the power at inputs of the second microdiversity SC receiver is higher than the power at inputs of the first microdiversity SC receiver. Therefore, level crossing rate of macrodiversity SC receiver output signal to interference ratio is:

$$
\begin{gather*}
N_{z}=\int_{0}^{\infty} d s_{1} \int_{0}^{\infty} d s_{2} p_{s_{1} s_{2}}\left(s_{1} s_{2}\right) \\
\left(\int_{0}^{\infty} d \Omega_{1} \int_{0}^{\Omega_{1}} d \Omega_{2} N_{z_{1} / \Omega_{1} s_{1}} p_{\Omega_{1} \Omega_{2}}\left(\Omega_{1} \Omega_{2}\right)+\right. \\
\left.+\int_{0}^{\infty} d \Omega_{2} \int_{0}^{\Omega_{2}} d \Omega_{1} N_{z_{2} / \Omega_{2} s_{2}} p_{\Omega_{1} \Omega_{2}}\left(\Omega_{1} \Omega_{2}\right)\right)=J_{1}+J_{2} \tag{19}
\end{gather*}
$$

Level crossing rate of $z_{1}$ is:

$$
\begin{gather*}
N_{z_{1}}=\frac{2 \sqrt{2 \pi} f_{m} m^{m-1 / 2} \Omega_{1}^{m-1 / 2} s_{1}^{1 / 2} z}{\left(\Omega_{1} m+z^{2} s_{1}\right)^{1 / 2}} . \\
 \tag{20}\\
\cdot\left(\frac{1}{\left(m \Omega_{1}\right)^{m}}-\frac{1}{\left(m \Omega_{1}+s_{1} z^{2}\right)^{m}}\right) .
\end{gather*}
$$

Level crossing rate of $z_{2}$ is:

$$
\begin{gather*}
N_{z_{2}}=\frac{2 \pi f_{m}}{(\kappa+1)^{1 / 2}} z \frac{2}{\Omega_{2}} \\
\cdot \frac{2(\kappa+1)}{e^{\kappa} s_{2}} \sum_{i_{1}=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{s_{2}}\right)^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} \Gamma\left(i_{1}+3 / 2\right) \\
\left(\Omega_{2} s_{2}\right)^{i_{1}+3 / 2} \frac{1}{\left((\kappa+1) \Omega_{2}+z^{2} s_{2}\right)^{i_{1}+1}} \\
\cdot \frac{(\kappa+1)}{e^{\kappa} \Omega_{2}} \sum_{i_{2}=0}^{\infty}(\kappa(\kappa+1))^{i_{2}} \frac{1}{\left(i_{2}!\right)^{2}} \Omega_{2}^{i_{2}+2} \Gamma\left(i_{2}+2\right) \frac{1}{i_{2}+1} \\
\left(\frac{1}{\left((\kappa+1) \Omega_{2}\right)^{i_{2}+1}}-\frac{1}{\left((\kappa+1) \Omega_{2}+z^{2} s_{2}\right)^{i_{2}+1}}\right)=J_{1}-J_{2} \tag{21}
\end{gather*}
$$

The integral $J_{1}$ is:

$$
\begin{gather*}
J_{1}=2 \sqrt{2 \pi} f_{m} m^{m-1 / 2} \frac{1}{\Gamma\left(c_{1}\right) \beta^{c_{1}}} z \\
\cdot \frac{2}{\Gamma(c)\left(1-\rho^{2}\right) \rho^{c-1} \Omega_{0}^{c+1}} \cdot \\
\cdot \sum_{i_{2}=0}^{\infty}\left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2 i_{2}+c-1} \cdot \frac{1}{i_{2}!\Gamma\left(i_{2}+c\right)} \cdot \\
\cdot\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{2}+c} \frac{1}{i_{2}+c} \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{2}+c}} \\
\cdot \sum_{j_{1}=0}^{\infty} \frac{1}{\left(i_{2}+c+1\right)\left(j_{1}\right)} \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} \\
\cdot \int_{0}^{\infty} d s_{1} \cdot s_{1}^{c_{1}-1+1 / 2} e^{-\frac{1}{\beta} s_{1}} \cdot \int_{0}^{\infty} d \Omega_{1} \cdot \Omega_{1}^{2 i_{2}+2 c+j_{1}+m-1 / 2} \\
\left(\frac{1}{\left(m \Omega_{1}\right)^{m}\left(m \Omega_{1}+s_{1} z^{2}\right)^{m}}-\frac{1}{\left(m \Omega_{1}+s_{1} z^{2}\right)^{2 m}}\right) \tag{22}
\end{gather*}
$$

The previous integral is:

$$
\begin{align*}
& \int_{0}^{\infty} d s_{1} \cdot s_{1}^{c_{1}-1+1 / 2} e^{-\frac{1}{\beta} s_{1}} \cdot \int_{0}^{\infty} d \Omega_{1} \cdot \Omega_{1}^{2 i_{2}+2 c+j_{1}+m-1 / 2} e^{-\frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}} \\
& \cdot\left(\frac{1}{\left(m \Omega_{1}\right)^{m}\left(m \Omega_{1}+s_{1} z^{2}\right)^{m}}-\frac{1}{\left(m \Omega_{1}+s_{1} z^{2}\right)^{2 m}}\right)=J_{11}-J_{12} \tag{23}
\end{align*}
$$

The integral $J_{2}$ is:

$$
\begin{gathered}
J_{2}=\frac{2 \pi f_{m}}{(\kappa+1)^{1 / 2}} z \frac{2}{\Omega_{2}} \frac{2(\kappa+1)}{e^{\kappa}} \\
\cdot \sum_{i_{1}=0}^{\infty}(\kappa(\kappa+1))^{i_{1}} \frac{1}{\left(i_{1}!\right)^{2}} \Gamma\left(i_{1}+3 / 2\right) \frac{1}{\Gamma\left(c_{1}\right) \beta^{c_{1}}} \\
\cdot \frac{2}{\Gamma(c)\left(1-\rho^{2}\right) \rho^{c-1} \Omega_{0}^{c+1}} \\
\sum_{i_{3}=0}^{\infty}\left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2 i_{3}+c-1} \cdot \frac{1}{i_{3}!\Gamma\left(i_{3}+c\right)}
\end{gathered}
$$

$$
\begin{gather*}
\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{3}+c} \frac{1}{i_{3}+c} \frac{1}{\left.\left(\Omega_{0}\left(1-\rho^{2}\right)\right)\right)^{i_{3}+c}} \cdot \\
\cdot \sum_{j_{2}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{2}\right)} \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{2}}} \\
\cdot \frac{(\kappa+1)}{e^{\kappa}} \sum_{i_{2}=0}^{\infty}(\kappa(\kappa+1))^{i_{2}} \frac{1}{\left(i_{2}!\right)^{2}} \Gamma\left(i_{2}+2\right) \frac{1}{i_{2}+1} \\
\int_{0}^{\infty} d s_{2} \cdot s_{2}^{c_{1}-1+1-i_{1}+i_{1}+3 / 2} e^{-\frac{1}{\beta} s_{2}} \cdot \int_{0}^{\infty} d \Omega_{2} \cdot \Omega_{2}^{2 i_{3}+2 c+j_{2}-1+i_{1}+3 / 2-1+i_{2}+2} \\
{ }_{0} \frac{1}{\left((\kappa+1) \Omega_{2}+s_{2} z^{2}\right)^{i_{1}+1}} \cdot \\
\cdot\left(\frac{1}{\left((\kappa+1) \Omega_{2}\right)^{i_{2}+1}}-\frac{1}{\left((\kappa+1) \Omega_{2}+z^{2} s_{2}\right)^{i_{2}+1}}\right) \tag{24}
\end{gather*}
$$

Previous integral can be written as:

$$
\begin{gather*}
\int_{0}^{\infty} d s_{2} \cdot s_{2}^{{c_{1}-1+5 / 2}^{-\frac{1}{\beta} s_{2}} \cdot \int_{0}^{\infty} d \Omega_{2} \cdot \Omega_{2}^{2 i_{3}+2 c+j_{2}+i_{1}+i_{2}+3 / 2} e^{-\frac{\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}}} \\
\left(\frac{1}{\left((\kappa+1) \Omega_{2}\right)^{i_{2}+1}\left((\kappa+1) \Omega_{2}+s_{2} z^{2}\right)^{i_{1}+1}}-\right. \\
 \tag{25}\\
\left.-\frac{1}{\left((\kappa+1) \Omega_{2}+z^{2} s_{2}\right)^{i_{1}+i_{2}+2}}\right)=J_{21}-J_{22}
\end{gather*}
$$

Previous two-fold integrals $J_{i j}$ can be solved by using the formulae [13]:

$$
\begin{align*}
& \frac{1}{(a \Omega+b s)^{n}} \int_{0}^{\infty} d s s^{p_{1}-1} e^{-\alpha_{1} s} \int_{0}^{\infty} d \Omega \Omega^{p_{2}-1} e^{-\alpha_{2} \Omega}= \\
& =\frac{a^{p_{1}-n}}{b^{p_{1}}} \frac{\Gamma\left(p_{2}\right)}{\alpha_{2}^{p_{1}+p_{2}-n}} \frac{\Gamma\left(p_{1}+p_{2}-n\right) \Gamma\left(p_{1}\right)}{\Gamma\left(p_{1}+p_{2}\right)} \\
& { }_{2} F_{1}\left(p_{1}+p_{2}-n, p_{1}, p_{1}+p_{2} ; 1-\frac{\alpha_{1} a}{\alpha_{2} b}\right) \tag{26}
\end{align*}
$$

where ${ }_{2} F_{1}(a, b, c ; z)$ is Gauss hypergeometric function [14] [15].

The integral $J_{i j}, i=1,2 ; j=1,2$ are solved as:

$$
\begin{gather*}
J_{11}=\frac{1}{m} m^{c_{1}+1 / 2-m} \frac{1}{z^{2\left(c_{1}+1 / 2\right)}} \\
\Gamma\left(2 i_{1}+2 c+j_{1}+m-1 / 2\right)\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{c_{1}+2 i_{1}+2 c+j_{1}} \\
\frac{\Gamma\left(c_{1}+2 i_{1}+2 c+j_{1}\right)}{\Gamma\left(c_{1}+2 i_{1}+2 c+j_{1}+m\right)} \Gamma\left(c_{1}+1 / 2\right) \\
{ }_{2} F_{1}\left(c_{1}+2 i_{1}+2 c+j_{1}, c_{1}+1 / 2, c_{1}+2 i_{1}+2 c+j_{1}+m ; 1-\frac{m \Omega_{0}\left(1-\rho^{2}\right)}{\beta z^{2}}\right) \tag{27}
\end{gather*}
$$

The next integral, $J_{12}$, is:

$$
\begin{gathered}
J_{12}=\frac{1}{m} m^{c_{1}+1 / 2-2 m} \frac{1}{z^{2\left(c_{1}+1 / 2\right)}} \\
\Gamma\left(2 i_{1}+2 c+j_{1}+m+1 / 2\right)\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{c_{1}+2 i_{1}+2 c+j_{1}-m+1} \\
\frac{\Gamma\left(c_{1}+2 i_{1}+2 c+j_{1}-m+1\right)}{\Gamma\left(c_{1}+2 i_{1}+2 c+j_{1}+m+1\right)} \Gamma\left(c_{1}+1 / 2\right)
\end{gathered}
$$

$$
\begin{equation*}
{ }_{2} F_{1}\left(c_{1}+2 i_{1}+2 c+j_{1}-m+1, c_{1}+1 / 2, c_{1}+2 i_{1}+2 c+j_{1}+m+1 ; 1-\frac{m \Omega_{0}\left(1-\rho^{2}\right)}{\beta z^{2}}\right) \tag{28}
\end{equation*}
$$

The next one is $J_{21}$ :

$$
\begin{gathered}
J_{21}=\frac{1}{(\kappa+1)^{i_{2}+1}}(\kappa+1)^{c_{1}+3 / 2-i_{1}} \frac{1}{z^{2\left(c_{1}+5 / 2\right)}} \\
\Gamma\left(2 i_{3}+2 c+j_{1}+i_{1}+1 / 2\right)\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{c_{1}+2 i_{3}+2 c+j_{2}+2} \\
\frac{\Gamma\left(c_{1}+2 i_{3}+2 c+j_{2}+2\right)}{\Gamma\left(c_{1}+2 i_{3}+2 c+j_{2}+i_{1}+3\right)} \Gamma\left(c_{1}+5 / 2\right)
\end{gathered}
$$

$$
\begin{equation*}
{ }_{2} F_{1}\left(c_{1}+2 i_{3}+2 c+j_{2}+2, c_{1}+5 / 2, c_{1}+2 i_{3}+2 c+j_{2}+i_{1}+3 ; 1-\frac{(\kappa+1) \Omega_{0}\left(1-\rho^{2}\right)}{\beta z^{2}}\right) \tag{29}
\end{equation*}
$$

The last one is $J_{22}$ :

$$
\begin{gather*}
J_{21}=(\kappa+1)^{c_{1}-i_{1}-i_{2}+1 / 2} \frac{1}{z^{2\left(c_{1}+5 / 2\right)}} \\
\Gamma\left(2 i_{3}+2 c+j_{2}+i_{1}+i_{2}+3 / 2\right)\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{c_{1}+2 i_{3}+2 c+j_{1}+2} \\
\frac{\Gamma\left(c_{1}+2 i_{3}+2 c+j_{1}+2\right)}{\Gamma\left(c_{1}+2 i_{3}+2 c+j_{2}+i_{1}+i_{2}+4\right)} \Gamma\left(c_{1}+5 / 2\right) \\
{ }_{2} F_{1}\left(c_{1}+2 i_{3}+2 c+j_{1}+2, c_{1}+5 / 2, c_{1}+2 i_{3}+2 c+j_{2}+i_{1}+i_{2}+4 ; 1-\frac{(\kappa+1) \Omega_{0}\left(1-\rho^{2}\right)}{\beta z^{2}}\right) \tag{30}
\end{gather*}
$$

## V. Numerical results

Level crossing rate of signal to interference ratio random process at output of macrodiversity SC receiver versus signal to interference ratio is shown in Fig. 2 for several values of Rician factor, Nakagami-m short term fading severity parameter, Gamma long term fading severity parameter and Gamma long term fading correlation coefficient.

Level crossing rate increases as resulting signal increases and level crossing rate decreases for higher values of resulting signal. The influence of resulting signal on level crossing rate is higher for lower values of resulting signal. Level crossing rate decreases when Rician factor increases or when dominant component increases.

The influence of Rician factor on level crossing rate is higher for lower values of Rician factor. Also, Rician factor has higher influence on level crossing rate for lower values of resulting signal.


Fig. 2. Level crossing rate versus signal to interference ratio


Fig. 3. LCR versus signal to interference ratio

Level crossing rate decreases when Nakagami-m severity parameter increases. The influence of Nakagami-m severity parameter on LCR is higher for lower values of Rician factor of Rician short term fading. Maximum of curves goes to higher values of resulting signal when Nakagami-m severity parameter increases.

In Fig. 3, level crossing rate is presented versus Rician factor for several values of fading parameters. Level crossing rate decreases when Rician factor of Rician short term fading increases. The influence of Rician factor on level crossing rate is bigger for lower values of Rician factor. Rician factor has greater values when dominant component power has higher value or scattering components power have lower values. The influence of Rician factor on LCR is higher for lower values of Gamma long term fading severity parameter. Level crossing rate decreases when Gamma long term fading severity parameter increases.

## VI. CONCLUSION

Macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers in the presence of Rayleigh short term fading, Rician short term fading, Nakagami-m short term fading, Gamma long term fading and co-channel interference is studied in this paper. Desired signal experiences Rayleigh small scale fading and Gamma large scale fading. Co-channel interference in the first microdiversity SC receiver experiences Nakagami- $m$ small scale fading and Gamma large scale fading. Co-channel interference in the second microdiversity SC receiver experiences Rician small scale fading and Gamma large scale fading.

Macrodiversity system reduces long term fading and short term fading effects on system performance simultaneously. Macrodiversity selects microdiversity SC receiver with higher power at inputs resulting in large scale fading effects reduction. Microdiversity SC receiver selects the branch with the highest signal to interference ratio resulting in small scale fading effects reduction.

In this paper, probability density function, cumulative distribution function and level crossing rate of the ratio of Rayleigh and Nakagami-m random process and also Rayleigh and Rician random process are derived. These expression are used for evaluation the level crossing rate of the signal to interference ratio at output of the first microdiversity SC receiver and at the output of the second microdiversity SC receiver and level crossing rate of the signal to interference ratio process at output of macrodiversity SC receiver.

The influence of Rician factor of Rician short term fading, Nakagami-m short term fading severity parameter, Gamma long term fading severity parameter and Gamma long term fading correlation coefficient on level crossing rate is analyzed. When correlation coefficient goes to one, macrodiversity system becomes microdiversity system.

Average fade duration (AFD) can be determined based on level crossing rate and AFD is better for lower values of level crossing rate.

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Dragana S. Krstić was born in Pirot, Serbia, in 1966. She received the BSc, MSc and PhD degrees in electrical engineering from Faculty of Electronic Engineering, Department of Telecommunications, University of Niš, Serbia, in 1990, 1998 and 2006, respectively. She works at the Faculty of Electronic Engineering in Niš since 1990. Her field of interest includes statistical communication theory, wireless communication systems. She participated in more projects which are supported by Serbian Ministry of Science and some international projects. She has written or co-authored near 240 papers, published in journals and at the international/ national conferences. She has reviewed many articles in well known journals. She is/ was reviewer of the papers for hundreds conferences and the member of technical program committees and international scientific committees of a large number of scientific conferences. Also, she is Associate Editor or member of Editorial Advisory Board/ Editorial Board of several journals: International WSEAS Transactions on Communications, International Journal of Communications, Journal on Advances in Telecommunications, International Journal of Communications (IARAS). She is IARIA Fellow.

Srdjan Milosavljevic was born in Kosovska Mitrovica, Serbia, in 1980. He received M.Sc. degrees in Electrical Engineering from the Faculty of Technical Sciences in Kosovska Mitrovica, University of Pristina, Republic of Serbia, in 2006. He is Ph.D. candidate in the Faculty of Electronic Engineering, University of Nis, Serbia.
He works at the Faculty of Economics in Kosovska Mitrovica as a system engineer, since 2006. He has published several scientific papers. His primary research interests are statistical communications theory and wireless communications.

Bojana Milosavljevic was born in Istok, Serbia, in 1982. She received M.Sc. degree in Electrical Engineering from the Faculty of Technical Sciences in Kosovska Mitrovica, University of Pristina, Republic of Serbia, in 2008. He is Ph.D. candidate in the Singidunum University of Belgrade, Serbia.
She works at the Technical Professional School in Zvecan, Republic of Serbia, as a teaching assistant, since 2015. Areas of her research include telecommunications and digital image processing. She has published several publications on the above subjects.

Suad N. Suljović received the BSc. degree in 1999 from the Faculty of Technical Sciences in Pristina, Department of Electronics and Telecommunications and MSc degree in 2009. at the Faculty of Electronic Engineering, University of Nis, Department of Telecommunications. He worked in Railway Training Centre in Sarajevo as a professor of vocational subjects in the field of electronics and telecommunications untill 2003. From 2005 to 2009 he was with the International University in Novi Pazar as an assistant. He is a professor at the Technical School in Novi Pazar since 2005. He is PhD student at Faculty of Electronic Engineering in Niš. Suljović has published several scientific papers in the field of telecommunications. His research interests are in the following areas: wireless communications under fading and interference influence; computer networks, as well as the area of programming and databases.

Mihajlo Č. Stefanović was born in Nis, Serbia, in 1947. He received the B. Sc., M. Sc. and Ph. D. degrees in electrical engineering from the Faculty of Electronic Engineering (Department of Telecommunications), University of Nis, Serbia, in 1971, 1976 and 1979, respectively. His primary research interests are statistical communication theory, optical, wireless and mobile communications. He has written or co-authored a great number of journal publications. He has written five monographs, too. He was a mentor to hundreds of graduates, for dozens of master's theses and doctoral dissertations, and many times a member of the committee for these works. Now, Dr. Stefanovic is a retired professor at the Faculty of Electronic Engineering in Nis


[^0]:    D. S. Krstic is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš (e-mail: dragana.krstic@elfak.ni.ac.rs).
    S. Milosavljevic, is with University of Pristina, Faculty of Economics, Kolašinska156, Kosovska Mitrovica (e-mail: srdjan.milosavljevic@pr.ac.rs)
    B. Milosavljević is with Higher Technical Professional School in Zvečan (e-mail: bojana_c11@yahoo.com)
    S. N. Suljovic is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš (suadsara@gmail.com).
    M. C. Stefanovic is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš e-mail: e-mail: misa.profesor@gmail.com).

