# Equal-Optimal Power Allocation and Relay Selection Algorithm Based on Symbol Error Probability in Cooperative Communication

Xin Song, Siyang Xu and Minglei Zhang

*Abstract*—To optimize power allocation and relay selection in a multi-relay cooperative communication network under a joint total power constraint, a low complexity equal-optimal power allocation and relay selection (EOPRS) algorithm is proposed in an Amplify-and-Forward (AF) cooperative network, which minimizes the symbol error probability (SEP). In the proposed algorithm, the equal power allocation among source and relay nodes is conducted. Then, we can derive an equivalent channel gain, which describes the compositive channel characteristics of two phases that are the source node to relay node and the relay node to destination in the cooperative process. With ascending order of the equivalent channel gain, the signal-to-noise ratio (SNR) can be taken as the threshold, and an optimal set of relay nodes is chosen to minimize the symbol error probability only by solving the arranged matrix. By combining with optimal power allocation in the chosen set, the power allocation factors are derived by the Lagrange multiplier and steepest descent methods, and thus the proposed scheme can further reduce the symbol error probability. The proposed algorithm need not know a large quantity of channel statistical information, which leads to a low complexity cost. In different sets of relay nodes, simulation results show that the proposed algorithm has better SEP performance and power efficiency compared with several traditional algorithms. Moreover, the simulation results can further verify the correctness of the theoretical analysis and the effectiveness of the proposed algorithm.

Key words: cooperative communication; optimal power allocation; symbol error probability; relay selection

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## I. Introduction

Cooperative relaying [1] is an efficient wireless transmission technique, which has been proposed to obtain spatial diversity by forming virtual antenna arrays without the use of multiple antennas at transmitters or receivers. It is especially suitable for small-size and antenna-limited wireless devices, because two main advantages are low radio frequency (RF) power requirement and spatial diversity gain [2]. However, the cooperative relaying selection schemes need to make a tradeoff between spectral and energy efficiencies in [3-4]. A cooperative diversity model is built in [5], in which two users act as partners and cooperatively communicate with a common destination. In the first time interval, each use transmits its own bits, and then in the second time interval, it estimates bits of its partner. In [6], the networks consisting of more than two users that employ space-time coding to achieve cooperative diversity are considered. Coded cooperation schemes are discussed in [7]-[8], where a user can transmit part of its partner’s code word. In order to reduce the computation complexity cost, several low-complexity cooperative protocols [9] are proposed, including fixed relaying, selection relaying and incremental relaying, in which the relay node can either amplify and forward or decode and forward the received signal. The relaying selection scheme based on SNR [10] is studied for multi-relay cooperative networks with distributed space-time coding. In the SNR-based scheme, the error propagation can be effectively reduced by employing appropriate thresholds on the relays.

Due to the limited transmission power of the relay and source nodes, power efficiency is a critical design consideration for wireless networks, such as sensor and ad-hoc networks. In order to improve power efficiency, it is important to select the appropriate relay that forward the source data, and to allocate transmit power levels of all the nodes. There are several efficient approaches that are known to solve the power allocation issue. One of the classic techniques is the optimal relay power allocation that can improve the sum-rate capacity and reduce the bit error rate (BER) [11]. The weighted sum-rate and multi-user scheme [12] has been used in two-way relaying. However the power allocation scheme in cooperative communication systems is usually limited to two-way relaying schemes [13]. In practice, the performance of multi-source and multi-relay cooperative vehicular networks largely depends on the power allocation and cooperative relay communication strategies [14-15]. The power allocation scheme for multi-source and multi-destination networks [16] is studied, in which the base station allocates one or more relays to each user and cooperative. In [17], auction-based power allocation scheme for multi-user relaying networks is proposed, in which the asymptotic expression of the outage probability is derived. The proposed scheme that combines Decode-and-Forward (DF) with space-time coding minimizes the total power in cooperative communication systems. In recent ten years, power allocation for the relay nodes has been investigated to reduce corresponding outage probability and BER. In [18], the hybrid decode-amplify-forward (HDAF) scheme can provide better output performance compared with AF and DF in a multi-relay cooperative system. All of these above algorithms can provide effective power transmission strategies. However, the principal disadvantages are that implementations of these algorithms require either the source or destination to have substantial information, including the channel state information (CSI) of all communication channels, topology of the network and received SNR at every node.

Such centralized power allocation and relay selection schemes may be unfeasible to implement due to the substantial feedback requirements, delay and overhead. In order to decrease the symbol error probability and complexity cost, we propose equal-optimal power allocation and relay selection algorithm. Firstly, an equivalent channel gain based on the statistic channel information and symbol error probability of the system is derived with equal power allocation. The equivalent channel gain describes the integrated channel characteristics of two phases that are the source node to relay node and the relay node to destination in the cooperative process. According to the SNR, the optimal set of relay nodes is selected only by solving the arranged matrix. Finally, the power allocation is conducted by the Lagrange multiplier and steepest descent methods, which leads to a low system symbol error probability. The rest of this paper is organized as follows. In Section II, the system model is described, and the closed-form expression of the symbol error probability in the AF scheme for the multi-node system is derived. In Section III, we derive an equivalent channel gain and propose the optimal power allocation scheme, which minimize the average SEP under the condition of limited power. In Section IV, simulation results are presented. Finally, Section V concludes the paper.

## II. System Model

In this paper, we consider a multi-node cooperative relaying system, as shown in Fig. 1. This system consists of one source, *N* relays and one destination. The link between any two nodes is modeled as a block Rayleigh fading channel with additive white Gaussian noise (AWGN). It is assumed that all the receiving nodes have the exact channel state information that is needed for demodulation. Thus, the relays have the CSI of the link from the source to themselves, and the destination has the CSI of all the links.

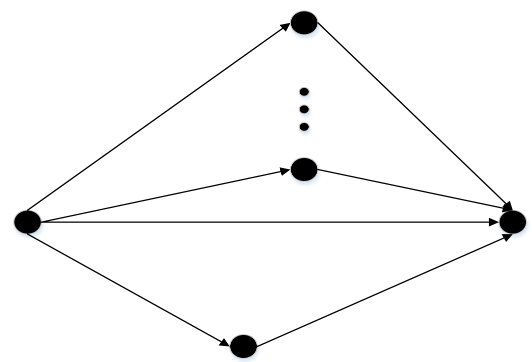

Fig. 1. Multi-node cooperative relay system

We assume that $h_{sd} \sim CN(0,\sigma_{sd}^2)$ is the fading coefficient of the channel from the source to the destination, $h_{si} \sim CN(0,\sigma_{si}^2)$ is the fading coefficient of the channel from the source to relay $R_i$, and $h_{id} \sim CN(0,\sigma_{id}^2)$ is the fading coefficient of the channel from relay $R_i$ to the destination. Similarly, we assume $n_{sd} \sim CN(0,N_0)$, $n_{si} \sim CN(0,N_0)$ and $n_{id} \sim CN(0,N_0)$ correspond to each additive Gaussian noise, respectively. We choose $M$ nodes from the $N$ relays to construct the set $\Phi_M$, then use the relays in $\Phi_M$ to transmit the information. Without loss of generality, we consider that all nodes have the same transmit power $p_s$. At the first time slot, the source broadcasts symbol $s(t)$ to both the destination and relay, and we can obtain

$$y_{sd} = \sqrt{p_s} h_{sd} s(t) + n_{sd} \tag{1}$$

and

$$y_{si} = \sqrt{p_s} h_{si} s(t) + n_{si} \tag{2}$$

where $y_{sd}$ and $y_{si}$ are the received signals at the destination and relay, respectively.

At the second time slot, the relay sends the processed signal $x(t)$ to the destination, i.e.

$$x(t) = \sqrt{\frac{p_s}{p_s |h_{si}|^2 + N_0}} y_{si} \tag{3}$$

The corresponding received signal $y_{id}$ at the destination can be written as

$$y_{id} = \sqrt{p_s} h_{id} x(t) + n_{id} \tag{4}$$

In the Rayleigh fading channel, the instantaneous SNRs of the S-D, S-R and R-D links can be expressed as

$$\gamma_{sd} = |h_{sd}|^2 p_s / N_0 \tag{5}$$

$$\gamma_{si} = |h_{si}|^2 \ p_s \ / \ N_0 \tag{6}$$

$$\gamma_{id} = |h_{id}|^2 \ p_s \ / \ N_0 \tag{7}$$

Therefore, the average SNRs can be obtained

$$\overline{\gamma_{sd}} = E(|h_{sd}|^2) p_s \ / \ N_0 = \sigma_{sd}^2 p_s \ / \ N_0 \tag{8}$$

$$\overline{\gamma_{si}} = E(|h_{si}|^2) p_s \ / \ N_0 = \sigma_{si}^2 p_s \ / \ N_0 \tag{9}$$

$$\overline{\gamma_{id}} = E(|h_{id}|^2) p_s \ / \ N_0 = \sigma_{id}^2 p_s \ / \ N_0 \tag{10}$$

where E(·) denotes the statistical average.
Then, we can obtain the SNR of the S-R-D link,

$$\gamma = \sum_{R_i \in \Phi_M} \frac{\gamma_{si}\gamma_{id}}{1 + \gamma_{si} + \gamma_{id}} \tag{11}$$

By using maximal ratio combining (MRC), the destination combines the received signals in the first phase and the second phase. The SNR of the combined signal can be expressed as

$$\gamma_{AF} = \gamma_{sd} + \gamma = \gamma_{sd} + \sum_{R_i \in \Phi_M} \frac{\gamma_{si}\gamma_{id}}{1 + \gamma_{si} + \gamma_{id}} \tag{12}$$

The symbol error probability of the AF scheme is derived as [19]

$$P_e = Q(\sqrt{K\gamma_{AF}}) \tag{13}$$

where $K$ is associated with modulation mode. When $K = 2$, $Q(x)$ is

$$Q(x) = (1 / \sqrt{2\pi}) \int_x^{+\infty} e^{-u^2/2} du \tag{14}$$

According to [19-20], if the first derivative to $t-1$ order derivative of $P_{\gamma_{AF}}(\gamma_{AF})$ is zero at high SNR, the approximate expression of the error probability is

$$P_e \to \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)K^{t+1}} \frac{1}{t!} \frac{\partial^t P_{\gamma_{AF}}}{\partial^t \gamma_{AF}}(0) \tag{15}$$

Theorem 1 [19] demonstrates that for a non-negative random variable sequence $\{X\} = \{X_0, X_1, ..., X_M\}$, the probability density functions can be expressed as $P_0, P_1, ..., P_M$. When the variable is zero, the probability density functions are not zero, that is to say $P_0(0), P_1(0), ..., P_M(0)$ are not zero. The random variable satisfies

$$V_M = \sum_{i=0}^{M} X_i \tag{16}$$

When the variable is zero, the first derivative to $M-1$ order derivative of $P_{V_M}$ is zero. The $M$ order derivative is

$$\frac{\partial^M P_{V_M}}{\partial V_M}(0) = \prod_{i=0}^{M} P_i(0) \tag{17}$$

It is shown that the above result is suitable for all kinds of possible diversity strategies, even if those outside the scope of the cooperative diversity network are also applicable. Now, applying the results to collaborative network of multiplier nodes, we can obtain

$$\frac{\partial^M P_{\gamma_{AF}}(\gamma_{AF})}{\partial^t \gamma_{AF}}(0) = P_{\gamma_{sd}}(0) \prod_{i=1}^{M} [P_{\gamma_{si}}(0) + P_{\gamma_{id}}(0)] \tag{18}$$

In the Rayleigh fading channel, by inserting (18) into (15), the symbol error probability can be approximately expressed as

$$P_e(M) \approx \frac{C(M)}{K^{M+1}(p_s\Gamma)^{M+1}} \frac{1}{\sigma_{sd}^2} \prod_{i=1}^{i=M} (\frac{1}{\sigma_{si}^2} + \frac{1}{\sigma_{id}^2}) \tag{19}$$

where $\Gamma = 1 / N_0$, $K = 2$ in phase shift keying (PSK), and the parameter $C(M)$ depends on the number of relay nodes

$$C(M) = \frac{\prod_{j=1}^{M+1}(2j\text{-}1)}{2(M+1)!} \tag{20}$$

## III. EQUAL-OPTIMAL POWER ALLOCATION AND RELAY SELECTION (EOPRS) ALGORITHM

The basic idea of the traditional exhaustive algorithm (full search algorithm) that solves the relay selection is as follows. The power distribution among all the possible relay collections is conducted and the BER is calculated to determine the collection of the minimum error rate via a global search. Then, we amplify and forward the data. When the number of potential relay nodes in the system is large, the complexity cost of the traditional algorithm is very high. To reduce the computational cost, we propose a novel the relay selection and power allocation algorithm. In this algorithm, the optimal set of relay nodes is chosen adaptively by solving the arranged matrix. The optimal power allocation is conducted to reduce the system error probability.

### 3.1 Relay selection scheme

From Eq. (19), we note that the symbol error probability is associated with $\frac{1}{\sigma_{si}^2} + \frac{1}{\sigma_{id}^2}$. Therefore, we define the equivalent channel gain (ECG) of relay $R_i$ as

$$m_j = (\frac{1}{\sigma_{ij}^2} + \frac{1}{\sigma_{jd}^2}) \tag{21}$$

With the ascending order of the equivalent channel gain $m_j$, i.e. $m_1 < m_2 < m_3 < ..... < m_N$, we can obtain the arranged matrix $\boldsymbol{\psi} = [R_1 R_2 ... R_K ... R_N]$, where $R_K$ represents the relay node whose ECG is in $K$th place in the ascending order. Thus, we can quickly complete the relay selection. When the SNR is $\Gamma$, we can select $M$ optimal ones from all the $N$ relays

$$G_{opt} = \arg\max_{M} \{\beta_{eq.M} < \Gamma\} \quad 1 \le M \le N \tag{22}$$

where

$$\begin{cases} \beta_{\text{eq, M}} = \dfrac{\text{g}(L)}{p_s} m_M \\ g(L) = \dfrac{2M+1}{(M+1)K} \end{cases} \tag{23}$$

Then, the optimal relay collection can be obtained

$$\Psi_M^{\text{opt}} = \{R_1, R_2, ..., R_M\} \tag{24}$$

Proof of Eq. (22):

From Eq. (22)-(24), we can obtain the conclusion that

$$P_{e,\psi_M^{opt}} < P_{e,\psi_{M-1}^{opt}} < ... < P_{e,\psi_0^{opt}} \tag{25}$$

$$P_{e,\psi_M^{opt}} < P_{e,\psi_{M+1}^{opt}} < ... < P_{e,\psi_N^{opt}} \quad (26)$$

Proof of Eq. (25):

According the above deduction, we can obtain

$$\frac{P_{e,\psi_M^{opt}}}{P_{e,\psi_{M-1}^{opt}}} = \frac{C(M)}{C(M-1)K\ p_s\Gamma} m_M < 1$$

$$\frac{P_{e,\psi_{M-1}^{opt}}}{P_{e,\psi_{M-2}^{opt}}} = \frac{C(M-1)}{C(M-2)K\ p_s\Gamma} m_{M-1} \quad (27)$$

The derivative of $\frac{C(M)}{C(M-1)}$ with respect to $M$ is written as

$$\frac{\partial \frac{C(M)}{C(M-1)}}{\partial M} > 0 \quad (28)$$

Therefore, we can obtain

$$\frac{P_{e,\psi_M^{opt}}}{P_{e,\psi_{M-1}^{opt}}} > \frac{P_{e,\psi_{M-1}^{opt}}}{P_{e,\psi_{M-2}^{opt}}} > .....\frac{P_{e,\psi_1^{opt}}}{P_{e,\psi_0^{opt}}} \quad (29)$$

It is obvious that

$$P_{e,\psi_M^{opt}} < P_{e,\psi_{M-1}^{opt}} < ... < P_{e,\psi_0^{opt}} \quad (30)$$

Therefore, Eq. (25) is proved.

Proof of Eq. (26):

$M$ is the number of optimal relay, and we can know that

$$\frac{P_{e,\psi_{M+2}^{opt}}}{P_{e,\psi_{M+1}^{opt}}} = \frac{C(M+2)}{C(M+1)K\ p_s\Gamma} m_{M+2} \quad (31)$$

From (31), we can get

$$\frac{P_{e,\psi_{M+1}^{opt}}}{P_{e,\psi_M^{opt}}} > 1 \quad (32)$$

Take the derivative of $\frac{C(M+2)}{C(M+1)}$ versus $M$, i.e.

$$\frac{\partial \frac{C(M+2)}{C(M+1)}}{\partial M} > 0 \quad (33)$$

Therefore, we can conclude

$$\frac{P_{e,\psi_{M+1}^{opt}}}{P_{e,\psi_M^{opt}}} < \frac{P_{e,\psi_{M+2}^{opt}}}{P_{e,\psi_{M+1}^{opt}}} < .....\frac{P_{e,\psi_N^{opt}}}{P_{e,\psi_{N-1}^{opt}}} \quad (34)$$

From (34), the following inequation is proved

$$P_{e,\psi_M^{opt}} < P_{e,\psi_{M+1}^{opt}} < ... < P_{e,\psi_N^{opt}} \quad (35)$$

Therefore, Eq. (26) is proved

Combining Eq. (25) and Eq. (26) with Eq. (19), we can obtain

$$m_j < \frac{p_s\Gamma}{g(j)} \qquad j = 1,2,...,M \quad (36)$$

$$m_j > \frac{p_s\Gamma}{g(j)} \qquad j = M, M+1,...,N \quad (37)$$

According (36) and (37), we can derive

$$\beta_{\text{eq,M}} = \frac{\text{g}(M)}{p_s} m_L < \Gamma < \frac{\text{g}(M)}{p_s} m_M = \beta_{eq,M+1} \quad (38)$$

Therefore, Eq. (22) is proved.

3.2 Power allocation scheme

After the optimal relay collection is chosen, we conduct optimal power allocation among the source and relay nodes in the optimal set. We can obtain the symbol error probability

$$P_e(M) = \frac{C(M)}{K^{M+1}\alpha_0(p\Gamma)^{M+1}} \frac{1}{\sigma_{sd}^2} \prod_{i=1}^{i=M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right) \quad (39)$$

where $p$ is the total power, $\alpha_0 p$ is the power of the source node, and $\alpha_i p$ is the power of relay $R_i$.

Under the limit of total transmit power, the power allocation problem can be optimized as

$$\begin{cases} \min P_e(M) = \frac{C(M)}{K^{M+1}\alpha_0(p\Gamma)^{M+1}} \frac{1}{\sigma_{id}^2} \prod_{i=1}^{i=M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right) \\ \text{s.t.} \qquad \sum_{i=0}^{i=M} \alpha_i \le 1 \end{cases} \quad (40)$$

The Lagrange multiplier function can be written as

$$\begin{aligned} J &= P_e(M) - \lambda(1 - \sum_{i=0}^{i=M} \alpha_i) \\ &= \frac{B}{\alpha_0} \frac{1}{\sigma_{id}^2} \prod_{i=1}^{i=M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right) - \lambda(1 - \sum_{i=0}^{i=M} \alpha_i) \end{aligned} \quad (41)$$

where $\lambda$ is the Lagrange multiplier and the parameter $B$ is

$$B = \frac{C(M)}{K^{M+1}(p\Gamma)^{M+1}} \quad (42)$$

By setting the derivative of $J$ versus $\alpha_0$ to zero, we have the following equation:

$$\begin{aligned} \frac{\partial J}{\partial \alpha_0} &= -\frac{B}{\alpha_0{}^2} \frac{1}{\sigma_{id}^2} \prod_{i=1}^{i=M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right) + \frac{B}{\alpha_0} \\ &\cdot \frac{1}{\sigma_{id}^2} \sum_{k=1}^{M} \left[\left(-\frac{1}{\alpha_0\ \sigma_{sk}^2}\right) \prod_{\substack{i=1 \\ i\ne k}}^{M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right)\right] + \lambda = 0 \end{aligned} \quad (43)$$

Then, we can obtain

$$\alpha_0 = \frac{P_e(M)}{\lambda} \left(1 + \sum_{i=1}^{M} \frac{\alpha_i\sigma_{id}^2}{\alpha_i\sigma_{id}^2 + \alpha_0\sigma_{si}^2}\right) \quad (44)$$

Let the derivative of $J$ with respect to $\alpha_i, (i = 1,...M)$ equal to zero, that is say

$$\frac{\partial J}{\partial \alpha_i} = \frac{B}{\alpha_0} \frac{1}{\sigma_{sd}^2} \prod_{\substack{i=1 \\ i\ne k}}^{M} \left(\frac{1}{\alpha_0\sigma_{si}^2} + \frac{1}{\alpha_i\sigma_{id}^2}\right)\left(-\frac{1}{\alpha_i{}^2} \frac{1}{\sigma_{id}^2}\right) + \lambda = 0 \quad (45)$$

Solve the equation (45) to get

$$\alpha_i = \frac{P_e(M)\sigma_{si}^2}{\lambda(\alpha_i\sigma_{id}^2 + \alpha_0\sigma_{si}^2)} \quad (46)$$

By setting the derivative of $J$ with respect to $\lambda$ to zero, we can obtain

$$\sum_{i=0}^{i=M} \alpha_i = 1 \quad (47)$$

By the steepest descent and the Lagrange multiplier method, the power allocation factors can be obtained, which leads to the least system symbol error probability [21].

$$\begin{cases} \alpha_i = \dfrac{-\alpha_0\sigma_{si}^2 + \sqrt{\alpha_0^{\,2}\sigma_{si}^4 + 4\alpha_0\ \sigma_{si}^2\sigma_{id}^2\ /\ (M+1)}}{2\sigma_{id}^2} \\ \sum\limits_{i=1}^{M} \dfrac{\sqrt{\alpha_0^{\,2}\sigma_{si}^4 + 4\alpha_0\ \sigma_{si}^2\sigma_{id}^2\ /\ (M+1)}}{\sigma_{id}^2} - \alpha_0 \sum\limits_{i=1}^{M} \dfrac{\sigma_{si}^2}{\sigma_{id}^2} + 2\alpha_0 = 2 \end{cases} \tag{48}$$

As a special case, we assume that all the relays are located on the same position, that is,

$$\frac{\sigma_{si}^2}{\sigma_{id}^2} = c \qquad i = 1,2,3,...,M \tag{49}$$

In this case, by substituting (49) to (48), we can obtain the following expression

$$\begin{aligned} \alpha_0 &= \frac{2 - \dfrac{cM}{M+1}\left(1 + \sqrt{\dfrac{4(1+M)}{c}}\right)}{2(1-cM)} \\ \alpha_i &= (1-\alpha_0)\ /\ M,\ i = 1,2,3,...,M \end{aligned} \tag{50}$$

Based on the above theoretical analysis, we can conclude that the proposed algorithm steps are as follows:

1) According to the current channel state information, we can calculate the equivalent path gain of each node and sort them in ascending order.

2) We can calculate $\beta_{eq,M}$ according to Eq. (22).

3) According to Eq. (22), $G_{\mathrm{opt}}$ and optimal relay collection $\Psi_M^{\mathrm{opt}} = \{\mathrm{R}_1, \mathrm{R}_2, ..., \mathrm{R}_M\}$ can be obtained.

4) Conduct optimal power allocation among the source and relays in the optimal set of relay nodes.

The full - search algorithm includes $2^N$ calculations of the symbol error probability and power allocation. With the increase of $N$, the computational complexity cost increases rapidly. However, the proposed algorithm only needs to calculate the equivalent channel gain of $N$ relay nodes, sort them in ascending order, and conduct once optimal power allocation. Compared with the full - search algorithm, the proposed algorithm greatly reduces the computation complexity.

## IV. Simulation Results

In this section, we present some simulation results to demonstrate the performance of our EOPRS scheme in a multi-node cooperative system. It is assumed that all the simulations are performed in the Rayleigh fading channels. In the simulation, we adopt BPSK modulation, $K = 2$, and select five relay nodes. The settings are as follows:

Table 1 Simulation parameters under different channel states

| parameters | value |
|---|---|
| $\sigma_{s1}^2$ | 1.00 |
| $\sigma_{s2}^2$ | 0.20 |
| $\sigma_{s3}^2$ | 0.50 |
| $\sigma_{s4}^2$ | 0.40 |
| $\sigma_{s5}^2$ | 2.00 |
| $\sigma_{1d}^2$ | 0.50 |
| $\sigma_{2d}^2$ | 0.10 |
| $\sigma_{3d}^2$ | 0.25 |
| $\sigma_{4d}^2$ | 0.20 |
| $\sigma_{5d}^2$ | 1.00 |
| $\sigma_{sd}^2$ | 1.00 |
| $p$ | 1.00 W |

According to Eq. (22), the experimental result gives the relation curve between the number of optimal relay $M$ and SNR, and thus we can obtain the optimal relay collection using the arranged matrix. As shown in Fig. 2, when $\mathrm{SNR} \le 11.8\mathrm{dB}$, the number of optimal relay is 1; when $11.8\mathrm{dB} < \mathrm{SNR} \le 15\mathrm{dB}$, the number of optimal relay is $M = 2$; when $15\mathrm{dB} < \mathrm{SNR} \le 16\mathrm{dB}$, $M = 3$; when $16\mathrm{dB} < \mathrm{SNR} \le 19.2\mathrm{dB}$, $M = 4$; and when $\mathrm{SNR} > 19.2\mathrm{dB}$, the optimal $M$ is equal to 5.

The system SEP of the EOPRS scheme with different relays versus SNR is shown in Fig. 3. As illustrated in Fig. 3, the SEP performance decreases with increasing SNR. At $\mathrm{SNR} \le 4\mathrm{dB}$, the SEP perfomance is the lowest with one relay. At $16\mathrm{dB} < \mathrm{SNR} \le 18.5\mathrm{dB}$, the system performance of EOPRS is optimal with three relay nodes. At $\mathrm{SNR} > 22.5\mathrm{dB}$, the proposed algorithm has the best SEP performance with five relays. It notes that when $\mathrm{SNR} \le 4\mathrm{dB}$, the SEP is greater than 1. That is because the SEP expression is approximate under a high SNR.

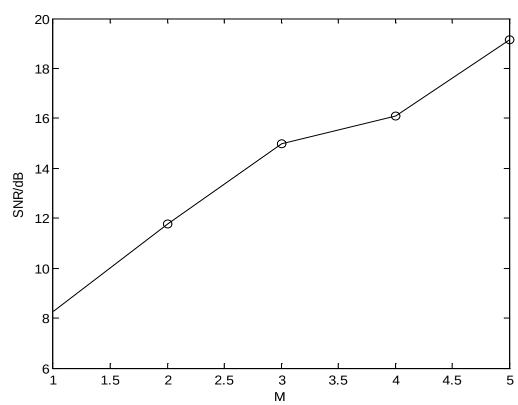


Fig. 2. Relation curve between optimal relay number M and SNR

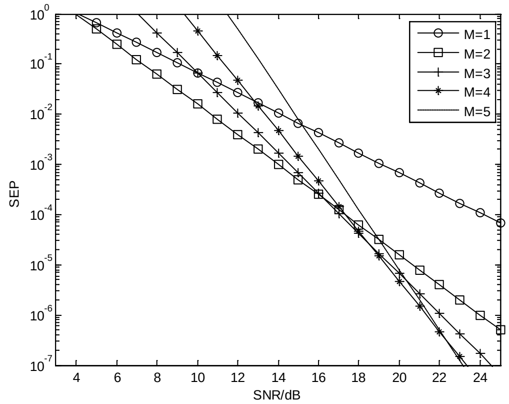


Fig. 3. System SEP of different relay nodes

Fig. 4 displays SEP performance of the EOPRS, EPRS and NPRS schemes versus the SNR. Each of the three schemes chooses different optimal relay selections with the SNR. From Fig. 4, it is shown that the output performance of NPRS is better than that of EPRS, while the SEP of EOPRS is lower than that of NPRS. Therefore, the SEP performance of the EOPRS scheme is the best in the three schemes.

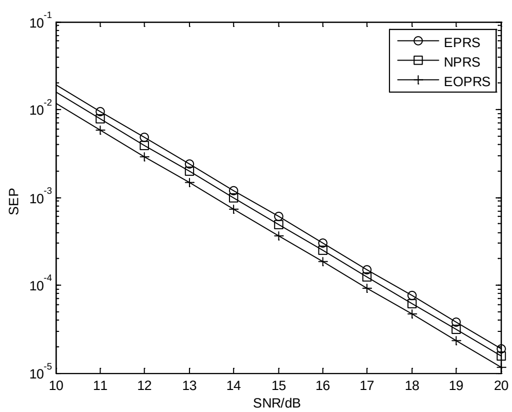


Fig. 4. System SEP of different schemes

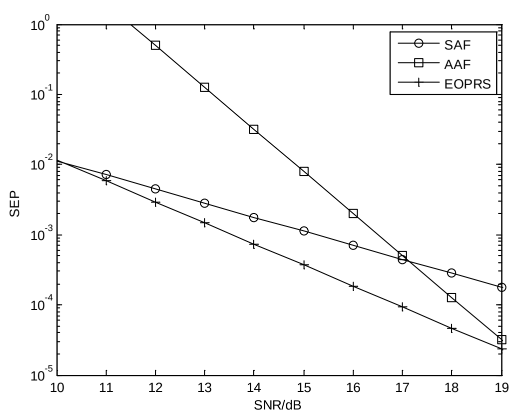


Fig. 5. System SEP of different schemes

The output performance of three schemes versus SNR is shown in Fig. 5. The three schemes are as follows: 1) the pre-select Single relay Amplify-and-Forward (SAF) scheme; 2) All relay Amplify-and-Forward (AAF) schemes; 3) the proposed EOPRS scheme. The SAF scheme chooses the optimal relay to forward to the single, while the AAF scheme chooses all the relays that participate in relaying. As illustrated in Fig. 5, at $\mathrm{SNR} < 17\mathrm{dB}$, the SEP of the SAF scheme is less than that of the AAF scheme, while at $\mathrm{SNR} \geq 17\mathrm{dB}$, the SEP of the AAF scheme is less than that of SAF. In the entire SNR range, the proposed algorithm has the lowest SEP. Therefore, the output performance of the EOPRS is better than those of the SAF and AAF schemes.

Fig. 6 displays the SEP performance of three schemes (i.e. EOPRS, equal-power-allocation-based relay selection (EPRS) and near-optimal-power-allocation-based relay selection (NPRS)) in the five-node cooperative system with different relay collections. As illustrated in Fig. 6, when $M = 1$, the SEP of the EOPRS is identical to those of the EPRS and NPRS schemes. When $M = 2$, the SEP of EOPRS is roughly equal to that of NPRS, and EOPRS improves the performance gain by 0.31dB compared with the EPRS scheme. When $M = 4$, EOPRS improves the performance gain by 0.30dB compared with the NPRS scheme and by 0.46dB compared with the EPRS scheme. In summary, the proposed EOPRS algorithm always achieves best performance among the three schemes in different relay collection.

## V. Conclusions

In this paper, we propose relay selection and power allocation algorithm namely EOPRS under limited transmission power in order to reduce the SEP performance and complexity cost. We can obtain an equivalent channel gain and sort the nodes with ascending order. According to the SNR, an optimal set of relay nodes is selected and the optimal factors of power allocation is derived by the Lagrange multiplier and the steepest descent methods. Then, the power allocation among source and relay nodes is conducted, and the proposed scheme further reduces the symbol error probability. This proposed algorithm analyzes the relation between the number of optimal relay and SNR, and gives theoretical derivation. The simulation results show that the proposed EOPRS algorithm can effectively reduce the system SEP and improve power efficiency, and thus it is superior to those of the EPRS and NPRS schemes.

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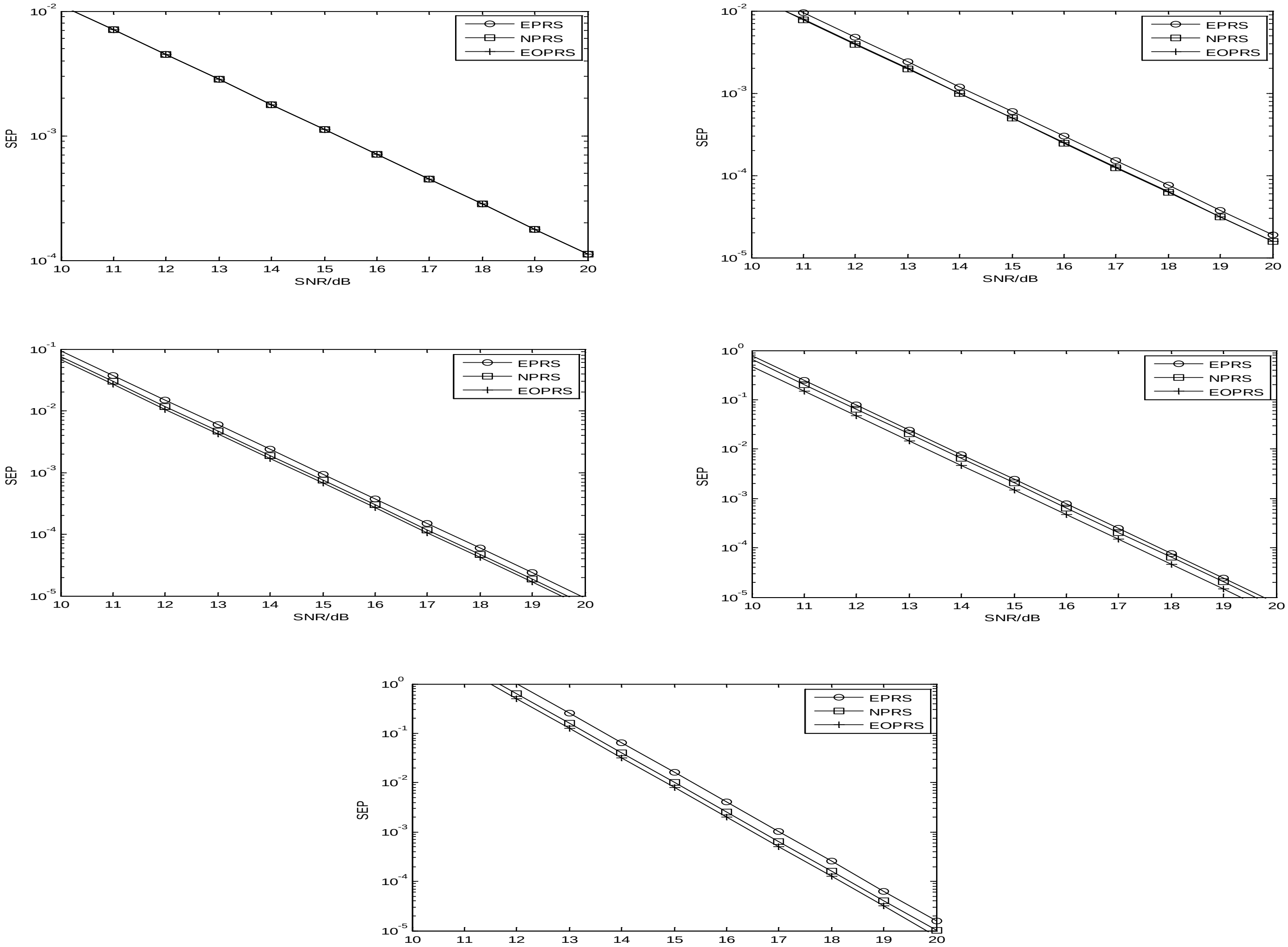


Fig. 6. SEP of three schemes with different relay collections

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