Analysis of the Level Crossing Rate of Wireless Communication System in the Presence of Nakagami-*m* fading and Double Nakagami-*m* Co-channel Interference

Dragana Krstić, Dušan Stefanović, Ivan Vulić, and Mihajlo Stefanović

***Abstract*—**In this article, the ratio of Nakagami-*m* random process (RP) and product of two Nakagami-*m* random processes is observed. This ratio presents signal-to-interference envelope ratio. Here is actually represented an interference limited Nakagami-*m* fading environment with Nakagami*Nakagami co-channel interference. Formula for the level crossing rate (LCR) of defined ratio is performed and graphically presented. The influence of the channel parameters on the LCR is discussed based on these graphs. The calculation of the performance of wireless systems is performed in order to achieve better reliability of wireless links used for command and information systems and for military purposes.

***Keywords*—**Co-channel interference, level crossing rate, Nakagami-*m* fading, random process.

I. Introduction

THE ratio of random processes (RPs) is important characteristic in performance analysis of wireless communication system in fading environments. The properties of the ratio of RPs make possible calculation of the first order (outage probability, bit error probability and wireless communication system capacity) and second order system performance (level crossing rate and average fade duration) [1, 2]. Also, the product of RPs is quite important and has been studied in the literature.

As the cascading fading models became important, the analysis was expanded. So, the model of the double Rayleigh (Rayleigh*Rayleigh) channel fading model, as convenient when transmitter and receiver are both moving, is considered in [3]. In [4], expressions for the level crossing rate (LCR) and the average fade duration (AFD) of the product of two Nakagami-*m* RPs are derived. They took advantage for calculation the second order statistics of multiple input multiple output (MIMO) keyhole fading channels.

A large number of distributions can well describe the behavior of the mobile radio signal, but the Nakagami-*m* distribution has special importance because of big range of applicability and good fitting with measuring in the mobile radio channels [1]. Therewithal, Nakagami-*m* distribution is general distribution. For different values of fading severity parameter *m*, other distributions can be obtained as special cases. For m=1, Nakagami-*m* RP becomes Rayleigh RP, for m=1/2, Nakagami-*m* turns into one sided Gaussian RP [5]. Therefore, higher order statistics of the Nakagami-*m* distribution is analyzed and exact closed-form formulas for the LCR and the AFD are derived in [6].

In this paper, we will derive an expression for the LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs in closed form. This means that the signal envelope is influenced by Nakagami-*m* fading, and co-channel interference (CCI) suffers double Nakagami-*m* fading. Because of generality of this distribution, the formula for the LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs becomes suitable for calculating the LCR of the ratio of Rayleigh RPs and product of two Rayleigh RPs, and some other combinations of fading distributions. This scenario is discussed in [7]. There, formula for the LCR of the ratio of Rayleigh random variable and product of two Rayleigh random variables was not performed in closed form. Using new formula, the AFD of wireless relay system operating over Rayleigh multipath fading environment in the presence of double Rayleigh co-channel interference is possible to be evaluated.

The paper is organized so that in section II, LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs is derived and graphically presented in Section 3. Analysis of graphical results is done to evaluate the impact of fading parameters and signal powers.

II. LCR of the Ratio of Nakagami-*m* Random Process and Product of Two Nakagami-*m* Random Processes

Random variables y_i, i=1,2,3, follow Nakagami-*m*

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distribution:

$$p_{y_i}\left(y_i\right) = \frac{2}{\Gamma\left(m_i\right)}\left(\frac{m_i}{\Omega_i}\right)^{m_i} y_i^{2m_i-1} e^{-\frac{m_i}{\Omega_i} y_i^2}, \ y_i \geq 0, \tag{1}$$

where Ω_i , i=1,2,3, indicates average powers of x_i; m_i is fading severity parameter of x_i and by $\Gamma(\cdot)$ the Gamma function is denoted.

Random variable y is defined as the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs:

$$y = \frac{y_1}{y_2 y_3}, \tag{2}$$

and y_1 is then:

$$y_1 = y y_2 y_3 . \tag{3}$$

The first time derivative of RP y is:

$$\dot{y} = \frac{\dot{y}_1}{y_2 y_3} - \frac{y_1 \dot{y}_2}{y_2^2 y_3} - \frac{y_1 \dot{y}_3}{y_2 y_3^2} . \tag{4}$$

The first time derivative of Nakagami-*m* RP is Gaussian RP. Thus, $\dot{y}_i$ are Gaussian RPs. Linear transformation of Gaussian RPs is Gaussian RP. Hence, $\dot{y}$ is distributed according to Gaussian distribution.

The average value of $\dot{y}$ is:

$$\overline{\dot{y}} = \frac{\overline{\dot{y}_1}}{y_2 y_3} - \frac{y_1 \overline{\dot{y}_2}}{y_2^2 y_3} - \frac{y_1 \overline{\dot{y}_3}}{y_2 y_3^2} = 0 , \tag{5}$$

as it applies already known fact that the Nakagami-*m* [6] fading envelope has zero-mean Gaussian distributed time derivatives:

$$\overline{\dot{y}_1} = \overline{\dot{y}_2} = \overline{\dot{y}_3} = 0 . \tag{6}$$

The variance of $\dot{y}$ is given by:

$$\sigma_{\dot{y}}^2 = \frac{1}{y_2^2 y_3^2}\sigma_{\dot{y}_1}^2 + \frac{y_1^2}{y_2^4 y_3^2}\sigma_{\dot{y}_2}^2 + \frac{y_1^2}{y_2^2 y_3^4}\sigma_{\dot{y}_3}^2 , \tag{7}$$

with:

$$\sigma_{\dot{y}_i}^2 = \pi^2 f_m^2 \frac{\Omega_i}{m_i} , \tag{8}$$

where f_m is maximal Doppler frequency.

After appropriate shifts, $\sigma_{\dot{y}}^2$ gets in shape:

$$\sigma_{\dot{y}}^2 = \pi^2 f_m^2 \left(\frac{1}{y_2^2 y_3^2}\frac{\Omega_1}{m_1} + \frac{y_1^2}{y_2^4 y_3^2}\frac{\Omega_2}{m_2} + \frac{y_1^2}{y_2^2 y_3^4}\frac{\Omega_3}{m} \right) =$$

$$= \pi^2 f_m^2 \frac{1}{y_2^2 y_3^2}\frac{\Omega_1}{m_1}\left(1 + \frac{y_1^2}{y_2^2}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1} + \frac{y_1^2}{y_3^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1} \right) =$$

$$= \pi^2 f_m^2 \frac{1}{y_2^2 y_3^2}\frac{\Omega_1}{m_1}\left(1 + y^2 y_3^2 \frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1} + y^2 y_2^2 \frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1} \right) \tag{9}$$

The joint probability density function (JPDF) of y, $\dot{y}$, y_2 and y_3 is:

$$p_{y\dot{y}y_2y_3}\left(y\dot{y}y_2y_3\right) = p_{\dot{y}}\left(\dot{y} / yy_2y_3\right) p_{yy_2y_3}\left(yy_2y_3\right) . \tag{10}$$

Here is valid:

$$p_{yy_2y_3}\left(yy_2y_3\right) = p_y\left(y / y_2y_3\right) p_{y_2y_3}\left(y_2y_3\right) =$$

$$= p_y\left(y / y_2y_3\right) p_{y_2}\left(y_2\right) p_{y_3}\left(y_3\right) , \tag{11}$$

$$p_y\left(y / y_2y_3\right) = \left|\frac{dy_1}{dy}\right| p_{y_1}\left(yy_2y_3\right) , \tag{12}$$

$$\frac{dy_1}{dy} = y_2 y_3 . \tag{13}$$

Now, the JPDF of y and $\dot{y}$ is obtained in the form of:

$$p_{y\dot{y}}\left(y\dot{y}\right) = \int_0^\infty \int_0^\infty dy_2 dy_3\, p_{y\dot{y}y_2y_3}\left(y\dot{y}y_2y_3\right) =$$

$$= \int_0^\infty dy_2 \int_0^\infty dy_3\, p_{\dot{y}}\left(\dot{y} / yy_2y_3\right) \cdot$$

$$\cdot y_2 y_3 p_{y_1}\left(yy_2y_3\right) p_{y_2}\left(y_2\right) p_{y_3}\left(y_3\right) . \tag{14}$$

The expected rate, N_y, at which the envelope crosses a given threshold in the positive direction is given by [8]:

$$N_y = \int_0^\infty d\dot{y}\, \dot{y}\, p_{y\dot{y}}\left(y\dot{y}\right) . \tag{15}$$

By inserting the corresponding formulas into the subintegral function of the upper integral, we have LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs:

$$N_y = \int_0^\infty dy_2 \int_0^\infty dy_3 \int_0^\infty d\dot{y}\, \dot{y}\, p_{\dot{y}}\left(\dot{y} / yy_2y_3\right)$$

$$\cdot y_2 y_3 p_{y_1}\left(yy_2y_3\right) p_{y_2}\left(y_2\right) p_{y_3}\left(y_3\right) =$$

$$= \int_0^\infty dy_2 \int_0^\infty dy_3\, y_2 y_3 p_{y_1}\left(yy_2y_3\right) p_{y_2}\left(y_2\right) p_{y_3}\left(y_3\right) \frac{1}{\sqrt{2\pi}}\sigma_{\dot{y}} =$$

$$= \int_0^\infty dy_2 \int_0^\infty dy_3\, y_2 y_3 \frac{1}{\sqrt{2\pi}} \pi f_m \cdot$$

$$\cdot \frac{1}{y_2 y_3} \frac{2}{\Gamma\left(m_1\right)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(yy_2y_3\right)^{2m_1-1} e^{-\frac{m_1}{\Omega_1}\left(yy_2y_3\right)^2} .$$

$$\cdot\frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} y_2^{2m_2-1} e^{-\frac{m_2}{\Omega_2}y_2^2} \cdot \frac{2}{\Gamma(m_3)}\left(\frac{m_3}{\Omega_3}\right)^{m_3} y_3^{2m_3-1} e^{-\frac{m_3}{\Omega_3}y_3^2}$$

$$\cdot\frac{\Omega_1^{1/2}}{m_1^{1/2}}\left(1+y^2y_3^2\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+y^2y_2^2\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2}=$$

$$=\frac{1}{\sqrt{2\pi}}\pi f_m\frac{\Omega_1^{1/2}}{m_1^{1/2}}\frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)}\cdot$$

$$\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\cdot y^{2m_1-1}\cdot$$

$$\int_0^\infty dy_2\int_0^\infty dy_3\, y_2^{2m_1+2m_2-2}y_3^{2m_1+2m_3-2}\cdot$$

$$e^{-\frac{m_1}{\Omega_1}(yy_2y_3)^2-\frac{m_2}{\Omega_2}y_2^2-\frac{m_3}{\Omega_3}y_3^2}\left(1+y^2y_3^2\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+y^2y_2^2\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2} \quad (16)$$

The double integral obtained in (16) can be solved using Laplace approximation formula [9]:

$$\int_0^\infty dy_2\int_0^\infty dy_3\, g(y_{20},y_{30})e^{\lambda f(y_{20},y_{30})}=$$

$$=\frac{\pi}{\lambda}\frac{g(y_{20},y_{30})}{A(y_{20},y_{30})}e^{\lambda f(y_{20},y_{30})}. \quad (17)$$

The matrix A is:

$$A(y_{20},y_{30})=\begin{vmatrix}\frac{\partial^2 f(y_{20},y_{30})}{\partial y_{20}^2} & \frac{\partial^2 f(y_{20},y_{30})}{\partial y_{20}\partial y_{30}}\\ \frac{\partial^2 f(y_{20},y_{30})}{\partial y_{20}\partial y_{30}} & \frac{\partial^2 f(y_{20},y_{30})}{\partial y_{30}^2}\end{vmatrix}. \quad (18)$$

The critical point is determining the values for which $\frac{\partial f(y_{20},y_{30})}{\partial y_{20}}=0$, and $\frac{\partial f(y_{20},y_{30})}{\partial y_{30}}=0$, i.e., the second derivatives are equal to zero.

Comparing (16) and (17), these functions are set:

$$g(y_2,y_3)=y_2^{2m_1+2m_2-2}y_3^{2m_1+2m_3-2}\cdot$$

$$\left(1+y^2y_3^2\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+y^2y_2^2\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2} \quad (19)$$

$$f(y_2,y_3)=-\frac{m_1}{\Omega_1}(yy_2y_3)^2-\frac{m_2}{\Omega_2}y_2^2-\frac{m_3}{\Omega_3}y_3^2 \quad (20)$$

In this manner, the LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs is derived. Actually, an expression for the LCR of wireless system operating in multipath fading environment under the influence of Nakagami-*m** Nakagami-*m* co-channel interference is obtained in closed form.

III. Analysis of Graphical Results

In this work, the level crossing rate of the ratio of Nakagami-*m* random processes and product of two Nakagami-*m* random processes is determined. The LCR is shown in two figures with different fading parameters and signal powers.

We use next fading parameters: m_1 for Nakagami-*m* fading envelope in numerator, and m_2 and m_3 for distributions in denominator, i.e. for Nakagami-*m**Nakagami-*m* co-channel interference.

It is possible to see from Figs. 1 and 2 that LCR increases for small values of resulting signal envelope, achieves maximum and start to decline for higher values of resulting signal envelope.

One can see from Fig. 1 the impact of signal powers on the LCR. For smaller values of Ω_1, and low level of y, LCR grows faster. For higher values of y, LCR is lower for less the value of Ω_1. For lower values of signal powers Ω_2 and Ω_3, LCR is smaller for small values of resulting y and increases for higher values of resulting signal envelope.

In Fig. 2, the influence of Nakagami-*m* fading severity parameter m on the LCR is presented. For increasing of m_1, for small values of envelope y, LCR has less value, while LCR getting bigger for higher y. For bigger resulting signal envelope y, and both fading severity parameter m_2 and m_3, LCR is lower. It is also lower for less the value of the resulting signal envelope y, and both fading severity parameters m_2 and m_3.

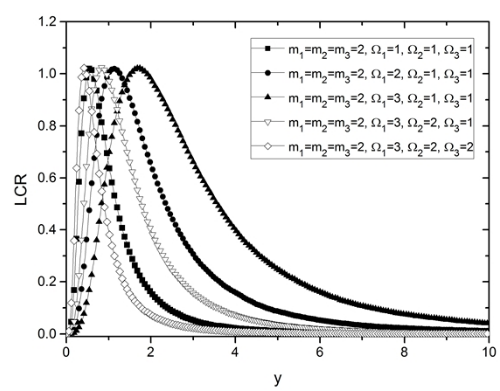

Fig.1. LCR of the ratio of Nakagami-*m* random process and product of two Nakagami-*m* random processes

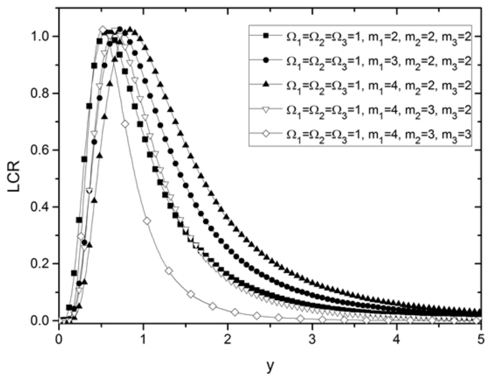

Fig.2. LCR of ratio of SIR in Nakagami-*m* fading channel with double Nakagami-*m* interference

The system performance is better for lower values of the level crossing rate.

Due to the generality of Nakagami distribution, for m_1 =1, the LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs converts to the LCR of the ratio of Rayleigh RP and product of two Nakagami-*m* RPs. For m_1 =1 and m_2=1 or m_3=1, the LCR of the ratio of Nakagami-*m* RP and product of two Nakagami-*m* RPs turns into the LCR of the ratio of Rayleigh RP and product of Rayleigh RP and Nakagami-*m* RP. Last case is for m_1=m_2=m_3=1. This means that LCR of the ratio of Nakagami-m RP and product of two Nakagami-m RPs becomes the LCR of the ratio of Rayleigh RP and product of two Rayleigh RPs, which case is earlier analyzed in [7].

IV.Conclusion

In this paper, the LCR of the ratio of Nakagami-*m* random process and product of two Nakagami-*m* random processes is obtained in closed form. This expression for LCR can be used for evaluation the AFD of wireless communication system operating in Nakagami-*m* multipath fading channel in the presence of double Nakagami-*m* co-channel interference. Beside, for special values of Nakagami-*m* fading severity parameters m_i, the LCR of the ratio of Rayleigh RP and product of two Rayleigh RPs can be evaluated.

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