A method of improving SCR for millimeter wave FM-CW radar without knowledge of target and clutter statistics

Fumio Nishiyama, and Hideo Murakami

Abstract—Frequency-modulated-continuous-wave (FM-CW) radars transmit a waveform whose frequency changes linearly in time. Received echoes of these radars can be categorized into two types due to either targets or clutter. Generally, the received target signals have a stronger correlation with respect to different carrier frequencies than that of the received clutter signals. This paper discusses a method of improving signal-to-clutter ratio (SCR) for millimeter wave FM-CW radar based on this statistical difference between the target and the clutter. The method first approximates an autocorrelation function from received signals using a numerical averaging. A power spectrum of received signals is obtained by taking discrete Fourier-transform of this autocorrelation. A target power spectrum is estimated from the power spectrum of received signals exploiting the statistical difference, and a matched filter is then designed from this target power spectrum. The matched filter is used to improve SCR. Performance of the method is evaluated experimentally for 60GHz band FM-CW radar. The simulation shows that the proposed method improves SCR better than competing with other methods.

Keywords—FM-CW radar, Ground clutter, Matched filter, Millimeter wave.

I. INTRODUCTION

Millimeter wave frequency-modulated-continuous-wave (FM-CW) radars are studied for use as an automotive sensor of the intelligent transportation system, and in fact mounted in several cars. FM-CW radar transmits a waveform whose frequency changes linearly in time. Such FM-CW radar can increase transmission energy without increasing peak power by employing continuous waves instead of pulses used in usual radar systems [1]. This property is particularly advantageous for the automotive radar [2], [3]. The automotive radar, being placed at a position near the ground, inevitably receives clutters from various objects such as road asphalt, sidewalk lines, and objects on the sidewalks. Whereas, the target such as an automobile is modeled as a relatively large plane with a smooth surface.

The clutter is a major factor that causes a false alarm in target detection. When the target is moving with respect to the ground clutter, discriminating the target from the clutter is relatively easy by exploiting the Doppler-effect. However, when the target is at rest, distinguishing it from the clutter is difficult. For such cases, improving signal-to-clutter ratio (SCR) is vitally important. For pulse radar system, a method using wavelet-transform exploits the statistical difference between the target and the clutter to improve SCR [4], and also integration processing is used [5].

When the clutter with respect to different carrier frequencies is assumed to be statistically white [6]-[8], the method of improving SCR exploiting the statistical difference has been proposed for FM-CW radar [9], [10]. However, in many practical cases, the clutter statistics is not white. Therefore, assuming that target signals with different carrier frequencies have a stronger correlation than that of clutter [7]-[11], we propose a method of improving SCR which does not require pre-knowledge of the correlation for the target and the clutter [11]. In this method, an autocorrelation function is first computed from received signals with different carrier frequencies. A power spectrum of received signals is then computed by taking discrete Fourier-transform (DFT) of the received signal autocorrelation function. A target power spectrum is estimated from the power spectrum of received signals using the statistical difference between the target and the clutter. Finally, a matched filter is designed from this target power spectrum [9]-[11].

This paper is organized as follows. In Chap. 2, the FM-CW radar system is briefly explained and received signal models are formulated as stochastic processes. In Chap. 3, the proposed methods of estimating the target power spectrum, and of a procedure of exploiting the estimated power spectrum for increasing SCR are described. In Chap. 4, performance of the proposed method is analyzed by using the data measured by 60GHz band FM-CW radar. In Chap. 5, the performance is analyzed in detail using a moving average (MA) model for the target and the clutter.

II. FM-CW RADAR

A. Stochastic Process for Received Signals

A block diagram of a double antenna FM-CW radar system studied in this paper is shown in Fig. 1. A signal generator generates a train of triangular pulses like a saw-tooth shape, and a modulator converts the train of pulses into a carrier frequency

Manuscript received March 15, 2007; Revised received October 29, 2007.
Fumio Nishiyama and Hideo Murakami are with the information and computer engineering, Kanazawa Institute of Technology, Nonoichi, Ishikawa, Japan (phone: +81-76-248-1100; e-mail: circus@venus.kanazawa-it.ac.jp, murakami@infor.kanazawa-it.ac.jp).
waveform sweeping its frequency according to the pulse train. Then the waveforms are radiated into space through a transmitting antenna.

Echoes reflected from objects, either the targets or the clutter, are received via a receiving antenna. The received carrier frequency waveform is converted into a base-band frequency waveform by a demodulator and then sampled. The sampled discrete signal is multiplied by a window function dividing it into blocks and then DFT of each block of samples is computed.

Fig. 2(a) shows frequency variation of the transmitted waveform for the FM-CW radar employing the saw-tooth modulation [5]. The carrier frequency linearly varies from \( f_{\text{min}} \) to \( f_{\text{max}} \) and repeats this cycle as shown in the figure. A time interval for the frequency change cycle is denoted as \( T \).

At the receiver, the sampler samples the base-band waveform with a faster rate than the difference \( f_{\text{max}} - f_{\text{min}} \). Fig. 2(b) shows a series of windows to be multiplied to the sampled signals. The window multiplication divides the received samples into blocks of \( N \) samples, and then the \( N \)-point DFT of each block is computed.

The \( r \)-th block samples are written as

\[
r_r(n) = r(IN+n), \quad 0 \leq n \leq N-1,
\]

where \( r(IN+n) \) is the sampled signal before the window multiplication. The \( N \)-point DFT of \( r \)-th block samples is denoted as \( R_r(k) \), \( 0 \leq k \leq N-1 \). The sampling rate being chosen to be faster than the difference between \( f_{\text{min}} \) and \( f_{\text{max}} \), there exist DFT values \( R_r(k_m) \) in such a way that \( k_m \) corresponds to the frequency

\[
f_m = f_{\text{min}} + \frac{f_{\text{max}} - f_{\text{min}}}{M-1} k_m, \quad 0 \leq m \leq M-1.
\]

These DFT values are arranged in the \( M \)-dimensional vector as

\[
x_r = [x_r(0) \quad x_r(1) \ldots \quad x_r(M-1)]
= [R_r(k_0) \quad R_r(k_1) \ldots \quad R_r(k_{M-1})].
\]

The \( m \)-th entry \( x_r(m) \) of the vector is the DFT value corresponding to the frequency \( f_m \). The small letters are used, because the vectors will be treated as a stochastic process with respect to \( m \) although the index \( m \) stands for the frequency \( f_m \).

Since only these vectors \( x_r \) are going to be used for improving SCR, they are simply called received signals in followings. Using DFT is particularly suitable because the transmitting waveform is periodic with the time interval \( T \), and a delay of the received waveform due to the distance between the antennas and the object does not change the absolute values of DFT.

Let \( y_T \) and \( y_C \) be stochastic processes corresponding to received sampled signals due to targets and clutter respectively. The numerical average of these \( L \) vectors is introduced as

\[
y = \frac{1}{L} \sum_{l=0}^{L-1} x_l = [y(0) \quad y(1) \ldots \quad y(M-1)].
\]

When \( L \) is large enough, \( y \) can be regarded as a stochastic process. It should be emphasized that the \( m \)-th entry of \( y \) corresponds to the carrier frequency \( f_m \), and does not indicate time as in a usual stochastic process. Under this interpretation, \( y \) is written as

\[
y = P_T y_T + P_C y_C,
\]

where \( P_T \) and \( P_C \) are probabilities of occurrence of the targets and the clutter.

**B. Power Spectrum of Target Signals**

The autocorrelation function of \( y \) is defined as

\[
\phi_T(\tau) = \mathbb{E}\{y(m) y(m+\tau)\}, \quad -M/2 \leq \tau \leq M/2-1,
\]

where \( \mathbb{E}\{\cdot\} \) denotes the expectation operation [12]. Because the length of the vectors is finite, the autocorrelation function depends on the variable \( m \). However, to avoid complexity in the following analysis, we concede that \( y \) is stationary, and that the
the autocorrelation function given by (6) is well-defined. The power spectrum of the process is defined as the M-point DFT of the autocorrelation function,

\[ \Phi_T(k) = \sum_{\tau=-M/2}^{M/2-1} \phi_T(\tau) \exp(-j2\pi\tau k/M), \quad 0 \leq k \leq M-1. \]  

(7)

The stochastic process \( y \) is composed of the target stochastic process \( y_T \) and the clutter stochastic process \( y_C \) as seen by (5). The autocorrelation functions of \( y_T \) and \( y_C \) are denoted as \( \phi_T \) and \( \phi_C \); the power spectra of \( y_T \) and \( y_C \) are denoted as \( \Phi_T \) and \( \Phi_C \).

As a wavelength of carrier changes, the reflection from the ground clutter having a rough surface scatters more than that of the target having a smooth surface. Therefore, the target signals having a smooth surface exhibit peaks at \(-k_p\) and \(k_p\) for obtaining \( k_p \). We use the second order numerical derivative, the bandwidth is obtained accordingly.

The problem is to estimate \( \Phi_T(k) \) from the received signal autocorrelation function. Suppose that the bandwidth is \( 2k_p \), that is, \( \Phi_T(k) < \varepsilon \) for \( k < -k_p \) or \( k > k_p \), where \( \varepsilon \) is a small positive real number. Then the second order derivative of the received signal power spectrum \( \Phi_T(k) \) would exhibit peaks at \(-k_p \) and \( k_p \). For obtaining \( k_p \), we use the second order numerical derivative,

\[ \Delta^2 \{ \Phi_T(k) \} = \frac{\Phi_T(k+2) - 2\Phi_T(k) + \Phi_T(k-2)}{4}, \]

\[ -M-4/2 \leq k \leq M-6/2. \]  

(14)

A constant \( k_p \) is estimated from peaks of \( \Delta^2 \{ \Phi_T(k_p) \} \), and thus the bandwidth is obtained accordingly.

Knowing \( k_p \), one may estimate the target power spectrum \( \Phi_T(k) \) directly from \( \Phi_T(k) \) by the ideal low-pass filter which has the transition region \( k_p \); namely, \( \Phi_T(k) \) is obtained from each value of \( \Phi_T(k) \) as

\[ \hat{\Phi}_T(k) = \begin{cases} \Phi_T(k), & -k_p \leq k \leq k_p, \\ 0, & -M/2 \leq k < -k_p, \quad k_p < k \leq M/2 - 1. \end{cases} \]  

(15)
An alternative method is to pass the received signal autocorrelation function through a low-pass filter with the bandwidth of \(2k_{\text{hp}}\) and then take the DFT of the filter output.

**B. Received Signal Norm**

Given the target power spectrum, it is now ready to process received signals to improve SCR. We prepare the matched filter \(G(k)\) given as

\[
G(k) = \frac{1}{M} \sum_{m=0}^{M-1} \Phi_T(m) \Phi_T(k)
\]

(16)

The matched filter \(G(k)\) is normalized so that the sum of \(|G(k)|^2\), \(0 \leq k \leq M-1\), becomes \(M\).

Using the matched filter \(G(k)\), we perform the steps described in Fig. 3. The input \(x_i(m)\), \(0 \leq m \leq M-1\), is assumed to be computed according to (3) in advance. The \(M\)-point DFT \(X_i(k)\) of this input is computed, and then each DFT value is multiplied by the matched filter to obtain its output \(Z_i(k)\) as

\[
Z_i(k) = G(k)X_i(k), \quad 0 \leq k \leq M-1.
\]

(17)

Finally, the norm of this matched filter output \(Z_i(k)\) is calculated by

\[
P_i = \frac{1}{M} \sum_{k=0}^{M-1} |Z_i(k)|^2.
\]

(18)

This norm \(P_i\) takes a large value when there is the target at the time of window \(i\).

The proposed method that has been explained so far is summarized as follows:

1) The matrix \(B\) is computed from received signals in accordance with (10). The autocorrelation function of received signals \(\Phi_T\) is approximated from the matrix \(B\) using the numerical average.

2) The power spectrum of received signals \(\Phi_T(k)\) is obtained by taking DFT of the autocorrelation function \(\Phi_T\).

3) The bandwidth of the target power spectrum is estimated from \(\Phi_T(k)\) using the second order numerical derivative. The target power spectrum \(\Phi_T(k)\) is estimated by filtering from \(\Phi_T(k)\) with the low-pass filter which has the same bandwidth as the target power spectrum.

4) Finally, the matched filter is designed given by (16). The received signal norm is computed by following the procedure in Fig. 3.

**IV. ANALYSIS ON SCR**

**A. Improvement Ratio of SCR**

The received signal \(x_i(m)\) becomes either the random variable \(y_T(m)\) or \(y_C(m)\) depending on whether there is the target or the clutter at the time of window \(i\). Based on this observation, we assign, in the place of \(P_i\), two random variables

\[
Q_i = \frac{1}{M} \sum_{k=0}^{M-1} |G(k)Y_i(k)|^2, \quad i = T, C,
\]

(19)

where \(Y_T(k)\) and \(Y_C(k)\) are the \(M\)-point DFT of \(y_T(m)\) and \(y_C(m)\), respectively. Therefore, \(Q_T\) and \(Q_C\) are the norm of the matched filter output when the target or the clutter is received. Evidently, phase components of the matched filter do not affect to the computation of the norm. From the relations of \(\Phi_T(k) = (1/M)E[|Y_T(k)|^2]\), the mean of the norm is given by

\[
E[Q_i] = \frac{1}{M} \sum_{k=0}^{M-1} |G(k)|^2 \Phi_T(k), \quad i = T, C.
\]

(20)

For evaluating the method, it is necessary to compare SCRs for the cases when the matched filter is used and not used. When the matched filter is used, SCR is given as

\[
SCR = \frac{E[Q_T]}{E[Q_C]}.
\]

(21)

Substituting \(G(k) = 1\) into (21), when the matched filter is not used, SCR is given by

\[
SCR_0 = \frac{1}{M} \sum_{k=0}^{M-1} \Phi_T(k).
\]

(22)

Therefore, the improvement ratio of SCR by using the matched filter is defined by

\[
\text{Improvement Ratio} = \frac{SCR}{SCR_0}.
\]
We evaluate the performance of the method by the improvement ratio \( R_{SCR} \). This equation means that when the power spectra of the target and the clutter are the same, SCR is equal to one, and cannot be improved by using the matched filter.

### B. Comparison with Conventional Methods

As a demonstration for measuring the improvement ratio, we have used a 60GHz band FM-CW radar system. Branches with leaves of a broadleaf tree are used as the clutter, and a flat board of aluminum as the target with its surface facing to the antenna. Table I shows specifications for the demonstration. Fig. 4 shows DFTs of the signals from the target and the clutter. The target signal has a narrower bandwidth than the clutter signal; the target signal has a stronger correlation than the clutter signal.

The target power spectrum is estimated by the method described in Sec. III. As the number of target signals in the ensemble of the received signals \( x_l \), \( 0 \leq l \leq L-1 \), contained in the matrix \( B \) given by (10) increases, the accuracy of the estimate of target power spectrum improves. The method approaches the expected performance. However, when the number is small, the performance degrades accordingly. The improvement ratio \( R_{SCR} \) is plotted as a function of the number of targets in Fig. 5. The improvement ratio is compared with two conventional methods: the integration processing method \([5]\) and the discrete wavelet-transform method \([4],[13]\). For the integration processing, the improvement ratio of SCR at 5 windows is computed. For the discrete wavelet-transform method, the scaling function of 12 orders of the Daubechies wavelet is employed. The improvement ratios of the conventional methods are also exhibited in Fig. 5. The conventional methods do not have capability of learning, and thus their performances are independent of the number of targets.

As seen in the figure, the improvement ratio increases as the number of targets increases, and then saturates after the number reaches five. The improvement ratio of the proposed method after the saturation is better than the conventional methods.

### V. SIMULATION

#### A. Target and Clutter Models

For simulation, we employ the MA models for both the targets and the clutter. That is, the target signal \( y_T(n) \) and the clutter signal \( y_C(n) \) are created by

\[
y_T(n) = \sum_{u=0}^{K-1} h_T(u) w(n-u) , i=T,C, \tag{24}
\]

where \( h_i(u) \) denotes MA parameter, and \( w(n) \) is a zero mean white Gaussian. \( K \) will be referred to as the order of the model. The power spectra are given by

\[
\Phi_i(k) = \sigma_w^2 |H_i(k)|^2 , i=T,C, \tag{25}
\]

where \( \sigma_w^2 \) is the variance of \( w(n) \), and \( H_i(k) \) is DFT of \( h_i(n) \) [12].

For the MA parameters, we consider the form given by

\[
h_i(n) = \exp(-a_i|n|) , i=T,C, \tag{26}
\]

where the value \( a_i \) is a positive constant. DFT of \( h_i(n) \) is obtained as

\[
H_i(k) = \sum_{n=0}^{N-1} h_i(n) \exp(-2\pi nk / N) = \frac{(1-\exp(-2a_i))(1-(1)^k \exp(-a_iN / 2))}{1-2\exp(-a_i) \cos(2\pi k / N) + \exp(-2a_i)}. \tag{27}
\]
These DFTs are shown in Fig. 6 when $a_i=0.15$ and $a_i=0.65$. As the constant $a_i$ increases, the bandwidth of $H_i(k)$ and thus the bandwidth of the power spectrum increase. These constants are deliberately chosen to find suitable MA models for the aluminum board target and the branch clutter shown in Fig. 4. Comparing Fig. 4 and Fig. 6, one can see that the aluminum board target and the branch clutter are modeled by selecting $a_T=0.15$, and $a_C=0.65$, respectively.

B. Improvement Ratio $R_{SCR}$

By substituting $\Phi_T(k)$ of $R_{SCR}$ in (23) for $\Phi_T(k)$ in (25) and $\Phi_C(k)$ for $\Phi_C(k)=\sigma_w^2|H_C(k)|^2$, the improvement ratio of SCR by using the matched filter is given as

$$R_{SCR} = \frac{\sum_{k=0}^{M-1}|H_C(k)|^2 \sum_{k=0}^{M-1}|G(k)|^2|H_T(k)|^2}{\sum_{k=0}^{M-1}|H_T(k)|^2 \sum_{k=0}^{M-1}|G(k)|^2|H_C(k)|^2}.$$  \hspace{1cm} (28)

Assuming that the estimation is accurate, Fig. 7 shows the improvement ratio $R_{SCR}$ as the function of $a_T$ when $a_C=0.65$, and also the ratio as the function of $a_C$ when $a_T=0.15$. The ratio increases as $a_T$ decreases or $a_C$ increases. In other words, the ratio increases as the target spectrum bandwidth decreases or the clutter spectrum bandwidth increases.

C. Learning Speeds for the MA Models

Simulations are performed to see how many targets are needed to acquire a desirable performance. The target and the clutter signals are created as the MA models according to (24). Simulation parameters used in the computer simulation are listed in Table II.

Fig. 8 shows the improvement ratio $R_{SCR}$ as the function of the number of targets used in the learning. When the number of target vectors is not large enough, the ratio does not reach its value given by (28) because the target power spectrum is not accurately estimated. The ratio $R_{SCR}$ improves by increasing the value $a_c$ as expected from Fig. 7. As explained at the end of Sec. V, $A$, the aluminum board target and the branch clutter are suitably modeled as the MA models with $a_T=0.15$, and $a_C=0.65$,
respectively. The plots for $a_T=0.15$ and $a_C=0.65$ in Fig. 8 indeed exhibit the similar tendency to the plots in Fig. 5.

VI. Conclusion

This paper has introduced the method for improving SCR for FM-CW radar systems. The simulations for the target and the clutter signals. The simulations of received signals was analyzed in terms of SCR. Moreover, the performance of the proposed method is analyzed using the MA models for the target and the clutter signals. The simulations verified that the proposed method is indeed useful for improving SCR for the FM-CW radar systems.

TABLE II
Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target MA parameter</td>
<td>0.15</td>
</tr>
<tr>
<td>Clutter MA parameter</td>
<td>0.65</td>
</tr>
<tr>
<td>Received data, LxM</td>
<td>100x64 matrices</td>
</tr>
<tr>
<td>Target data vectors</td>
<td>64 dimensions</td>
</tr>
<tr>
<td>Clutter data vectors</td>
<td>64 dimensions</td>
</tr>
<tr>
<td>Number of target data vector</td>
<td>0-20</td>
</tr>
<tr>
<td>Number of clutter data vector</td>
<td>100-80</td>
</tr>
</tbody>
</table>

Fig. 8 $R_{SCR}$ as a function of the number of targets when $a_T=0.15$

References


Fumio Nishiyama received the B.E. degree in electrical engineering from Tokyo University of Science, Tokyo, in 1995, and the M.E. degree in information engineering from Kanazawa Institute of Technology, Kanazawa, in 2005. He has been in Technical Research and Development Institute, Ministry of Defense, since 1990. He is also currently with Kanazawa Institute of Technology as a doctorate student. His research interests include digital signal processing and millimeter-wave radar systems.

Hideo Murakami received the B.S. degree in electrical engineering from Kanazawa University, Kanazawa, in 1968, and the M.S. and Ph.D. degrees in electrical engineering from the University of Southern California, Los Angeles, U.S.A., in 1973 and 1977, respectively. From 1968-1970, he was employed at Mitsubishi Electric Corporation, Hyogo. Since 1977, he has been with Kanazawa Institute of Technology, Kanazawa, where he is an Associate Professor, and is currently a Professor there. His research interests include digital signal processing and coding theory.