Cavity Dynamics of a Figure of Eight Fiber laser

Zheng Zheng, Muddassir Iqbal, and Tian Yu

***Abstract*—** Pulse propagation in a birefringent medium carries due weightage and importance in passive modelocking of fiber lasers. We carried out study of nonlinear effects in birefringent medium by simulating pulse propagation using adaptive step size method, which is globally an efficient algorithm for solving Schrodinger type equations using split step Fourier method. Nonlinear optical loop mirror was realized; the results hereby are used in forming a figure of eight fiber laser using NOLM. Where NOLM behaves as a fast saturable absorber.

***Keywords*—** Nonlinear Schrodinger equation (NLSE), Self Phase modulation (SPM), cross phase modulation (XPM), Nonlinear Optical Loop Mirror (NOLM).

I. INTRODUCTION

Even a single mode fiber supports two orthogonally polarized modes with the same spatial distribution. In an ideal fiber the effective refractive indices n_x, and n_y of both the modes are identical [1]. It is pertinent to mention here that all fibers exhibit some modal birefringence ($n_x \neq n_y$); this is due to unintentional variations in the core shape and anisotropic stresses along the fiber length. However the degree of modal birefringence, $B_m = |n_x - n_y|$ and the orientation of x and y axes changes randomly over a length scale $\approx$10m unless it is catered for [1,2].

A variety of linear and nonlinear fiber properties enable new applications in photonic devices as elements of a fiber laser cavity. One of the most interesting fiber elements is the loop interferometer, also termed as Sagnac interferometer; it has transmission characteristics that can be controlled by outside influences, such as strain birefringence, current birefringence and temperature variations [3]. The nonlinear optical loop mirror (NOLM) [4] which is an example of Sagnac interferometer, has been used in a number of applications such as optical switching, passive modelocking of fiber lasers, logic gates, fiber sensors and all optical demultiplexing of data streams [5-9]. The basic concept of the conventional NOLM is based on the differential nonlinear phase shift between the counter propagating, linearly polarized light beams in the loop interferometer. Such NOLM has high/low power reflection coefficient and provides nonlinear switching by an asymmetrical coupler at the loop's input, i.e. one in which coupling coefficient differs from 50/50.

The transmission properties of the NOLM can be modified by using birefringent fiber and different polarization orientations between the counter propagating beams. The difference in mode propagation constant for different polarizations is defined as the modal birefringence [1] or phase birefringence B.

$$B = \left|n_x - n_y\right| = \frac{\left|\beta_x - \beta_y\right|}{2\pi/\lambda_0} = \frac{\left|\beta_x - \beta_y\right|}{k_0} \qquad (1)$$

n_x, n_y, β_x and β_y are the effective mode indices and mode propagation constants of the two orthogonal principle axes respectively. Since the effective mode index of the two polarizations are not the same, by injecting a linearly polarized light at 45^o to one principle axis, the polarization will evolve periodically from linear to elliptical, elliptical to circular and vice versa along the length of the fibre. The length of fibre for the light to exit with the exact same polarization as at the input is called the beat length L_B.

$$L_B = \frac{2\pi}{\left|\beta_x - \beta_y\right|} = \frac{\lambda}{B} \qquad (2)$$

Figure 1 shows a schematic diagram of how the polarization varies along a birefringent fiber.

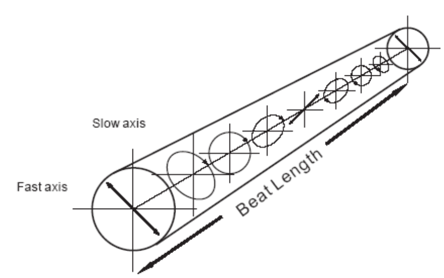

Fig. 1: A schematic diagram of how the polarization varies along a birefringent fibre when the input was polarized at 45^o to one axis.

Two pulses propagating in two orthogonal axes of a birefringent fibre at different group velocities (due to the difference in the propagation constants); they give rise to group velocity mismatch (GVM), and can be quantified by the group birefringent parameter. Group birefringence is

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proportional to the difference of the group index δn of the two orthogonal modes. δn is related to the difference in propagation constant $\delta\beta$, and hence by knowing $\delta\beta$; we can determine the group birefringence, and $\delta\beta$ is given by.

$$\delta\beta = k_0 B \tag{3}$$

Applying Taylor expansion on equation 3 results into following expression:

$$\delta\beta(\omega) = \delta\beta_0 + \delta\beta_1(\omega - \omega_o) + \frac{\delta\beta_2}{2!}(\omega - \omega_0)^2 + \frac{\delta\beta_3}{3!}(\omega - \omega_0)^3 + \ldots \tag{4}$$

$$\delta\beta_m = \left.\frac{d^m \delta\beta}{d\omega^m}\right|_{\omega=\omega_0}, (m = 0,1,2,3\ldots) \tag{5}$$

The parameter $\delta\beta_1$ is called the group birefringent parameter, and is proportional to the difference in group index of the axes. Differentiating $\delta\beta_1$ with respect to ω gives the difference in the GVD parameters $\delta\beta_2$. Rest of the paper is organized as follows: Part two comprising of discussions regarding various passive modelocking mechanisms; part three describes pulse propagation along with introducing some equations which are to be used in this work, part four discusses simulation process and results, whereas part five comprises of conclusion.

II. PASSIVE MODE LOCKING TECHNIQUES

Several different physical mechanisms in use for passive mode-locking: saturable absorption (SA) {slow and fast}, Nonlinear Polarization Rotation (NLPR), Additive Pulse mode-locking, Colliding Pulse, and Kerr lens mode-locking.

A. Saturable Bragg Reflector (SBR)

It can be thought of as intensity and wavelength-dependent mirror constructed using a semiconductor material. The wavelength dependence comes from a Bragg grating; that is fabricated in the semiconductor host and acts as a mirror. This grating also includes a saturable absorption (SA) medium, in the form of quantum wells (or quantum dots); imparting intensity dependent reflection properties to the saturable Bragg reflector. If low-intensity light (at Bragg wavelength) is incident on such a saturable Bragg reflector, it will all be absorbed. However, a high enough intensity (at Bragg wavelength) incident on the SBR, will be almost completely reflected. Both of these characteristics are a direct result of the saturable absorber located at the front of the Bragg grating.

B. Slow saturable absorber mode-locking

It uses a saturable absorber, with the exception that it is not able to saturate on a time scale local to the pulse. The slow saturable absorber saturates after some leading part of the pulse has pumped it. Longer life time of slow saturable absorber makes it stays saturated for some finite time after the peak of the pulse has passed, hence asymmetrically shortening the pulse by only clipping off its leading edge. This effect is then combined with the saturation of the gain medium which changes the amount of available gain. If the gain medium saturation is visualized (thus reducing the available gain) after the saturable absorber saturates; it is able to clip off the trailing edge of pulse. It is the combined action of both saturable absorption mechanisms, which forms the basis of slow saturable absorption mode-locking. Rare-earth- doped fiber-based gain media have relatively long lifetimes, hence resulting pulse widths are large.

C. Nonlinear polarization Rotation

Fiber exhibits a nonlinear birefringence that depends on the local intensities of the two orthogonally polarized field components. As a result, an elliptically polarized pulse will have its 'x' and 'y' components experience different phase shifts, thus rotating the polarization ellipse. Since this is an intensity-dependent process, it rotates the polarization of a pulse by different amounts depending on the pulse's local intensity. Nonlinear mechanisms of self phase modulation (SPM) and cross phase modulation (XPM) action on an electric field can be approximated by:

$$SPM \rightarrow A_x(z+\Delta z, t) = A_x(z,t)\, e^{i\gamma |A_x(z,t)|^2 \Delta z} \tag{6}$$

$$XPM \rightarrow A_x(z+\Delta z, t) = A_x(z,t)\, e^{i\gamma \frac{2}{3} |A_y(z,t)|^2 \Delta z} \tag{7}$$

Noting that fields propagate with an e^{ikz} dependence (where $k = 2\pi n/\lambda$), it is realized that the SPM and XPM effects may be interpreted as introducing a polarization-dependent, intensity-dependent refractive index. Considering the superposition of two linearly polarized waves, electric field in phasor notation can be written as:

$$E(z) \approx \begin{bmatrix} A_x(z,\tau)\, e^{ikz} e^{i\Delta k_x z} \\ A_y(z,\tau)\, e^{ikz} e^{i\Delta k_y z} \end{bmatrix} \tag{8}$$

Where

$$\Delta k_x = \frac{2\pi}{\lambda}\gamma\left(|A_x(z,\tau)|^2 + \frac{2}{3}|A_y(z,\tau)|^2\right) \tag{9}$$

and

$$\Delta k_y = \frac{2\pi}{\lambda}\gamma\left(|A_y(z,\tau)|^2 + \frac{2}{3}|A_x(z,\tau)|^2\right) \tag{10}$$

Real fields a_x and a_y and the phase ϕ accounts for any initial phase difference between the two. Polarization of such a field is governed by the following ellipse:

$$\left(\frac{E_x(z,\tau)}{a_x(z,\tau)}\right)^2 + \left(\frac{E_y(z,\tau)}{a_y(z,\tau)}\right)^2 - 2\left(\frac{E_x(z,\tau)}{a_x(z,\tau)}\right)\left(\frac{E_y(z,\tau)}{a_y(z,\tau)}\right) \cos\left(\left[\Delta k_y - \Delta k_x\right]z + \phi\right) = \sin^2\left(\left[\Delta k_y - \Delta k_x\right]z + \phi\right) \tag{11}$$

Angle of the ellipse can be written as:

$$\alpha = \frac{1}{2}\tan^{-1}\left[\left(\frac{2a_x(z,\tau)a_y(z,\tau)}{a_x^2(z,\tau)-a_y^2(z,\tau)}\right)\cos\left(\left[\Delta k_y-\Delta k_x\right]z+\phi\right)\right] \quad (12)$$

Polarization at a fixed location L_1 is shown in figure 2(a), polarization due to a slightly perturbed location L_2 is shown in figure 2(b) (assuming $\tau_2 \sim \tau_1$)

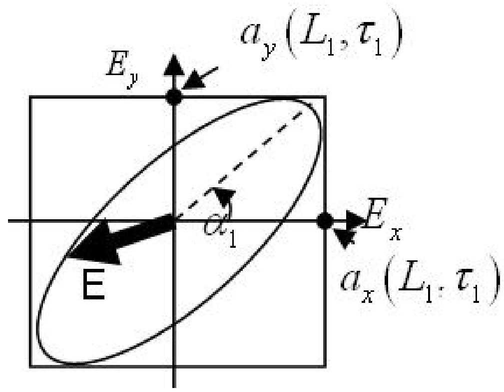

Fig. 2(a): Tilted ellipse of a plane wave at L_1.

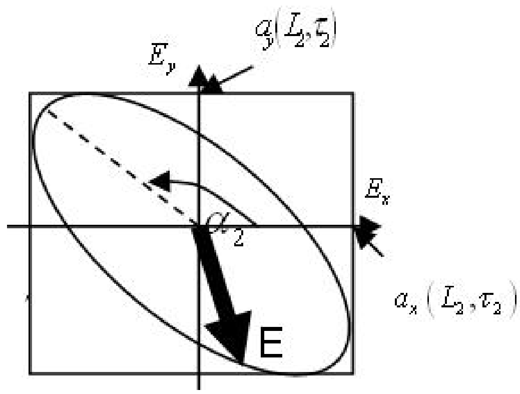

Fig. 2(b): Tilted ellipse of a plane wave at L_2.

D.Kerr lens mode-locking

It is used most notably in Ti: Sapphire laser cavities, relies on a crystal's nonlinear refractive index to create a device that acts as an intensity-dependent lens. As a result, inserting an aperture behind a suitable Kerr material effectively filter continuous wave light out of a laser cavity as it does not have a high enough intensity to be tightly focused. Consequently, short pulses, which possess high intensities, are focused more acutely and end up with a lower threshold than continuous wave light.

E. Additive pulse mode-locking

It relies on the nonlinear process of self phase modulation in a passive optical fiber cavity, coupled to the main cavity by using a beam splitter. The pulse in the fiber cavity interferes with the pulse circulating in the main cavity at the beam splitter. Self phase modulation action on pulses in the fiber cavity, made them to interfere constructively at their peaks and destructively on their wings.

F. Colliding pulse mode-locking

It relies on pulse interference in a saturable absorber (SA) medium. A saturable absorber with high saturation intensity can only be saturated by allowing more pulses incident on the saturable absorber at the same time. This can be made practical only in a linear or ring cavity, where counter-propagating pulses can be made to saturate a dye jet (located in the center of the cavity) when they overlap in it.

III. EQUATIONS GOVERNING PULSE PROPAGATION

Based on the birefringent fiber nonlinear optical loop mirror can be developed just by joining the two ends of a coupler using birefringent fiber [10]. (See figure 3 below)

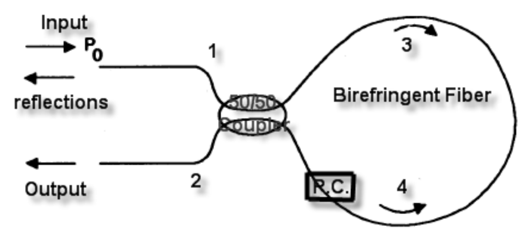

Fig. 3: Schematic of a Nonlinear Optical Loop Mirror.

The input is provided from port number 1 of coupler and after the pulse is propagated in the loop from two opposite ends the output is achieved at the port number 2. We had simulated NOLM based on birefringent fiber, and results were published in WSEAS SMO-07 held in Beijing [11]. Figure of eight fiber laser can be developed using nonlinear optical loop mirror (NOLM). In order to complete the loop to make a laser cavity, port number 1 and port number 2 of the coupler can also be joined, (see figure 4) [10]. Erbium doped fiber has been realized as gain medium in between port number 1 and 2, output coupler is between gain medium and port number 1; where as input source was provided in the other half of the F8FL.

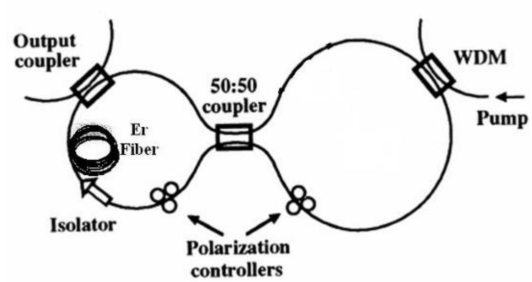

Fig.4: Schematic of a figure of eight fiber laser.

Pulse propagation in the birefringent fiber loop is governed by the following coupled equations [11]:

$$\frac{\partial A_x}{\partial z}+\beta_{1x}\frac{\partial A_x}{\partial t}+\frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2}+\frac{\alpha}{2}A_x = i\gamma\left(\left|A_x\right|^2+B\left|A_y\right|^2\right)A_x \quad (13)$$

$$\frac{\partial A_y}{\partial z}+\beta_{1y}\frac{\partial A_y}{\partial t}+\frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2}+\frac{\alpha}{2}A_y = i\gamma\left(\left|A_y\right|^2+B\left|A_x\right|^2\right)A_y \quad (14)$$

The coupling parameter B depends on the ellipticity angle θ and can vary from 2/3 to 2 for values of θ in the range $0 \rightarrow \pi/2$; for a linearly birefringent fiber $\theta = 0$, and B = 2/3. For a circularly birefringent fiber $\theta = \pi/2$ and B = 2; wave number difference between two modes = $2\beta = 2\pi\Delta n/\lambda$. For the gain medium; pulse propagation through erbium doped fiber has been simulated using a NLS equation including gain and gain dispersion. The modified NLS equation is in fact a complex Ginzberg Landau equation of the following form [7]:

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + igT_2^2\frac{\partial^2 A}{\partial T^2} + \frac{g}{2}A + i\gamma\left|A^2\right|A \quad (15)$$

The gain dispersion term in Eq. (15) incorporates the finite gain bandwidth of the erbium fiber. The inversion of erbium ions and gain g was assumed to be constant along the length of erbium fiber for each round trip. The gain g is governed by the following expression:

$$g - \frac{g - g^{eq}}{T_1} = \frac{g}{T_1}\frac{I}{I_{sat}} \quad (16)$$

g^{eq} is the unsaturated gain, T_1 is the population relaxation time. I is the average optical intensity over the fiber area and I_{sat} is the average saturation intensity over the fiber mode area. The population relaxation time is in milli seconds, while a round trip is in nano seconds, hence in our simulations we assumed a constant gain term for each round trip. The input pulse is linearly polarized at 45 degrees to the principal axis, its phase is as follows:

$$ux = u.*\exp(i\pi/4); uy = u.*\exp(-i\pi/4) \quad (17)$$

Input coupling in 50/50 splitter is as follows ($\alpha = 0.5$):

$$\begin{aligned} u_3^{xy} &= \sqrt{\alpha} * u_1^{xy} + i(1-\alpha) * u_2^{xy} \\ u_4^{xy} &= i\sqrt{(1-\alpha)} * u_1^{xy} + \sqrt{\alpha} * u_2^{xy} \end{aligned} \quad (18)$$

As the pulse propagating counter clockwise from point 4 passes through a polarization controller that rotates the polarization by 90^o before being launched in the birefringent fiber, hence the phase of signal u_4^{xy} will be changed as follows:

$$u_4^x = u_4^x.*\exp(i\pi/2); u_4^y = u_4^y.*\exp(-i\pi/2) \quad (19)$$

IV. SIMULATIONS AND RESULTS

Coupled non-linear Schrödinger equations responsible for propagation in loop were solved in Matlab using Symmetrized split step Fourier algorithm. In case of coupled mode equations output signal from one equation is to be fed in the other equation. This has been catered for very carefully. In Symmetrized split step Fourier method (SSSFM) a fiber is divided into small parts (not necessarily of equal length) and then non-linearity is considered at the center of the segment, while dispersion is considered at the two halves. Fig.5 below describes the process.

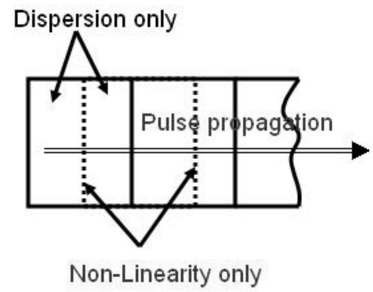

Fig.5: Symmetrized Split Step Fourier Method (SSSFM).

Coupled equations had been solved at various input values. The fiber nonlinearity is .01/W/m, 2nd order dispersion is -20 e-27 s2/m. The clockwise rotating pulse will experience the birefringence bias rotation by 90^O after passing around the loop:

$$u_3^x = u_3^x.*\exp(-i\pi/2); u_3^y = u_3^y.*\exp(i\pi/2) \quad (20)$$

Output coupling in 50/50 splitter is as follows:

$$\begin{aligned} u_{signal}^{xy} &= \sqrt{\alpha} * u_3^{xy} + i\sqrt{1-\alpha} * u_4^{xy} \\ u_{reflections}^{xy} &= i\sqrt{1-\alpha} * u_3^{xy} + \sqrt{\alpha} * u_4^{xy} \end{aligned} \quad (21)$$

The nonlinear optical loop mirror length was kept comparable with beat length; the beat length used in our simulations is 150 meters. Using 10% output coupler, the outpulse recorded at each round trip for fifty turns showed a promising decrease in pulse width (see figure 6) and increase in pulse amplitude (see figure 7).

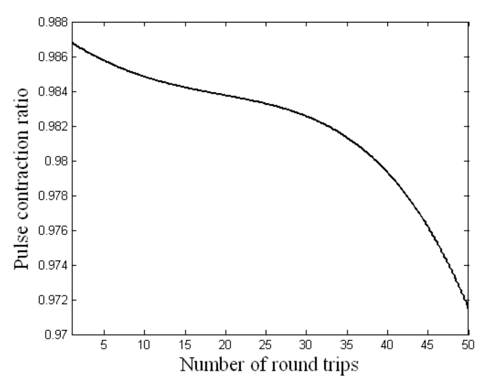

Fig.6: Pulse width ratio at each round trip.

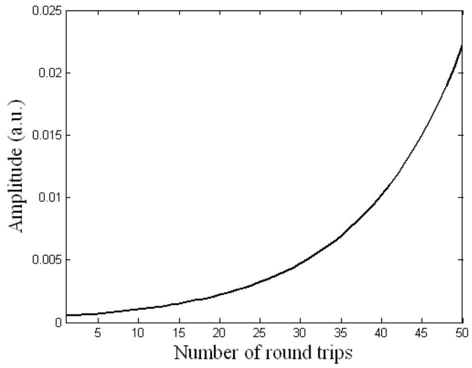

Fig.7: Pulse amplitude at each round trip.

Pulse characteristics and shape at each round trip of laser cavity was recorded and analyzed. The pulses were plotted together in time domain; this gives a true picture of how the pulse is evolved along the number of round trips. It has been found that a pulse of large width is contracted by some extent; however erbium doped fiber used as gain medium increased the pulse amplitude considerably. Pulse evolution in figure of eight fiber laser in time domain is shown in figure 8 below.

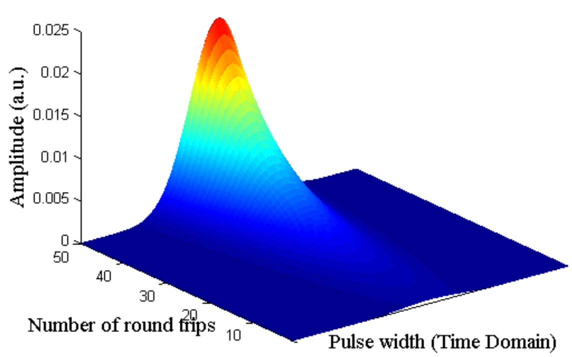

Fig.8: Pulse evolution in figure of eight fiber laser

V. CONCLUSION

It is pertinent to mention that pulse propagation through a nonlinear medium (though it is single mode) cause the pulse under go various nonlinear phenomenon, like Self phase Modulation (S.P.M.), cross phase modulation (X.P.M.) and Four Wave Mixing (F.W.M.). The combined and managed effects of the above mentioned non-linear effects give rise to passive mode locking mechanism. The combined action of nonlinear effects inside NOLM acts similar to fast saturable absorber action. The pulse compression ratio in NOLM has been under the action of nonlinear phenomenon.

Study of pulse propagation in nonlinear optical loop mirror based upon birefringent fiber medium shown us a way to exploit the architecture in forming a laser source. Pulse compression is happening due to the application of nonlinear effects. A figure of eight fiber laser source based upon NOLM had been successfully demonstrated. The output pulse of this laser is dependent upon the birefringence parameter *B* of the birefringent fiber used in the NOLM. This easy to realize laser source will help us in demonstrating such a source practically hence proves the importance of research work carried out.

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