

Joint Partial Crosstalk Cancellation and Modified Iterative Water-Filling for Upstream VDSL

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Abstract— *Far-end crosstalk (FEXT) is the major limitation to achieve high data-rates in VDSL systems. Full crosstalk cancellation (FCC) techniques have been proposed to mitigate the effect of crosstalk. However, these are too complex. Considering the fact that there are only a few strongest cross-talkers, the crosstalk cancellation can be simplified to line selection (LS), tone selection (TS) and joint line-tone selection (JLTS) partial crosstalk cancellation (PCC). These all result in much less online complexity. In this paper first, we propose a new modified JLTS algorithm with significantly lower initialization complexity than the optimal JLTS (OJLTS) algorithm and approximately with similar performance. Then a novel joint algorithm is presented that combines the multi-user power control technique with the PCC scheme to achieve near the capacity rates. We also consider dynamic online complexity constraint for each user. This joint algorithm leads to much larger online complexity reduction and higher data-rates at the expense of slightly higher initialization complexity. The performance of the proposed algorithm is verified by some simulation results and compared with current algorithms that use flat PSDs and other existing joint algorithms.*

Keywords—Far-end crosstalk, crosstalk cancellation, VDSL, power control, complexity, water-filling

I. INTRODUCTION

THE VDSL offers multi-ten-Mbps services by using as high as 12 MHz in the ordinary telephone twisted pairs [1, 2]. The twisted pairs are distributed within large binders which typically contain 20-100 individual twisted pairs. As a result of the close distance between lines and high frequencies used in VDSL systems, there is significant FEXT among near-by twisted pairs [1].

Multi-user power control techniques and FCC schemes are the two general strategies to deal with FEXT destructive effects [2, 3]. While the former tries to vary the transmit power spectral densities (PSDs) of the users in order to reduce generation of crosstalk, the latter attempts to perfectly remove crosstalk after it has occurred. However, FCC schemes require huge online complexity.

Fortunately, the main part of crosstalk seen by each user comes from a few numbers of its neighboring lines in a binder and in a small subset of tones [3]. Thus a large online

complexity reduction can be achieved via PCC by removing only the strongest cross-talkers in space and frequency domains.

In [3], some PCC schemes based on resource allocation are presented. These schemes consider fixed online complexity budget per user and try to find a set of dominant cross-talkers based on line and tone selection while considering flat transmit PSDs. However, these are sub-optimal and have considerable online complexity. Recently, some algorithms have been proposed that solve independently the multi-user power control and PCC problems [3, 4, 5]. First a multi-user power control technique chooses PSDs to minimize the crosstalk, and then a PCC scheme is used to cancel the remaining crosstalk. This approach is not efficient since the independent solution first runs the power control algorithm such that the crosstalk is minimized. Due to this process, especially in strong crosstalk scenarios, the PSDs result in long and short lines to occupy different frequency bands and hence no much crosstalk left to be cancelled by the PCC algorithm. Therefore, only a limited online complexity budget can be used effectively. A better solution with lower online complexity can be obtained if the multi-user power control and PCC problems are solved jointly [4]. In [4], the joint problem is formulated as a constrained optimization problem. The Lagrange dual decomposition method is used to decouple the constrained optimization problem into a series of per-tone unconstrained optimization problems. However, this problem suffers from per-tone exhaustive search method due to non-convexity of its per-tone optimization problem. Thus, each per-tone problem still has an exponential complexity in terms of the number of users [4]. In [5], the proposed PCC schemes in [3] are combined with conventional iterative water-filling (IWF) algorithm [1] in an iterative fashion. However, IWF tends to be highly suboptimal in near-far or strong crosstalk scenarios.

In this paper, first a new modified PCC scheme based on resource allocation [4] is presented that has a performance quite close to optimal performance and much lower initialization complexity. Then a novel low complexity joint algorithm is presented that combines the multi-user power control technique with the PCC scheme to achieve near the capacity rates. We use modified IWF algorithm (MIWF) [6] as multi-user power control technique that is suitable and efficient in strong crosstalk scenarios. MIWF can achieve

local optimal solutions where it is appeared to be fairly close to the global optimum in practical xDSL scenarios [6]. Moreover, we consider dynamic online complexity budget per user in contrast to the fixed online complexity budget per user which was used in [3, 5]. This allows us to trade-off between the bit-rates of different users in a binder. The proposed joint algorithm leads to larger online complexity reduction and higher data-rate at the expense of slightly higher initialization complexity compared to the results in [3, 5]. The proposed iterative algorithm converges after a few iteration steps.

II. SYSTEM MODEL

Consider a VDSL network with N users in a binder each with K tones. The upstream transmission of a single DMT symbol in this network can be modelled as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k, \quad (1)$$

where x_k^n and y_k^n are the transmitted and received signals by user n on tone k , respectively. z_k^n is the additive white Gaussian noise experienced by user n on tone k . \mathbf{H}_k is the $N \times N$ channel transfer matrix where $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from customer premises (CP) transmitter m into central office (CO) receiver n . The vector \mathbf{x}_k is the set of QAM symbols transmitted by all of the CP modems. The vector \mathbf{y}_k represents the set of received signals on all of the CO modems. The vector \mathbf{z}_k shows the vector of additive noise on tone k . The transmit PSD on tone k is \mathbf{S}_k with $s_k^n \triangleq [\mathbf{S}_k]_{n,n}$. The received noise power is σ_k^n . We denote the tone spacing as Δ_f and DMT symbol-rate as f_s . We assume a spectral mask constraint $s_k^{n,mask}$ for each tone. The achievable bit-loading of user n on tone k is [2]

$$b_k^n \triangleq \log_2 \left(1 + \frac{1}{\Gamma} \frac{|h_k^{n,n}|^2 s_k^n}{(\sigma_k^n + \sum_{m=1, m \neq n}^N |h_k^{n,m}|^2 s_k^m)} \right), \quad (2)$$

where Γ denotes the SNR-gap to capacity [1]. The data-rate on line n is

$$R^n = f_s \sum_{k=1}^K b_k^n \text{ bits/sec} \quad n = 1 \dots N. \quad (3)$$

Each modem is typically subject to a total power constraint P_n^{\max} .

III. FCC TECHNIQUES

Several FCC techniques have been proposed to remove crosstalk. When both the transmitters and receivers are coordinated, the channel capacity can be achieved by means of

pre and post filtering [1]. When this level of coordination is not available, successive interference cancellation or pre-compensation can be used if there is only coordination at the receivers or transmitters, respectively [1].

IV. PCC TECHNIQUES

While the benefits of FCC are large, the online complexity is extremely high and grows with the square of the number of users [3]. Therefore, the PCC schemes have been proposed to provide a practical balance between bit-rate and complexity by cancelling dominant cross-talkers in each frequency tone [4, 6].

First, the crosstalk to each loop is usually dominated from a limited number of lines, e.g. those in close proximity or shorter loops. This is known as the space-selectivity of crosstalk. Second, crosstalk coupling depends heavily on the frequency. This is known as the tone-selectivity of crosstalk. Since most of the crosstalk originates from a limited number of lines on a limited number of tones, a fraction of the complexity of the FCC suffices to cancel most of crosstalk. This is called the PCC algorithm [3]. In [3] three PCC schemes have been presented based on resource allocation, namely LS, TS, and JLTS. Note that the proposed PCC schemes in [3] consider only ρK multiplications/DMT-symbol/user in contrast to the NK multiplications required for FCC. Therefore, PCC schemes consider fixed online complexity budget for each user. Moreover, they assume flat transmit PSDs during dominant cross-talkers selection process.

A. Line Selection scheme (LS)

For user n , LS assumes N crosstalk powers are sorted in descending order. Based on fixed online complexity budget of ρK , the LS simply cancels the first ρ strongest cross-talkers in each tone for each line [3].

B. Tone Selection scheme (TS)

Here $b_k^n(\rho)$ is defined as the bit-rate achieved by user n on tone k when the ρ strongest cross-talkers are cancelled. Define the gain of FCC ($\rho = N$) as $g_{k,n} \triangleq b_k^n(N) - b_k^n(0)$ and sort the tone indices in descending order by this gain. Therefore, for each user the $\rho K / N$ important tones are selected when assuming all of their cross-talkers are cancelled and no cancellation on all other tones [3].

C. OJLTS scheme

Define the value of $v_k^n(\rho) = (b_k^n(\rho) - b_k^n(0)) / \rho$; $\rho = 1 \dots N$ as the average increase in bit-rate per allocated complexity on a certain tone. Given ρK multiplications/DMT-symbol/user as the online complexity constraint for each user, now crosstalk powers are sorted in each tone. Then a table can be formed with entries

representing the $v_k^n(\rho)$ values. Then the algorithm finds the maximum entry in the above table. Based on the selected value $v_{k_s}^n(\rho_s)$, the ρ_s largest cross-talkers on tone k_s are determined. Then all values $v_{k_s}^n(\rho)$ with $\rho \leq \rho_s$ are set to zero and all values $v_{k_s}^n(\rho)$ with $\rho > \rho_s$ are recalculated as $v_{k_s}^n(\rho) = (b_{k_s}^n(\rho) - b_{k_s}^n(\rho_s)) / (\rho - \rho_s)$. This is repeated until all available online complexity budgets are allocated [3].

D. Simple JLTS scheme (SJLTS)

It defines the gain of canceling the cross-talker m on tone k in the detection of user n , and in the absence of other cross-talkers. The cross-talkers are sorted in each tone. Thus, a table with these gains for sorted cross-talkers on each tone is created and the ρK largest entries are selected.

E. New scheme: Modified JLTS scheme (MJLTS)

Here, we propose a new modified method which is quite close to the OJLTS algorithm. This is similar to the SJLTS algorithm with the exception of considering the effect of other remained cross-talkers in computation of the bit-increments. First cross-talkers are sorted in each tone. Then we calculate the true natural bit-rate increment after removing the i cross-talkers. Thus a table with entries of these increments is created for all sorted cross-talkers on each tone and then the ρK largest entries are selected.

F. Complexity Budget Distribution between Users

In above PCC schemes the online complexity was limited to fixed ρK multiplications/DMT-symbol per user. If the total multiplications/DMT-symbol, i.e. $\rho K(N+1)$ for $N+1$ users, can be distributed between users based on their target rates, hence the higher bit-rates or larger rate-regions would be achieved [3, 4]. The problem of distribution of multiplications/DMT-symbol between users is formulated as the following optimization problem:

$$\begin{aligned} \max \quad & \sum_n w_n R^n \text{ subject to:} \\ & \sum_n \kappa_n \leq \rho K(N+1) \end{aligned} \quad (4)$$

Where κ_n is the number of online complexity budget allocated to user n . We can write down $\kappa_n = \mu_n \rho K(N+1)$ where $\sum_n \mu_n = 1$. For particular values of μ_n and hence κ_n and under fixed transmit PSDs for each user, the dominant cross-talkers for each line on each tone can be found by using one of LS, TS or JLTS PCC schemes. Then, if the target rates are not achieved, we can update the value of μ_n using bisection method or sub gradient method [2, 4]. This process is repeated until convergence is achieved.

V. MULTI-USER POWER CONTROL SCHEMES

In a multi-user channel, since the transmit PSD of each user influences the crosstalk that it induces on the other lines, it must be assigned in an appropriate way to reduce crosstalk effect. This is known as spectrum management which has static and dynamic types [2]. Dynamic spectrum management or multi-user power control chooses the transmit PSDs such that crosstalk is avoided. Advanced optimization techniques have been developed that take into account the network topology to determine optimal transmit PSDs [2]. The IWF is one of the first so-called multi-user power control algorithms [1]. In this algorithm each user obtains its most favorable PSD by iteratively performing single user WF until its PSD is stabilized [1]. However, IWF converges to a selfish optimum point and it is suboptimal for CO/remote terminal (RT) mixed deployments [2].

The MIWF algorithm solves the Karush-Kuhn-Tucker (KKT) system of the sum rate maximization problem to find local optimal solutions of the problem efficiently [6]. The KKT system of this optimization problem dualized with respect to the power constraint can be found by taking the derivative of its objective function of with respect to PSDs [6].

VI. PROPOSED ALGORITHM: JOINT PCC AND MULTI-USER POWER CONTROL

In all the previous PCC schemes, cross-talkers must be sorted in terms of their powers. In receiver the cross-talker power is computed by transmitter PSD multiplied by crosstalk channel transfer function. Since the transmitter power is an unknown a priori; the PCC considers flat or equal transmit PSD for each tone to estimate the crosstalk power and select dominant cross-talkers subject to the given online complexity budget. This is simple but dominant cross-talkers selection may not be precise when the true real transmitter PSDs are not taken into account.

Using multi-user power control technique causes to enhance space- and tone-selectivity. Then PCC can exploit this merit to increase the bit-rate compared to the existing PCC that employs flat PSD. Now, the problem is how to assign the dynamic PSD to users in order to increase the bit-rate in the existing PCC subject to a given total complexity budget. Since selection of dominant cross-talkers depends on transmitter PSD and also the transmitter PSD assignment depends on dominant cross-talkers selection, then we must solve this optimization problem jointly. Here we suggest a new iterative PCC that helps to select the true real cross-talkers by exploiting the multi-user power control technique joint with PCC algorithm. For example the OJLTS scheme which is one of the PCC schemes and the MIWF as one of the multi-user power control algorithms can be jointly employed to make crosstalk cancellation as efficient as possible. We consider variable online complexity budget for each user. A simplified

form of this algorithm is illustrated below. The algorithm starts with an initial set of transmit PSDs and proportional complexity budget μ_n for the users. First, one of the PCC schemes can be used which selects the strongest cross-talkers subject to the current allocated online complexity budget for each user. Second, MIWF algorithm is run while virtually discards the selected cross-talkers for each user. Then, new PSD for each user is obtained. In the final phase, the current data-rates is computed and compared with the target rates. If the target rates are not achieved, the proportional number of online complexity and the weights assigned to the users must be updated to enforce the constraint. The proposed sub-gradient descent form for updating Lagrange multipliers used in [4] is exploited. This procedure is repeated until each user achieves its target rate.

Although the analytical convergence of the algorithm has been discussed in the next part, many simulation results on various practical scenarios have shown that in all cases the proposed algorithm converges after 3 or 4 iterations. It is noted that the existing PCC algorithms exploit space- and tone-selectivity to reduce complexity and hence, increase the bit-rate. Our proposed algorithm tends to enhance space- and tone-selectivity through using power control techniques in an efficient manner.

Algorithm: Iterative joint PCC and MIWF algorithm

1. Initialize PSDs (s_k^n), w_n and μ_n .
 2. repeat
 3. repeat % PCC phase (e.g. OJLTS PCC scheme)
 4. for $n = 1 \dots N$
 5. run one of PCC scheme
 6. end for
 7. until $\kappa_n = \sum_k \kappa_n^k \leq \mu_n \rho KN$
 8. run MIWF % multi-user power control scheme
 9. % update w_n and μ_n
 9. $w_n = (w_n - \varepsilon(\sum_k b_k^n - R^{n,\text{target}}))^+$;
 10. $\mu_n = (\mu_n - \varepsilon(\sum_k b_k^n - R^{n,\text{target}}))^+$;
 11. until convergence
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A. Convergence Analysis

When the PCC algorithm is exploited; the true strongest cross-talkers are selected and discarded. So there is not much crosstalk left to be cancelled. Thus SINR in each tone grows rapidly due to less remained crosstalk. Consider a binder with N lines and m cross-talkers. In the worst case let the cross-talkers have identical crosstalk channels h_k^n to user n . Canceling the cross-talkers leads to a SINR increase.

when the MIWF algorithm is exploited; the transmit PSDs are chosen such that the effect of remained crosstalk that has not yet been cancelled is now efficiently reduced. Hence, this

reduction of the remained crosstalk by use of multi-user power control will also lead to extra increase of SINR in each tone.

In addition, MIWF algorithm puts more power in the high SINR tones and less power in the tones with low SINR. This causes a non-uniform PSD assignment to the tones which in turn enhances the space- and tone-selectivity. Now, this process makes the situation better for the PCC algorithm to select the strongest cross-talkers more properly. Consequently, PCC cancels the dominant cross-talkers more properly which leads to more increase of the system SINR. This gives more opportunity to MIWF algorithm to accomplish the bit-allocation process better and consequently achieve higher bit-rates. In conclusion, exploiting joint use of the PCC and MIWF algorithms not only increases the SINR by each one individually, they also have mutual effects that help each other to perform more efficiently and consequently more increase of the system SINR.

It is clear that convergence of our algorithm mainly relies on the convergence of the MIWF [6]. In [6] it has been proved that under high SINR approximation, the MIWF globally converges to the unique optimum point with only few iterations. Where, this condition is well provided in our algorithm under the joint use of PCC and MIWF algorithms. This means that the convergence of MIWF in the proposed algorithm is accelerated by the use of PCC and the overall convergence is guaranteed with fewer iteration steps. Moreover, the MIWF solves the KKT system of weighted sum rate maximization problem directly, when it converges it always converges to a KKT point, which is guaranteed to be at least a local optimum solution [6].

VII. NUMERICAL RESULTS

All VDSL lines are considered to be 0.5 mm twisted pairs with a background noise level of -140 dBm/Hz. The maximum transmit power is 11.5 dBm. Γ is set to 12.9 dB [1, 2]. The tone-spacing Δ_f is set to 4.3125 kHz and the DMT symbol-rate $f_s = 4$ kHz. The modems use 4096 tones. We have used initial flat PSD equal to -60 dBm/Hz in all tones. Online complexity budget is shown as a percentage relative to FCC.

A. A Near-Far Scenario

The upstream transmission in VDSL with 8 users consisting of 4×300 m and 4×1200 m lines is simulated. The performance of MJLTS, OJLTS and SJLTS joined with MIWF and joined with IWF [5] for near and far users are depicted in figures 1 and 2. The MJLTS+MIWF outperforms the pervious JLTS schemes, especially in low complexity budgets and for near users. It has surprisingly increased the bit-rate to more than 200% particularly for far users and 30% for near users. Since near users are the strongest source of crosstalk; this space-selectivity assists the PCC algorithms to reduce complexity. Thus all line and tone selections are assigned to shorter lines as candidates to be cancelled. By this cancellation the overall crosstalk drops down and far users can take advantage of the provided condition to employ multi-user

power control to estimate true PSD when they encounter the lower crosstalk. As a result, they achieve higher bit-rates. It is seen that with 50% complexity our algorithm achieves 99% of the performance gain on near users and achieves 97% of performance gain on far users with 25% complexity. With a target rate 2.5 Mbps on far users, the required complexity is 8% for MIWF and 32% for IWF. This shows a significant gain of 400% in online complexity reduction which goes to 250% for near users.

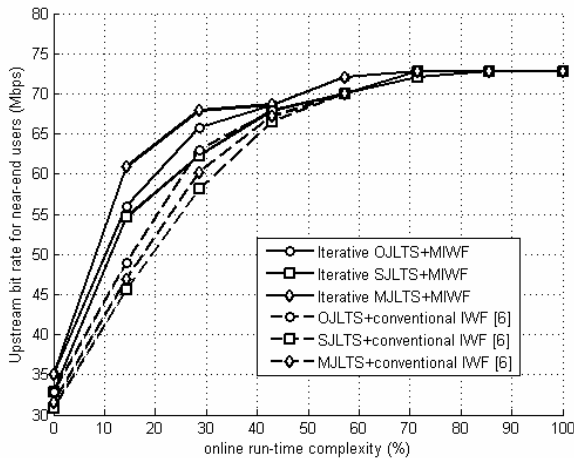


Fig. 1 OJLTS, SJLTS and MJLTS comparison for near users

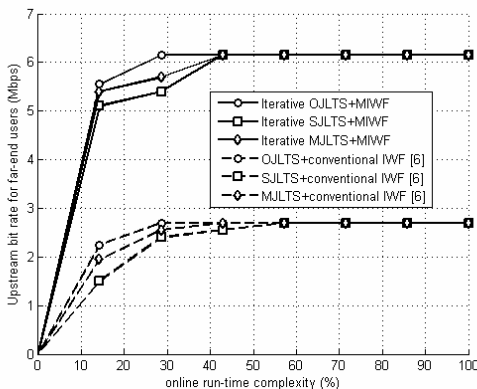


Fig. 2 OJLTS, SJLTS and MJLTS comparison for far users

B. Complexity Analysis

The previous PCC algorithms are not iterative and use flat PSD while the proposed algorithm is iterative. It consists of two loops, where in the outer loop it is initialized with flat PSD and repeated while PCC and MIWF algorithms are performed as the inner loop. The total complexity would be the product of the number of iterations and the complexity required to execute each iteration step. The complexity within each iteration step (inner loop) is sum of the complexity of PCC scheme and that of the MIWF. The complexity of the PCC scheme is $O(KN^2)$ for OJLTS and $O(KN)$ for other

schemes [3]. The complexity of MIWF is approximately linear in terms of the number of users and tones and is $O(KN)$ [6] which is smaller or almost equal to that of the PCC schemes [4]. Since PCC scheme needs some more operations for taking logarithm, comparison and sorting processes, hence the PCC scheme is counted as a major component of the complexity of the algorithm. If the proposed algorithm converges after ν iteration steps, its complexity will be $O(\nu KN^2)$ for OJLTS+MIWF and $O(\nu KN)$ for other schemes. The proposed algorithm typically converges after 3 or 4 iterations. Thus, exploiting MIWF into PCC process will make the initialization complexity only ν times larger which is effectively very small. Moreover, because of the static nature of DSL environment, all initialization algorithms including the proposed algorithm run only once for a long timescale. Hence, this initialization complexity increase will be negligible.

Besides, the proposed non-iterative MJLTS algorithm gives a performance quite close to that of the OJLTS algorithm and has lower complexity compared to OJLTS. It will also cause excessive reduction in initialization complexity.

Moreover, we note that the PCC requires ρK multiplications/DMT-symbol/user for any DMT symbol [3]. To achieve similar bit-rate, the proposed joint PCC algorithm exploits much lower number of cross-talkers to be canceled; it means that the value of ρ decreases here compared to that of the existing PCC schemes, particularly for far users. The average decrease of ρ is around 300% as it is shown in the simulation results.

VIII. CONCLUSIONS

In low distance high frequency modems like VDSL networks, FEXT is the main source of performance degradation. The FCC techniques require too much complexity. The PCC schemes achieve most of their performance by canceling the strongest cross-talkers subject to the given complexity budget. In this paper, first we have presented a new modified PCC scheme named as MJLTS. Then we have presented a combination of MIWF and the PCC schemes. This new approach achieves higher bit-rates in much less complexity compared to the existing PCC algorithms, especially in longer lines. The cost of these gains is slight increase in initialization complexity which would be very negligible due to almost static nature of DSL channels. Furthermore, we have used dynamic online complexity for users in the proposed joint algorithm which allows flexibility in choosing their bit-rates leading to maximum weighted sum rate. The proposed MJLTS+MIWF scheme has almost the same performance as the OJLTS+MIWF scheme and the presented algorithm in [4] but with much less initialization complexity. Even in near users, it shows superior performance.

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