Markovian Chain Analysis in ALOHA for Satellite Applications

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***Abstract*—** In this paper, the average delay in ALOHA is calculated for digital satellite system. The main parameters to analyze the system are the throughput, the average number of retransmission and the average packet delay. A complete mathematical analysis is presented where a Markov Chain is used.

***Keywords*—** Satellite, Multiple Access, ALOHA, Markov Chain, Average Packet Delay.

I. Introduction

THE system with a lot of lightly loaded users can be optimized with a Random Multiple Access (RMA) [3] [4] [5]. In this kind of system, the transmission media is employed at the time the information is being generated. Other Multiple Access Schemes have a better performance if the information generation is increased [1] [2] [6] [7] [8] [9].

In Figure 1, the line flow illustrated corresponds to a pure ALOHA channel. The users produce a non-uniform traffic, then the packages are transmitted in random times. This features are satisfied in many rural applications, with VSAT terminals [14] [15].

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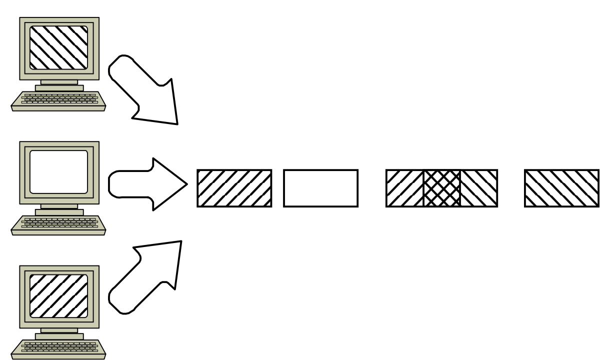

Fig. 1 Satellite Multiple Access System.

The throughput of the pure ALOHA technique can be increased significantly if the packages are sent in a slot. This RMA variation is named slotted ALOHA [16].

The most important advantage of ALOHA protocol in comparison to other Multiple Access technique is the average packet delay [13] [16].

In this paper the average packet delay is calculated using a Markov Chain. The results are compared with another computations in [10] [11] [12].

The paper has been organized in the following sections: section two is dedicated to define the Markov chain employed, section three is completely related to evaluate the performance of the average packet delay in this kind of system, in the section four the main results of simulations are shown and, the conclusions are presented in the section five.

It is important to emphasize that the introduction to pure ALOHA and slotted ALOHA was widely analyzed in the reference [16], which paper also includes the throughput for both RMA variations and other procedure for the average package delay computation.

II. Markov Chain

The Markov Chain used has the following features: the state of the system changes at regularly spaced intervals, this interval is determined by the slot size in the slotted ALOHA variation and the probability distribution of the state after the next transition depends only on the present state.

It is important to point out, that the state of the system is represented by a stochastic transition matrix.

III. Average Packet Delay using a Markov Chain

The binomial distribution of probabilities determines the probabilities for j transmission or arrivals in i backlogged terminals, se equations (1) and (2).

$$P\left(j \text{ transmissions in a slot}/i \text{ backlogged terminals}\right) = \\ = \binom{i}{j}\alpha^{j}(1-\alpha)^{i-j} \qquad j = 0,1,\ldots,i \tag{1}$$

Where: α is the probability of a retransmission in a succeeding interval.

$$P\left(j \text{ arrivals in a slot}/i \text{ backlogged terminals}\right) = \\ = \binom{N-i}{j}\sigma^{i}(1-\sigma)^{N-i-j} \qquad j = 0,1,\ldots,N-i \tag{2}$$

Where: σ is the probability of a message generation in a available slot, and N is the number of terminals in the ALOHA channel.

The state transition matrix is built with the following parameters, see equation (3).

$$t_{ij} = \\ P\begin{pmatrix} j \text{ backlogged terminal slot } n/\ldots \\ \ldots/i \text{ backlogged terminal slot } n\text{ - }1 \end{pmatrix} \tag{3}$$

One of the most important step in the calculation is to find the values for the steady-state transition matrix. We assume that the first row of the matrix with only no backlogged terminals, and a system empty will be in this state.

$$t_{00} = (1-\sigma)^{N} + N\sigma(1-\sigma)^{N-1} \tag{4}$$

Of course, if two or more users in the channel generate a package in the same slot interval there will be a collision, but these packages remain in the system. Then the rest of the values in the first row is calculated with equation (5).

$$t_{0j} = \binom{N}{j}\sigma^{j}(1-\sigma)^{N-j} \qquad j = 2,3,\ldots,N \tag{5}$$

Equation (6) allows to evaluate the coefficients when there are two or more arrivals in a single slot.

$$\begin{aligned} t_{ii} &= (N-i)\sigma(1-\sigma)^{N-i-1}(1-\alpha)^{i} + \\ &+ (1-\alpha)^{i}(1-\sigma)^{N-i} + \\ &+ (1-\sigma)^{N}\left(1-(1-\alpha)^{i} - i\alpha(1-\alpha)^{i-1}\right) \end{aligned} \qquad 0 < i \le N-1 \tag{6}$$

With the assumptions mentioned before, the form of the transition matrix is determined by equation (7).

$$t = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{B-1} & \sum_{j=B}^{\infty} a_j \\ a_0 & a_1 & a_2 & \cdots & a_{B-1} & \sum_{j=B}^{\infty} a_j \\ 0 & a_0 & a_1 & \cdots & a_{B-2} & \sum_{j=B-1}^{\infty} a_j \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_1 & \sum_{j=2}^{\infty} a_j \\ 0 & 0 & 0 & \cdots & a_0 & \sum_{j=1}^{\infty} a_j \end{bmatrix} \tag{7}$$

In order to solve the matrix described before, we use the following equations (8) and (9).

$$P_i = \begin{cases} \sum_{j=0}^{i+1} P_j t_{ij} & 0 \le i \le N-1 \\ \sum_{j=0}^{N} P_j t_{iN} & i = N \end{cases} \quad (8)$$

The other probabilities are found using the equation (9).

$$P_{i+1} = \frac{P_i - \sum_{j=0}^{i} P_j t_{ji}}{t_{i+1i}} \quad (9)$$

Where P_1, P_2, P_3, P_4, …, P_N; are the steady-state probabilities for the backlogged terminals in the system.

The average number of backlogged terminals in the system is determined by equation (10).

$$\overline{K} = \sum_{i=0}^{N} iP_i \quad (10)$$

The average number of the messages waiting the next slot for their transmission is find by equation (11).

$$S_{in} = \sigma\left(N - \overline{K}\right) \quad (11)$$

Then the average packet delay of the slotted ALOHA channel can be calculated by equation (12).

$$\overline{D} = 1 + \frac{\overline{K}}{\sigma\left(N - \overline{K}\right)} \quad (12)$$

The parameter defined is an approximation to the real value, because the time for collision detection has not been considered. In a practical system, this approximation should be taken in account.

IV. Results

The calculation of the probability in a slot σ is illustrated in Figure 5. The arrival rates for the three lines 1, 2 and 3 plotted are 1×10^6, 1×10^5 and 1×10^4 messages per second respectively.

The Figure 3 shows the probability of binomial distribution for the number of transmissions generated in a slot time interval, where there are *i* backlogged terminals.

The main result of this paper has been plotted in Figure 4, where the average packet delay is shown for σ = 0.003, 0.005 and 0.01 (lines 1, 2 and 3 respectively).

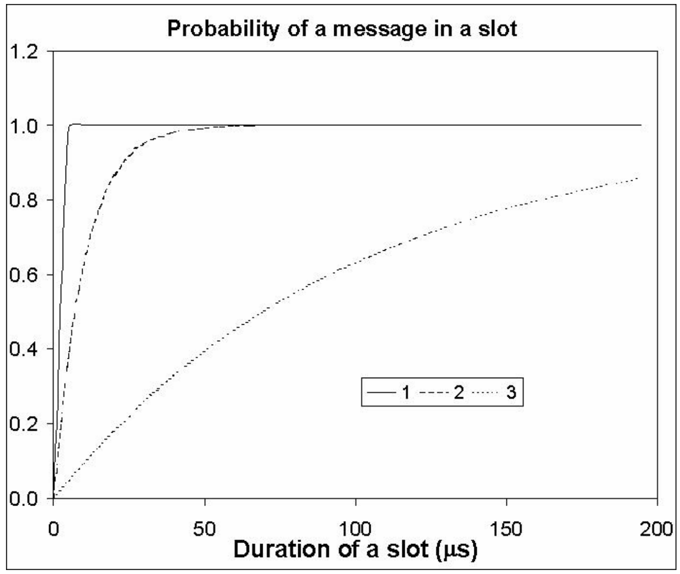

Fig. 2 Pure ALOHA throughput.

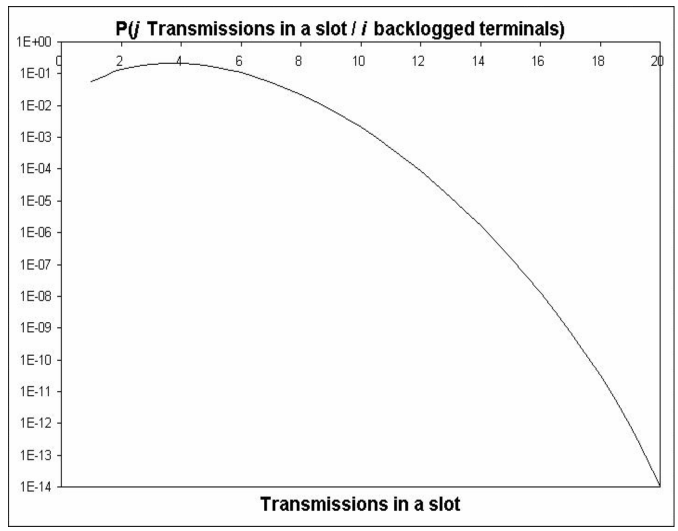

Fig. 3 Slotted ALOHA throughput.

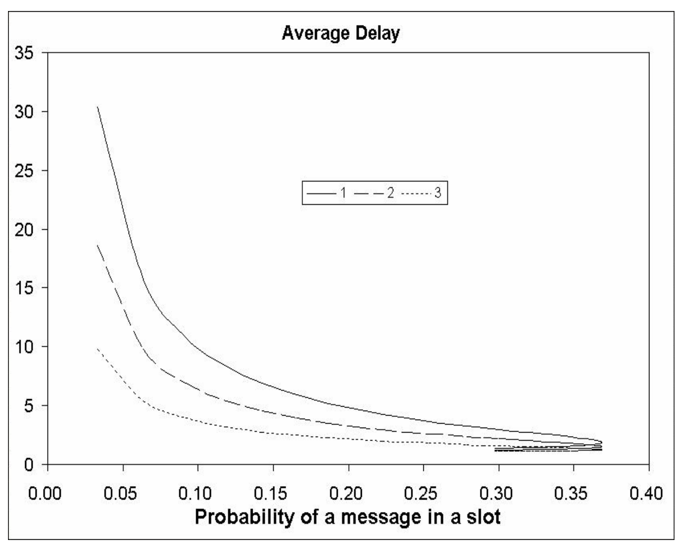

Fig. 4 Average Packet Delay in Slotted ALOHA system.

V. CONCLUSIONS

The main parameter of performance in RMA is the average packet delay, because it is the most important advantage of this multiple access technique in comparison with TDMA or CDMA. In this paper this parameter has been evaluated.

The work presented here demonstrates that RMA is the best solution in packet networks, where the number of the sources is very high and the traffic is bursty and when the satellite channel capacity is limited.

Now, the results of this article are quite similar in comparison with the propagation delay procedure is used [16] instead of using a Markov chain embedded at the slot boundaries. Nevertheless is necessary to explore the possibilities of the model varying the number of users and the considering the time dedicated to detect the package collisions.

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