Study of Parameters Effect on the Performance of Precoding and Equalizer Sharing Systems

Mutamed Khatib, Mohd Fadzil. Ain, Farid Ghani and Syed Idris Syed Hassan

***Abstract*—**This paper studies the effect of the block length and the channel impulse response on the performance of three block transmission systems. The main idea of the block transmission system is to transmit the data in blocks of certain length, and to use signal processing networks either in the transmitter or the receiver to eliminate the effect of the multipath channel. So, both the block length and the channel impulse response will play an important roll in the system performance. The first studied system moves all the signal processing operations from the receiver to the transmitter and leaves the receiver quite simple (pre-coding). The other two systems make some sharing of the signal processing between the transmitter and the receiver in different ratios to obtain some enhancement on the performance of the pre-coding system.

***Keywords*—** block length, block transmission, channel response, precoding, sharing.

I. Introduction

SCATTERING, or the reflection of signal from objects when transmitted between transmitter and receiver, is one of the major problems that affect the communication systems. The transmitted signal arrives at the receiver end as superposition of delayed versions of the original signal. Due to the phase difference, these 'replicas' may change the contents of the original signals if they are continuous-waves or sinusoidal waveforms, Conventional communication systems use high carrier frequencies in order to have a wider bandwidth. The propagation losses are proportional to the frequencies of these signals. However, Researchers are working to design systems to give signals greater penetrating ability through obstacles such as walls than conventional signals while having the same data transmitting rate [1].

We are standing few steps away from the Fourth Generation of mobile systems (4G). Various services support the principal requirements of the 4G system; therefore, the need to improve integration of heterogeneous networks is significant [2].

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Block transmission is one of the effective systems that reduce the effect of the channel. The main idea is to transmit the bits in groups with the assumption that the channel is constant during the transmission of a sufficiently short message. This implies that the information symbols are transmitted in the corm of blocks of sufficiently short duration. Channel estimation is updated for each block with the help of known transmitted symbols that separate the information blocks. Moreover, decision symbols are then used to re-estimate the channel. and the new estimate is used to obtain a new detection. The process of channel estimation and symbol detection is thus repeated until a reliable symbol detection is reached [3-6].

Improving the performance of the block transmission system was the goal of many researchers. In [6], we have proposed a pre-coding system to simplify the receiver -i.e., the mobile unit- by complicating the transmitter, i.e., the base station. Also in [7], we have modified the pre-coding system in [6] by introducing two systems that share the equalization process between the transmitter and the receiver in a certain ratios to get around 4dB enhancement in the system performance. But that approach was depending on mathematical representation only, and we didn't focus on the effect of the system parameters on the performance of the system.

The main difference between the systems proposed in [7] is length of the transmitted vector. Both of them may transmit the same vector at the input of the transmitter, but after coding, the first system in [7] generates a longer code than the second one. This will give the first system two guardband areas after and before the transmitted block. This will be useful in environments with many obstacles that usually cause duplicate versions of the transmitted signal, and finally cause what is called Inter-Symbol Interference ISI. [8]

Unfortunately, we can't get all the advantages in one system. The immunity against ISI will cause increase in bandwidth in an unaccepted ratios in some applications. For examples, transmission in codes of 4 elements in an environment with a baseband channel of length 3, will cause a transmitted block of length 8 at the first system in [7] and of length 6 at the second system.

All the previously mentioned system is assumed to operate in an environment where the base-band channel $y(t)$ is either time invariant or varies slowly with time. White Gaussian noise, with zero mean and variance σ^2, is assumed to be added to the data signal at the output of the transmission path, giving the Gaussian waveform $w(t)$ added to the data signal. The sampled impulse-response of the base-band channel in Fig.1 is given by the $(g+1)$ component row vector:

$$y_i = y(iT) = y_o \quad y_1 \quad \dots \quad y_g \tag{1}$$

where $(g+1)$ is the length of the channel impulse response and $y_o \neq 0$, $y_i = 0$ for i < 0 and i > g.

In this paper, we have built a simulation model of the previously mentioned systems to prove the mathematical models, and to study the effect of the variables. Also, we have managed to study the effect of each variable, such as the block length, the guardband length, the channel characteristics and the Signal to Noise Ratio SNR. This paper is organized as follows: in Section 2, we present the system models. The simulation results are compared with those obtained mathematically and presented in Section 3. Finally, Section 4 presents the conclusions of our study.

II. Systems Models

A. Precoding System

We developed a technique for CDMA downlink in synchronous multipath fading channel that reduces the complexity of the receiver in which the detection process needs only a threshold decision to retrieve the transmitted data, no match filtering or any other processing is needed [6]. In the base station, a precoder is used to generate a code from the transmitted signal that makes it immune to the channel, so, there is no need for any further equalization process in the receiver that reduces the mobile unit receiver to a decision process due to a certain threshold testing. It depends on the channel's prior knowledge at the base station, so, the channel characteristics are assumed to be known both at the transmitter and the receiver. When comparing the cost of adding a coder at the base station with the savings at the receiver units, it will be acceptable because few base stations serve many receiver units in the downlink.

The system considered is shown in Figure 1. The signal at the input to the transmitter is a sequence of *k*-level element values $\{s_i\}$, where *k* = 2, 4, 8,… and the $\{s_i\}$ being statistically independent and equally likely to have any of the possible values. The buffer-store at the input to the transmitter holds *m* successive element values $\{s_i\}$. In the coder, the *m* $\{s_i\}$ are converted into the corresponding *m* coded signal-elements. The coder performs a linear transformation on the *m* $\{s_i\}$ to generate the corresponding sequence of impulses that is fed to the baseband channel $y(t)$ which is assumed that it is either time invariant or varies slowly with time.

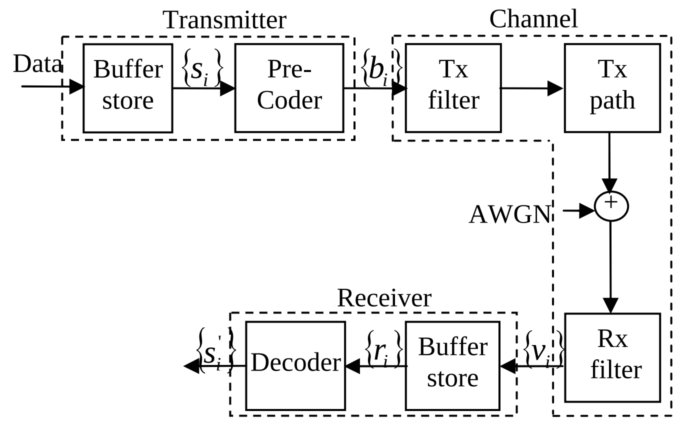

Fig. 1: The downlink of the precoding system.

White Gaussian noise, with zero mean and variance σ^2, is assumed to be added to the data signal at the output of the transmission path, giving the Gaussian waveform $w(t)$ added to the data signal.

The received waveform $r(t)$ at the output of the baseband channel is sampled at the time instants $\{iT\}$, for all integers $\{i\}$. The $\{r_i\}$ are fed to the buffer store which contains two separate stores. While one of these stores holds a set of the received $\{r_i\}$ for a detection process, the other store is receiving the next set of $\{r_i\}$ in preparation for the next detection process. A group of *m* multiplexed signal-elements are detected simultaneously in a single detection process, from the set of $\{r_i\}$ that depend only on these elements. The receiver uses the knowledge of the $\{y_i\}$ and the possible values of $\{s_i\}$ in the detection of the *m* element values $\{s_i\}$ from the received samples $\{r_i\}$. A period of *nT* seconds is available for the detection process, *n* is given by:

$$n = m + g \tag{2}$$

where *m* is the block length, and *g* is the channel length -1.

Except where otherwise stated, the decoder in Figure 1 determines from the appropriate set of received $\{r_i\}$ the *m* estimated $\{x_i\}$ of the *m* element-values $\{s_i\}$ in a received group of elements. Each x_i is an unbiased estimate of the corresponding s_i such that:

$$x_i = s_i + u_i \tag{3}$$

where u_i is a zero mean Gaussian random variable. The detector detects each s_i by testing the corresponding x_i against appropriate thresholds. The detected value of s_i is designated as s_i'.

In our design, there is no need for dividing the channel matrix into the fading gain and the delay, because the coder design depends only on the channel matrix itself. Also, at the mobile unit, no match filtering or any other processing is needed.

In this system, using buffer store, an $1 \times m$ vector $\mathbf{S} = [s_1 \quad s_2 \quad \cdots \quad s_m]$ is formed from the symbols to be transmitted. This vector is coded at the transmitter. The coder accepts the input vector **S** and codes it to form the $1 \times n$ signal vector **B**, which is the convolution between the input vector **S** and the $m \times n$ coder matrix **F**, i.e.:

$$\mathbf{B} = \sum_{i=1}^{m} \mathbf{s_i F_i} = \mathbf{SF} \tag{4}$$

where $\mathbf{F_i}$ is the *n* component row vector.

This convolution process will add a time gap of gT seconds between each pair of adjacent groups of *m* signal-elements. Then, the output values from the coder multiplexer are fed to the baseband channel. The sampled impulse response of the baseband channel is given by the $g+1$ component row vector as given in equation 1

At the receiver, the sample values of the received signal, corresponding to a single group of *m* signal elements, will normally be a sequence of $n+g$ non-zero sample values. The sequence of these $n+g$ sample values in the absence of noise is:

$$v_i = \sum_{j=1}^{n} b_j y_{i-j} \qquad i = 1, 2, \ldots, n+g \tag{5}$$

In vector form, it may be written as:

$$\mathbf{V} = \mathbf{BC} \tag{6}$$

where $\mathbf{V} = [v_1 \quad v_2 \quad \cdots \quad v_{n+g}]$ is the $1 \times (n+g)$ received signal, and **C** is the $n \times (n+g)$ channel matrix and its i^th^ row is:

$$\mathbf{C_i} = \overbrace{0 \quad \ldots \quad 0}^{i-1} \quad \overbrace{y_o \quad y_1 \quad \ldots \quad y_g}^{g+1} \quad \overbrace{0 \quad \ldots \quad 0}^{n-i} \tag{7}$$

Assume now that successive groups of signal-elements are transmitted and one of these groups is that just considered, where the first transmitted impulse of the group occurs at time *T* seconds. Figure 2 shows the $n+g$ received samples which are the components of **V**.

Due to the Inter Block Interference (IBI), the first *g* components of V are dependent in part on the preceding received group of *m* signal-elements, and the last *g* components of **V** are dependent in part on the following received group of *m* elements. Thus there is Intersymbol Interference (ISI) from adjacent received groups of elements in both the first and the last *g* components of **V**. However, the central *m* components of **V** depend only on the corresponding transmitted group of *m* elements, and can therefore be used for the detection of these elements with no ISI from adjacent groups.

The central *m* components of the vector **V**, $v_{g+1} \quad v_{g+2} \quad \ldots \quad v_{g+m}$, can be obtained by introducing a new matrix $\mathbf{BD^T}$ where **D** is the $m \times n$ matrix of rank *m* whose i^th^ row is:

$$\mathbf{D_i} = \overbrace{0 \quad \ldots \quad 0}^{i-1} \quad \overbrace{y_g \quad y_{g-1} \quad \ldots \quad y_o}^{g+1} \quad \overbrace{0 \quad \ldots \quad 0}^{m-i} \tag{8}$$

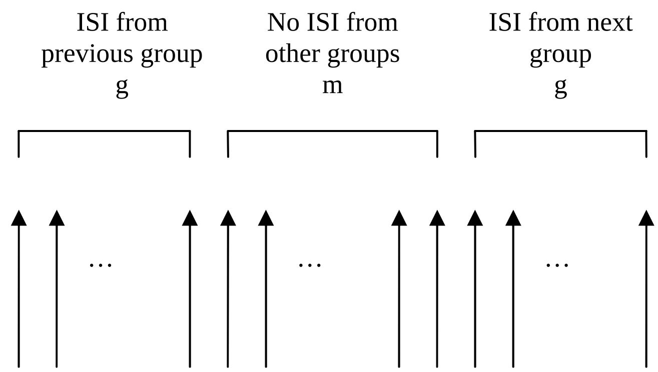

Fig. 2: Sequence of $n+g$ samples for one received block.

Thus,

$\mathbf{BD^T}$ is an $1 \times m$ vector where each row of it gives information about the received symbols at that row:

$$\mathbf{BD^T} = [v_{g+1} \quad v_{g+2} \quad \ldots \quad v_{g+m}] \tag{9}$$

When noise is present, the received vector is:

$$\mathbf{R} = \mathbf{BD^T} + \mathbf{W} \tag{10}$$

where **W** is the zero mean AWGN

Thus the detector can now detect the values of the signal elements by comparing the corresponding $\{r_i\}$ with the appropriate thresholds.

To maximize the tolerance to noise at the detector input, the elements of **B** should be selected such that the total transmitted energy of all the symbols is minimized. i.e. $\mathbf{BB}^T = |\mathbf{B}|^2$ must be minimized for the given vector **S**. Thus the problem is to find an $m \times n$ linear network **F** representing the coder, which minimizes the transmitted element energy and -at the same time- satisfies $\mathbf{BD}^T = \mathbf{S}$.

As shown in Appendix A, the coder matrix **F** has to be:

$$\mathbf{F} = \left(\mathbf{DD}^T\right)^{-1} \mathbf{D} \tag{11}$$

Thus, under the assumed conditions, the linear network **F** representing the transformation performed by the coder is such that it makes the *m* signal elements of a group orthogonal at the input to the detector and also maximizes the tolerance to additive white Gaussian noise in the detection of these signal elements.

Assume that the possible values of s_i are equally likely and that the mean square value of **S** is equal to the number of bits per element. Suppose that the *m* vectors $\{\mathbf{D}_i\}$ have unit length. Since there are *m k*-level signal elements in a group, the vector **S** has k^m possible values each corresponding to a different combination of the *m k*-level signal-elements. So, the vector **B** whose components are the values of the corresponding impulses fed to the baseband channel, has k^m

possible values. If e is the total energy of all the k^m values of the vector **B**, then in order to make the transmitted signal energy per bit equal to unity, the transmitted signal must be divided by:

$$\ell = \sqrt{\frac{e}{nk^m}} \tag{12}$$

The m sample values of the received signal from which the corresponding $\{s_i\}$ are detected, are the components of the vector:

$$\mathbf{R}' = \frac{1}{\ell}\mathbf{BD}^T + \mathbf{W} \tag{13}$$

Then, the m sample values which are the components of the vector **V** (after taking only the central m components), must first be multiplied by the factor ℓ to give the m-component vector:

$$\begin{aligned}\mathbf{R} &= \ell\mathbf{V} \\ &= \mathbf{S} + \mathbf{U}\end{aligned} \tag{14}$$

where **U** is an m component row vector that represents the AWGN vector after being multiplied by ℓ.

The mean of the new noise vector **U** is zero and its variance is:

$$\eta_T^2 = \ell^2\sigma^2 \tag{15}$$

Thus, the tolerance to noise of the system is determined by the value of η_T^2. When there is no signal distortion from the channel, $\left(\mathbf{DD}^T\right)^{-1}$ is an identity matrix. Under these conditions, $\ell = 1$, so that $\eta_T^2 = \sigma^2$.

Note that the $m \times n$ linear network transforms the transmitted signal such that under the assumed conditions, the corresponding sample values at the receiver are the best linear estimates of the $\{s_i\}$.

The bit error rate may be calculated as:

$$\begin{aligned}P_e &= \frac{1}{2} erfc\left[\frac{\sqrt{\xi_b}}{\sqrt{2}\eta_T}\right] \\ &= \frac{1}{2} erfc\left[\frac{1}{\ell}\sqrt{\frac{\xi_b}{N_o}}\right]\end{aligned} \tag{16}$$

The result of equation 16 will not show that enhancement on the performance of the system –but at least not worse– in comparison with other systems that depend on block transmission. The main goal obtained in this system is that no processing to eliminate the effect of the channel is done in the receiver. That leaves the receiver quite simple, and will save a lot through the designing process. Although there will be a little complication in the transmitter (i.e. base station), but comparing the savings in the manufacturing of the receivers (i.e. handsets) will show that the precoding in the base station is nothing.

B. *Tx-Sharing System*

In some application, where the transmitted signal faces a badly scattering channel, or in systems that need very high signal to noise ratio, receiver simplicity is not a place of concern. In these systems, one can accept some processing in the receiver in order to increase the performance of the system. We have developed a sharing strategy between the transmitter and the receiver for the downlink of a synchronous multiuser communication system in fading multipath environment. The sharing is such that some equalization is done at the transmitter, while the rest of the process is done at the receiver. This results in an enhancement in comparison with the precoding system, where all the equalization process is done at the transmitter and leaves the receiver quite simple. Also, as in the precoding case, It is assumed that the transmitter has prior knowledge of the multipath channels.

Figure 3 shows the basic model of the sharing system considered. The signal at the input to the transmitter is the same as precoding system. It is a sequence of k-level element values $\{s_i\}$, where k = 2, 4, 8,… and the $\{s_i\}$ being statistically independent and equally likely to have any of the possible values. Also, The buffer-store at the input to the transmitter holds m successive element values $\{s_i\}$ to form the $1 \times m$ data vector **S**. The transmitter's processor, $\mathbf{F_1}$, is an $m \times n$ matrix, so, in this processor, **S** is converted into the corresponding n elements vector **B** which is the convolution between **S** and $\mathbf{F_1}$ that is fed to the baseband channel.

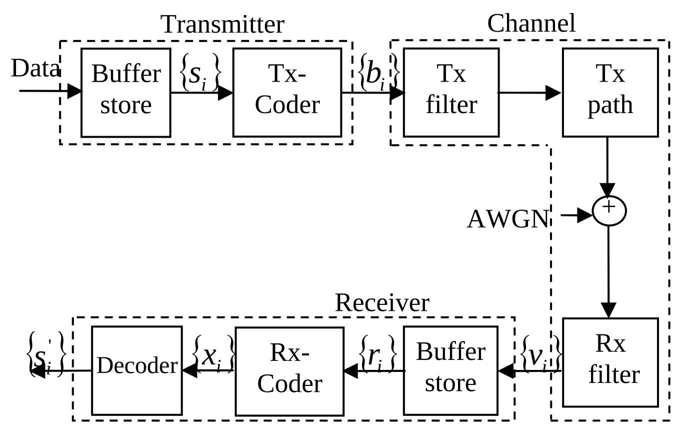

Fig. 3- Basic model of the sharing system with GB.

The value of n is chosen than it is the Algebraic sum of the length of the input data vector m and the channel's length g.

The linear baseband channel has an impulse response $y(t)$ and includes all transmitter and receiver filters used for pulse shaping and linear modulation and demodulation. White Gaussian noise is introduced at the output of the transmission path. The noise has zero mean and a two sided power spectral density of σ^2, giving the zero mean Gaussian waveform $w(t)$ at the output of the receiver filter. The sampled impulse-response of the baseband channel was given in equation 1.

As explained in the precoding system, the baseband channel can be represented in matrix form by the $n \times (n+g)$ matrix **C** and its i[th] row as given in equation 7:

The output of the channel will be the $1 \times (n+g)$ vector **V**, which is the convolution between **B** and **C** and also being given in equation 6.

Note that in this system also, we have two guard band of length g each at the beginning and the end of the transmitted vector at the input of the receiver. This is why we called it a sharing system with guard band. In the next system, we'll introduce another technique that looks the same, but without this guard band to reduce the used bandwidth of the system.

The received vector $1 \times (n+g)$ **V** will be the $1 \times (n+g)$ vector **BC** with the $1 \times (n+g)$ AWGN vector **W** added on. i.e.

$$\mathbf{V} = \mathbf{BC} + \mathbf{W} \tag{17}$$

Until now, we did not introduce anything that differs from the precoding system. The difference between the two models can be seen obviously in the receiver. The receiver buffer store chooses the central *m* component of the vector **V** to form the vector **R**, which will be fed to the receiver's processor matrix $\mathbf{F_2}$. This block is new, it was not mentioned in the precoding system (where the receiver contains only a comparator), and this is the main difference between the two systems.

In the sharing process studied here, the transmitter's processor operates as a precoding scheme on the transmitted signal, and the receiver's processor completes the detection process on the received vector to obtain the detected value of **S**. In each case, it has an exact prior knowledge of the channel characteristics **Y**, derived from the knowledge of the sampled impulse response of the channel. In the case of a time-varying channel, the rate of change in **Y** is assumed to be negligible over the duration of a received group of *m* signal elements, and sufficiently slow to enable **Y** to be correctly estimated from the received data signal [4].

Or goal from this system is to present a new technique with better performance than the precoding system. The channel characteristics have no effect on the behavior of the precoding system. The only effective element is the AWGN as shown in equation 14. So, let us make a look on the variance distribution of the precoding system to see how could we improve it. The variance at the output of the system is shown in Figure 4 and given in the equation 15. In order to reduce the power of the noise at the output of the system, we need to reduce η_T^2. Because we can't control the variance of the AWGN σ^2, the only factor we can modify is the transmitter coder (the precoder).

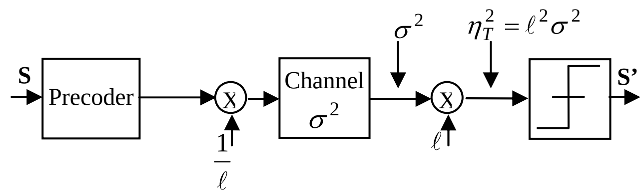

Fig. 4: Variance distribution in the precoding system.

The main idea proposed here is to split the precoding process between the transmitter and the receiver, so that the transmitter's share of the process is the $m \times n$ matrix:

$$\mathbf{F}_1 = \left(\mathbf{DD}^T\right)^{-p}\mathbf{D} \tag{18}$$

where:

$$0 \le p \le 1 \tag{19}$$

and the receiver's share of the process is the $m \times m$ matrix:

$$\mathbf{F}_2 = \left(\mathbf{DD}^T\right)^{-q} \tag{20}$$

where:

$$q = p - 1 \tag{21}$$

So, the total equation of the system from the input to the output is:

$$\mathbf{X} = \mathbf{SF}_1\mathbf{CF}_2 \tag{22}$$

$$= \mathbf{S}\left(\mathbf{DD}^T\right)^{-p}\mathbf{DD}^T\left(\mathbf{DD}^T\right)^{-q} = \mathbf{S} \tag{23}$$

As mentioned earlier, the assumption that $\mathbf{C} = \mathbf{D}^T$ is because that only the central *m* components of the vector V, i.e., $v_{g+1} \quad v_{g+2} \quad \ldots \quad v_{g+m}$, will be taken into consideration because they give information about the transmitted data without ISI.

In absence of AWGN, it is clear from the equation 23 above that there is no need for any further processing after the receiver's share of the equalization process, but when noise is present,

$$\mathbf{X} = \left(\mathbf{SF}_1\mathbf{C} + \mathbf{W}\right)\mathbf{F}_2 \tag{24}$$

$$= \mathbf{SF}_1\mathbf{CF}_2 + \tilde{\mathbf{W}} = \mathbf{S} + \tilde{\mathbf{W}} \tag{25}$$

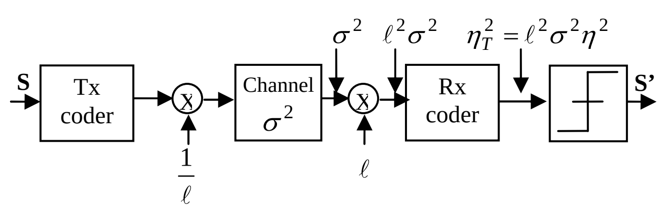

Fig. 5: Variance distribution in the sharing system with GB.

Thus the detector can now detect the values of the signal elements by comparing the corresponding $\{x_i\}$ with the appropriate thresholds.

The variance distribution of the sharing system is shown in Figure 5.

Using the same assumptions as in the precoding system, that the possible values of $\{s_i\}$ are equally likely and that the mean square value of **S** is equal to the number of bits per element. Suppose that the *m* vectors $\{\mathbf{D}_i\}$ have unit length. Since there are *m* *k*-level signal elements in a group, the vector **S** has k^m possible values each corresponding to a different combination of the *m* *k*-level signal-elements. So, the vector **B** whose components are the values of the corresponding impulses fed to the baseband channel, has k^m possible values. If *e* is the total energy of all the k^m values of the input data vector **S**, then in order to make the transmitted signal energy per bit equal to unity, the transmitted signal must be divided by

$$\ell = \sqrt{\frac{e}{nk^m}} \tag{26}$$

The *m* sampled values of the received signal from which the corresponding $\{s_i\}$ are detected, are the components of:

$$\mathbf{R}' = \frac{\mathbf{BD}^T}{\ell} + \mathbf{W} \tag{27}$$

Then, the *m* sample values which are the components of the vector **R'**, must first be multiplied by the factor ℓ to give the *m*-component vector

$$\begin{aligned} \mathbf{R} &= \ell\mathbf{R}' \\ &= \mathbf{S} + \mathbf{U} \end{aligned} \tag{28}$$

where **U** is an *m* component row vector whose components are sample independent Gaussian random variables with zero mean and variance $\ell^2\sigma^2$. Thus, the tolerance to noise of the transmitter's share is determined by the value of $\ell^2\sigma^2$.

In the receiver, the tolerance to noise can be calculated by:

$$\eta^2 = \frac{1}{m}\sum_{j=1}^{m}\sum_{i=1}^{m}(f_2)_{ij}^2 \tag{29}$$

and, the total tolerance to noise from both the transmitter's and the receiver's shares is

$$\eta_T = \sqrt{\ell^2\eta^2\sigma^2} \tag{30}$$

In case of no distortion, the signal to noise ratio $(\mathrm{SNR})_{\mathrm{ND}}$ is given by:

$$SNR_{ND} = \frac{\xi_b}{\sigma^2} \tag{31}$$

while the signal to noise ratio in the real channel (with noise) is:

$$\begin{aligned} SNR_C &= \frac{\xi_b}{\eta_T^2} \\ &= \frac{\xi_b}{\ell^2\eta^2\sigma^2} \end{aligned} \tag{32}$$

In order to understand the behavior of the system, we calculated the signal to noise ratio relative to no distortion channel

$$\begin{aligned} SNR_{relative} &= \frac{SNR_C}{SNR_{ND}} \\ &= \frac{1}{\ell^2\eta^2} \end{aligned} \tag{33}$$

or in dB:

$$SNR_{relative} = 10\log_{10}\left(\frac{1}{\ell^2\eta^2}\right) \tag{34}$$

The bit error rate equation may be written as:

$$\begin{aligned} P_e &= \frac{1}{2}erfc\left[\frac{\sqrt{\xi_{\mathrm{b}}}}{\sqrt{2}\eta_T}\right] \\ &= \frac{1}{2}erfc\left[\frac{1}{\ell\eta}\sqrt{\frac{\xi_{\mathrm{b}}}{N_o}}\right] \end{aligned} \tag{35}$$

C. Rx-Sharing System

The main difference between this system and the previous one is length of the transmitted vector. Both of them may transmit the same vector at the input of the transmitter, but after coding, the previous system generates a longer code than this one. This will give that system two guard band areas after and before the transmitted block, which will be useful in environments with many obstacles that usually cause duplicate versions of the transmitted signal, and finally cause Inter Symbol Interference (ISI).

Unfortunately, we can't get all the advantages in one system. The immunity against ISI will cause increase in bandwidth in an unaccepted ratios in some applications. For example, transmission in codes of 4 elements in an environment with a baseband channel of length 5 ($g = 4$), will cause a transmitted block of length 12 at the previous system and of length 8 at this one.

So, we introduced this system as a system for efficient use of available bandwidth in applications with less obstacles, or where ISI may be accepted in certain ratios.

Figure 6 shows the basic model of the sharing system without guard band considered. The signal at the input to the transmitter will not differ from the two previous systems. It is a sequence of *k*-level element values $\{s_i\}$, where *k* = 2, 4, 8,… and the $\{s_i\}$ being statistically independent and equally likely to have any of the possible values. The buffer-store at the input to the transmitter holds *m* successive element values $\{s_i\}$ to form the $1\times m$ data vector **S**. The difference will be clear in the size of the transmitter's processor, $\mathbf{F_1}$, in this case, it is an $m\times m$ matrix instead of $m\times n$, so, in this processor, **S** is converted into the corresponding *m* elements vector **B** which is the convolution between **S** and $\mathbf{F_1}$ that is fed to the baseband channel.

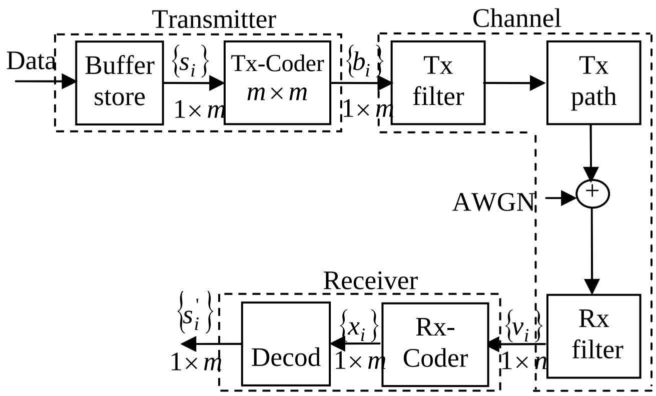

Fig. 6: Basic model of sharing system without GB.

The linear baseband channel has an impulse response $y(t)$ and includes all transmitter and receiver filters used for pulse shaping and linear modulation and demodulation. White Gaussian noise is introduced at the output of the transmission path. The noise has zero mean and a two sided power spectral density of σ^2, giving the zero mean Gaussian waveform $w(t)$ at the output of the receiver filter. The sampled impulse-response of the baseband channel is given in equation 1.

The baseband channel now may be represented in matrix form by the $m\times n$ matrix **Y** and its i[th] row is:

$$Y_{\mathbf{i}} = \overbrace{0 \quad \dots \quad 0}^{i-1} \quad \overbrace{y_g \quad y_{g-1} \quad \dots \quad y_o}^{g+1} \quad \overbrace{0 \quad \dots \quad 0}^{m-i} \tag{36}$$

where n is the sum of the length of the input vector m and the channel's length g.

Note that **Y** is derived and arranged in the same manner as done for **C** in the previous systems, but here, because that the transmitted block contains only m elements instead of n elements, the size of the channel matrix is $m\times n$ instead of $n\times n+g$.

The output of the channel will be the $1\times n$ vector **V**, which is the convolution between **B** and **Y**, with the $1\times n$ AWGN vector **W** added on.

$$\mathbf{V} = \mathbf{BY} + \mathbf{W} \tag{37}$$

This vector **V** will be fed to the receiver's processor matrix $\mathbf{F_2}$ to complete the detection process on the received vector to obtain the detected value of **S**.

As we did in the previous system, we are going to split the equalization process between the transmitter and the receiver. Here, the size of the channel output vector is $1\times n$, and the size of the receiver's coder matrix is $n\times m$, which means that all the vector is needed for the coding process, and no way to choose only the central components to reduce ISI. So, no need here to introduce a new matrix to represent the channel in the coders design as we did earlier.

The transmitter's share of the process will be the $m\times m$ matrix:

$$\mathbf{F}_1 = \left(\mathbf{YY}^T\right)^{-p} \tag{38}$$

and the receiver's share of the process is the $n\times m$ matrix:

$$\mathbf{F}_2 = \mathbf{Y}^T\left(\mathbf{YY}^T\right)^{-q} \tag{39}$$

where:

$$0 \le p \le 1 \tag{40}$$

and:

$$q = 1 - p \tag{41}$$

So, the total equation of the system from the input to the output is:

$$\mathbf{X} = \mathbf{SF}_1\mathbf{YF}_2 \tag{42}$$

$$= \mathbf{S}(\mathbf{YY}^T)^{-p}\,\mathbf{YY}^T(\mathbf{YY}^T)^{-q} = \mathbf{S} \tag{43}$$

In absence of AWGN, it is clear from the equation above that there is no need for any further processing after the receiver's share of the equalization process, but when noise is present,

$$\mathbf{X} = (\mathbf{SF}_1\mathbf{Y} + \mathbf{W})\mathbf{F}_2 \tag{44}$$

$$= \mathbf{SF}_1\mathbf{YF}_2 + \tilde{\mathbf{W}} = \mathbf{S} + \tilde{\mathbf{W}} \tag{45}$$

Thus the detector can now detect the values of the signal elements by comparing the corresponding $\{x_i\}$ with the appropriate thresholds.

As discussed earlier, the channel's impulse response has no effect on the total performance of the system, so, the only effective element is the AWGN. In order to study the performance of the system, we must find the tolerance to noise from the transmitter's and the receiver's shares.

Assume that the possible values of **S** are equally likely and that the mean square value of s is equal to the number of bits per element. Suppose that the m vectors $\{\mathbf{Y}_i\}$ have unit length. Since there are m k-level signal elements in a group, the vector **S** has k^m possible values each corresponding to a different combination of the m k-level signal-elements. So, the vector **B** whose components are the values of the corresponding impulses fed to the baseband channel, has k^m possible values. If e is the total energy of all the k^m values of the input data vector **S**, then in order to make the transmitted signal energy per bit equal to unity, the transmitted signal must be divided by:

$$\ell = \sqrt{\frac{e}{mk^m}} \tag{46}$$

Note here that this equation is not the same as equations 12 and 26 for the previous systems because of the difference in the transmitted block size.

The n sampled values of the received signal from which the corresponding $\{s_i\}$ are detected, are the components of:

$$\mathbf{V}' = \frac{\mathbf{BY}}{\ell} + \mathbf{W} \tag{47}$$

Then, the n sample values which are the components of the vector $\mathbf{V}'$, must first be multiplied by the factor ℓ to give the m-component vector

$$\mathbf{V} = \ell\mathbf{V}'$$

$$= \mathbf{S} + \mathbf{U} \quad (48)$$

where $\mathbf{U}$ is an m component row vector whose components are sample independent Gaussian random variables with zero mean and variance $\ell^2\sigma^2$. Thus, the tolerance to noise of the transmitter's share is determined by the value of $\ell^2\sigma^2$.

In the receiver, the tolerance to noise can be calculated by:

$$\eta^2 = \frac{1}{m}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(f_2\right)_{ij}^2 \quad (49)$$

So, the total tolerance to noise from both the transmitter's and the receiver's shares is:

$$\eta_T = \sqrt{\ell^2\eta^2\sigma^2}$$
$$= \ell\eta\sigma \quad (50)$$

In case of no distortion, the signal to noise ratio $(\mathrm{SNR})_{\mathrm{ND}}$ is given by:

$$SNR_{ND} = \frac{\xi_b}{\sigma^2} \quad (51)$$

while the signal to noise ratio in the real channel (with noise) is:

$$SNR_C = \frac{\xi_b}{\eta_T^2} \quad (52)$$

In order to understand the behavior of the system, we calculated the signal to noise ratio relative to no distortion channel

$$SNR_{relative} = \frac{SNR_C}{SNR_{ND}}$$
$$= \frac{1}{\ell^2\eta^2} \quad (53)$$

or in dB:

$$SNR_{relative} = 10\log_{10}\left(\frac{1}{\ell^2\eta^2}\right) \quad (54)$$

The bit error rate may be calculated as:

$$P_e = \frac{1}{2}erfc\left[\frac{\sqrt{\xi_\mathrm{b}}}{\sqrt{2}\eta_T}\right] \quad (55)$$

III. Numerical Results

We've built a Matlab program to simulate the three systems. The input of the program is a random data $\{+1 \quad -1\}$ (one million bit is used as input data), and the data will be processed in blocks of different lengths through different channels. Matlab built-in generator is used to add AWGN to the transmitted signal with different values of SNR, and ISI is taken into considerations. All these results have been compared with the results obtained mathematically for the systems in [6, 7].

Figure 7 shows comparisons between the three systems for both mathematical and simulation results for $m = 8$ and $Y = [0.5 \quad 1 \quad 0.5]$.

Now we proved that the models presented earlier are correct. So, now, we can study the effect of each effective variable of the systems, such as the block length, the guardband length, the channel characteristics and the SNR.

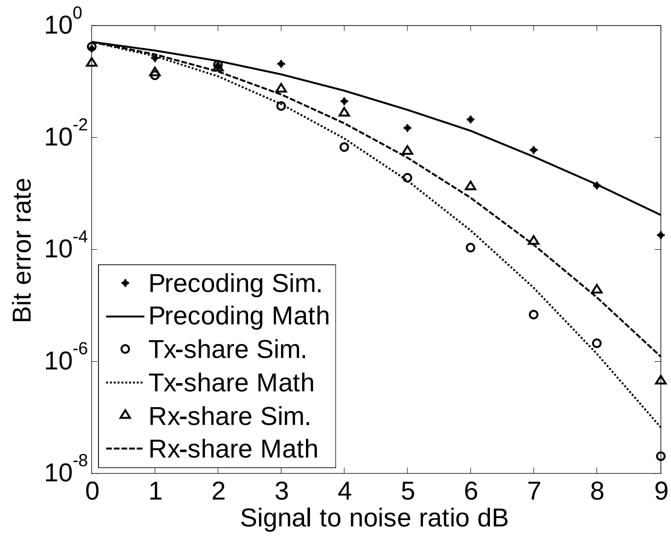

Fig. 7: Comparison between systems.

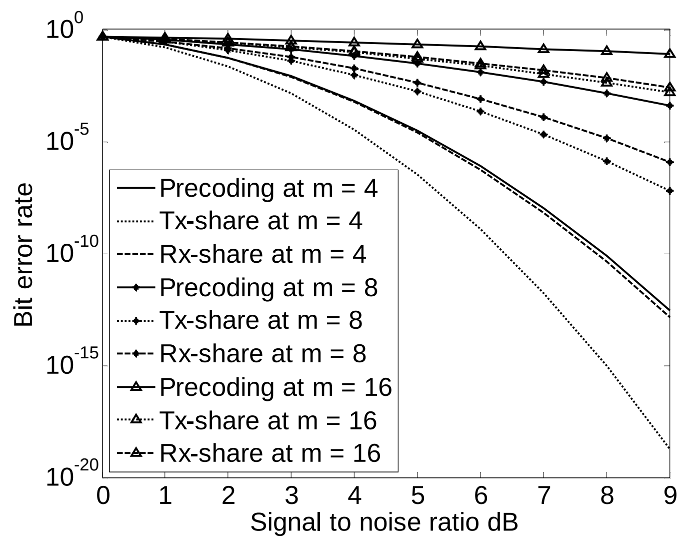

Fig. 8: Effect of block length m

The proposed systems depend on transmitting the input data in blocks. The source of these data may be serial, i.e. from the same source, or even parallel from different parallel sources. So, the length on the block itself is expected to have a great effect on the performance of the system. Figure 8 shows the bit error rate of the systems for different values of SNR using three different lengths of the block, i.e. $m = 4$, $m = 8$ and $m = 16$, the channel here is assumed to have impulse response $Y = [0.5 \quad 1 \quad 0.5]$.

From this figure, we can notice that increasing the block length will reduce the performance of the system and the probability of error becomes worse. This result is expected because increasing the block length will increase the variance of the coder matrices at the transmitter which maximize the noise variance at the output of the systems. Also, increasing the block length will increase the inter-symbol interference inside the block itself (ISI between the blocks is removed by using guardband in the precoding and Tx-sharing systems but still effective in the Rx-share system).

Theoretically, the best results will be for $m = 1$, which means transmitting each bit separately, and this is not accepted because in this case, each bit will use g bits as a guardband, and this is a great loss in the bandwidth. So, one must find an optimum solution for the block length. Comparing the system results with other used systems, we can say that $m = 8$ is a good choice.

In Figure 9, we studied the effect of the channel length on the performance of the systems. Here, we used two different channels with different lengths $g = 2$ and $g = 4$, but with the same norm values.

Although channels with longer vectors give the system more guardband bits to reduce ISI between blocks, but it will increase the variance η^2_{F-Tx} of the $m \times n$ coder matrices at the transmitter too, affecting an increase in the total noise variance η^2_T at the receiver side of the systems.

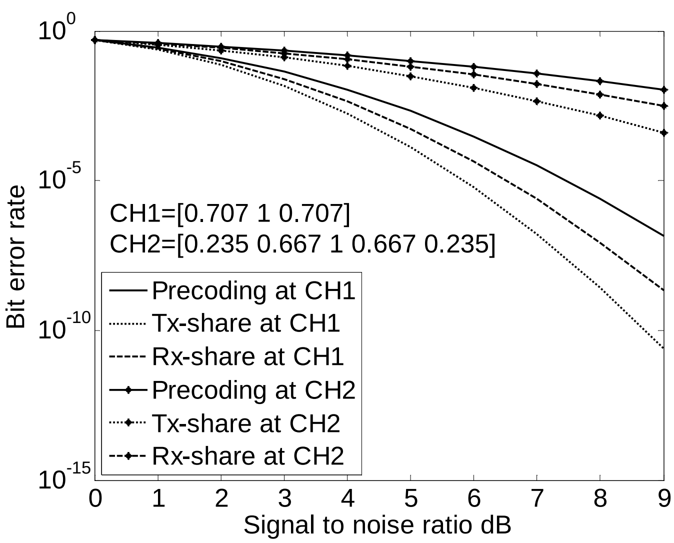

Fig. 9: Effect of channel length g

Then we tested the effect of the channel norm value of the performance of the system as shown in Figure 10. Here, we used two channels that differ in variance, but similar in length. It is clear that the channels with higher variance (norm) have better performance than those with lower variance. Note that the variance of the channel has no direct effect on the system, it affects the variance of the pre-coder, and that affects the total performance of the systems.

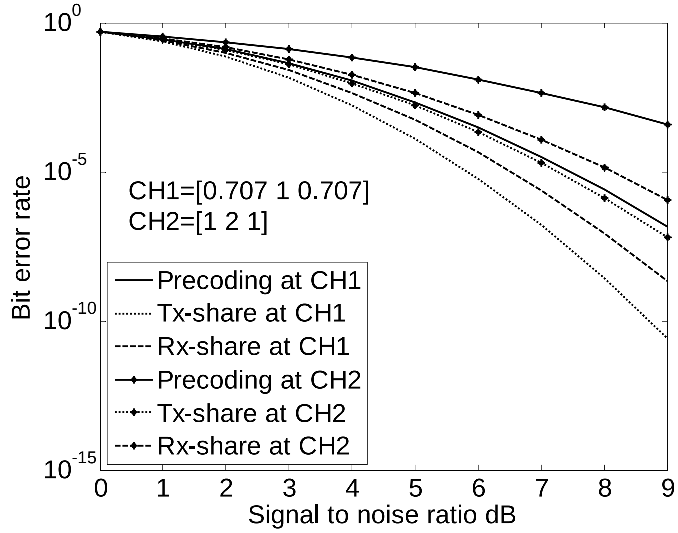

Fig. 10: Effect of channel norm

In Figure 11, we used typical channels, but we reversed the sign of one of them at one side. The effect was great. Asymmetric channels gave much better performance than symmetric one. It is not strange because the symmetric channel increased the coder variance four times more than the asymmetric.

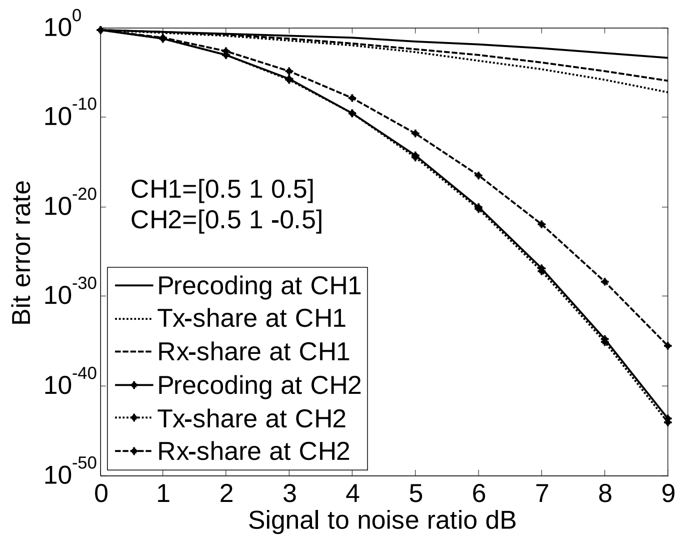

Fig. 11: Effect of channel symmetry

The amplitude of the channel will have no effect because if we use typical channels, but different in amplitude, that will lead us to two channels with the same normalized vector, which means the same performance. To clarify this point, let us make a look on the equation of the system The channel itself has no effect of the output signal; it affects only the coder matrices at the transmitter too. Figure 12 is an exampl

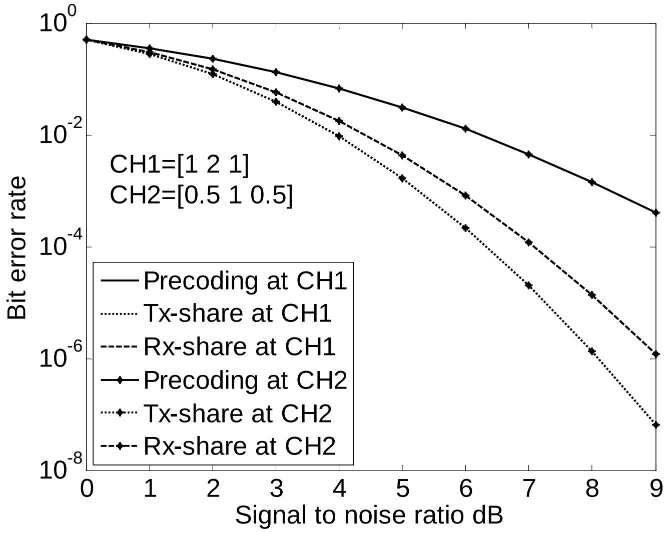

Fig. 12: Effect of channel amplitude

IV. CONCLUSION

In this paper, we have studied the parameters effects on some block transmission strategies in the transmitter and the receiver for the downlink of a synchronous multiuser communication system in fading multipath environment. We proved the mathematical models presented earlier. Those systems are promising ones, in the precoding system, no equalization process is needed in the receiver, while the sharing is 75% of the equalization is done at the transmitter, while 25% of the process is done at the receiver for the Tx-share system, and the Rx-share system has 25% in the transmitter and 75% in the receiver. This results in a 4 dB enhancement in comparison with the precoding system.

We studied the effect of the system parameters, and we get the following conclusions:

- The block length has a great effect on the performance of the system, the performance of the system increases while reducing the block length. We suggested to use a block with length $m = 8$ as an optimum solution.
- The channels with higher variance have better performance than those with lower variance.
- Short channels are much better in performance than long ones despite of the extra guardband they offer.
- Channel amplitude has no effect on the system.
- Asymmetric channels gave much better performance than symmetric ones.

It is assumed that the transmitter has prior knowledge of the multipath channels. There are many available estimation techniques in the literature that may fit this system.

APPENDIX

Appendixes, if needed, appear before the acknowledgment.

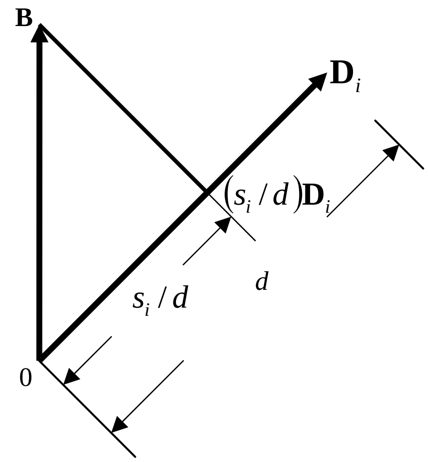

Fig. 13: Vectors **B** and $\mathbf{D}_i$ for $d > 1$ and $s_i = 1$.

Assuming that d is the length of the vector $\mathbf{D}_i$, where $i = 1, 2, \ldots, m$, (i.e. the distance of the point $\mathbf{D}_i$ from the origin in an *nT* dimensional vector space). It can be easily shown that $\mathbf{D}_i$ is independent of *i*. $\mathbf{BD}_i^T$ is the inner product of the vectors **B** and $\mathbf{D}_i$ so that it is *d* times the value of the orthogonal projection of **B** onto the vector $\mathbf{D}_i$ [9]. Thus, **B** lies on the hyper plane ($n-1$ dimensional subspace) which contains the point $(s_i / d)\mathbf{D}_i$ and which is orthogonal to the vector given by this point, so that the hyper plane is orthogonal to the line joining the origin to $(s_i / d)\mathbf{D}_i$. The vectors **B** and $\mathbf{D}_i$ are shown in Figure 13, for the case where $d > 1$ and $s_i = 1$. The vector **B** must, therefore, lay on each of the *m* hyper planes and as illustrated in Figure 13. Thus, the required vector **B** is the point on these *m* hyper planes at the minimum distance from the origin. By the Projection Theorem [9], **B** is the orthogonal projection of the origin on to the $nT - m$ dimensional subspace formed by the intersection of the *m* hyper planes. Thus **B** is the intersection of the *m* dimensional subspace spanned by the *m* $\{\mathbf{D_i}\}$ (each of which is orthogonal to the corresponding hyper plane) with the $nT - m$ dimensional subspace formed by the intersection of the *m* hyper planes. Clearly, **B** can be represented as a linear combination of the *m* $\{\mathbf{D_i}\}$, so that:

$$\mathbf{B} = \sum_{i=1}^{m} e_i \mathbf{D_i} \tag{56}$$

where $\mathbf{E} = [e_1 \quad e_2 \quad \cdots \quad e_m]$

Then, it can be easily shown that

$$\mathbf{S} = \mathbf{BD}^T = \mathbf{E}\left(\mathbf{DD}^T\right) \tag{57}$$

Thus,

$$\mathbf{E} = \mathbf{S}\left(\mathbf{DD}^T\right)^{-1} \tag{58}$$

and,

$$\mathbf{B} = \mathbf{S}\left(\mathbf{DD}^T\right)^{-1}\mathbf{D} \tag{59}$$

From the previous equation, knowing that $\mathbf{B} = \mathbf{SF}$, it is clear that $\mathbf{F}$ can be given by

$$\mathbf{F} = \left(\mathbf{D}\mathbf{D}^T\right)^{-1}\mathbf{D} \tag{60}$$

So,

$$\begin{aligned}\mathbf{R} &= \mathbf{B}\mathbf{D}^T + \mathbf{W} \\ &= \mathbf{S}\left(\mathbf{D}\mathbf{D}^T\right)^{-1}\mathbf{D}\mathbf{D}^T + \mathbf{W} \\ &= \mathbf{S} + \mathbf{W} \end{aligned} \tag{61}$$

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