A Study of the Multi-Scale WOFDM Transmission in Time Variant Channels

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Abstract— WOFDM (Wavelet-based Orthogonal Frequency Division Multiplexing) reshapes the multi-carrier transmission concept, by using wavelet carriers instead of OFDM's complex exponentials. The modulator and the demodulator rely on the Discrete Wavelet Transform (DWT) computation. One crucial parameter of this transform is the number of iterations, tightly related to the time-scale nature of the wavelet transform. Practically, the number of DWT iterations coincides with the number of transmission scales used. The influence of this parameter on the WOFDM transmission performance is studied in this paper. The considered scenario refers to a flat, time variant Rayleigh fading channel. Our simulations show that, by increasing the number of transmission scales, the BER performance degrades. This result is explained by the fact that a small number of iterations keeps the duration of the transmitted symbols significantly shorter than the coherence time of the channel.

***Keywords*—**WOFDM, DWT, scales, time-variance.

I. Introduction

Wavelets represent a successful story of the last decade in signal processing applications. Thus, these signals, with some remarkable properties, are widely used in various domains as compression, denoising, segmentation or classification. Despite their popularity in signal processing, few wavelets applications were found for data transmission. By the other hand, in data communications, the same successful story can be assigned to multi-carrier modulation techniques. Practically, most of the data transmission systems nowadays use Orthogonal Frequency Division Multiplexing (OFDM) or some versions of it. We can mention here WiFi (IEEE 802.11), WiMAX (IEEE 802.16) or ADSL. Due to its capabilities, OFDM is the preferred technique even for most of the power-line transmission systems [1]. It is especially the excellent OFDM resilience to the inter-symbol interference (ISI) that makes from this technique a reliable candidate for transmission in any dispersive channel.

The WOFDM technique, sometimes referred to as wavelet modulation, is the point where the above concepts meet with each-other. Although they are widely used in signal processing, few wavelets applications are known in data transmission. The idea that gathers the two concepts is to use wavelet signals as carriers in a multi-carrier data transmission.

Despite its undoubted advantages, OFDM presents some drawbacks too [2]. Recent research has shown that, by associating the multi-carrier concept and the wavelet signals, some of the OFDM's classical drawbacks can be counteracted. Thus, the sidelobes of the OFDM spectrum contain an important amount of energy, causing interference in the adjacent bands. The interference is a consequence of the sinc spectrum of every modulated carrier in OFDM. This shape of the spectrum is a result of gating the sine carrier (having, by its nature, infinite duration) with a rectangular waveform, which localizes the symbol on the time axis. In the WOFDM transmission, due to the waveform of the carriers, significant out-of-band rejection is provided by comparison to OFDM [3]-[5]. Thus, wavelets, by their nature, have finite duration. From this point of view, we may say that wavelets, unlike the sine carriers, are localized on the time axis "by themselves". This property leads to a spectrum which has significant supplementary out-of-band rejection compared to the sinc spectrum from OFDM.

Furthermore, since wavelets meet Nyquist's criterion for zero ISI, they can be used as pulse-shaping waveforms [6],[7]. Due to the fact that the WOFDM symbol is a linear combination of modulated wavelets, this technique can be seen as a combination of multicarrier transmission and pulse-shaping.

Another well known drawback of OFDM is the high value of the Peak-to-Average-Power Ratio (PAPR) [8]. There are even applications which combine the wavelet-pulse shaping and OFDM, commonly referred to as filtered multi-tone modulations [9]. In these systems, superior out of band rejection is provided, but the Peak-to-Average Ratio remains a challenge.

It is well known that the orthogonality of the OFDM carriers relies on their precise positioning on the frequency axis [10]. It is the Doppler shift effect that occurs in the time variable channels, which particularly attacks this orthogonality. Unlike in OFDM, the orthogonality of the wavelet carriers relies on both their time position and their frequency localization (scale). This makes the WOFDM transmission less sensitive to Doppler, leading to noticeable BER improvements, under certain circumstances [11]. All the above considerations prove that an extensive investigation of WOFDM is worthwhile.

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An important detail related to the two techniques (OFDM and its wavelet based version) is that their practical implementation relies on digital signal processing algorithms. This reduces the implementation complexity, and leads to simple and cost-effective solutions for the multicarrier based techniques. At the time, the idea of using the Fast Fourier Transform (FFT) algorithm for a software-based implementation of the OFDM's modulator and demodulator, transformed this technique from an attractive, but difficult to implement idea, into a incredibly successful story of the data transmission. Happily, the same kind of helpful principle can be used for WOFDM. Thus, an inverse transform implements the modulator and the direct transform will be the key point of the demodulator.

In the case of WOFDM, the implementation is based on the famous Mallat's filter bank algorithm, which computes the DWT [12]. This transform has two important parameters: the wavelets mother and the number of iterations used in computation. Previous research [13] has shown that the BER performance of WOFDM systems is significantly influenced by the wavelets mother choice. Now, we will focus on the other parameter of the WOFDM transmission, the number of DWT iterations. In the next section, we will review the principles of WOFDM, focusing on its practical implementation. The transmission chain it's described in section 3. In section 4, simulation results are presented and commented. Last section is dedicated to some conclusions and possible advances on the subject.

II. WOFDM PRINCIPLES

In any multi-carrier modulation, the orthogonality of the multiple carriers is the key point that allows their separation at receiver. The multicarrier approach has the advantage of the long symbol duration, provided by the simultaneous transmission of several low-rate parallel streams. OFDM relies on this idea, and employs complex exponential carriers having frequencies which are multiples of f_0 (1):

$$subc(k) = e^{jk2\pi f_0 t} \tag{1}$$

where *subc(k)* denotes the k-th subcarrier used. The idea which links OFDM and wavelets is that, in the same manner that the complex exponentials are orthogonal to each-other, the members of a "wavelet family" will satisfy the same property:

$$<\Psi_{j,k}(t), \Psi_{m,n}(t)> = \begin{cases} 1, \ if \ \ j = m \ and \ k = n \\ 0, \ otherwise \end{cases} \tag{2}$$

Such a family can be obtained by translating and scaling a unique function called wavelets mother and denoted by $\Psi(t)$ (3):

$$\psi_{j,k}(t) = s_0^{-j/2}\psi(s_0^{-j} \cdot t - k\tau_0) \tag{3}$$

Equation 3 corresponds to a sampled version of a wavelet family, the discrete variables being s_0 (the scale) and k (the position within the scale). In the following we will consider $s_0=2$ and $\tau_0=1$. It is of particular interest for our research to see what is the effect of changing the scale factor j. Therefore, in figure 1, some members of such a family are shown, at different scales and locations.

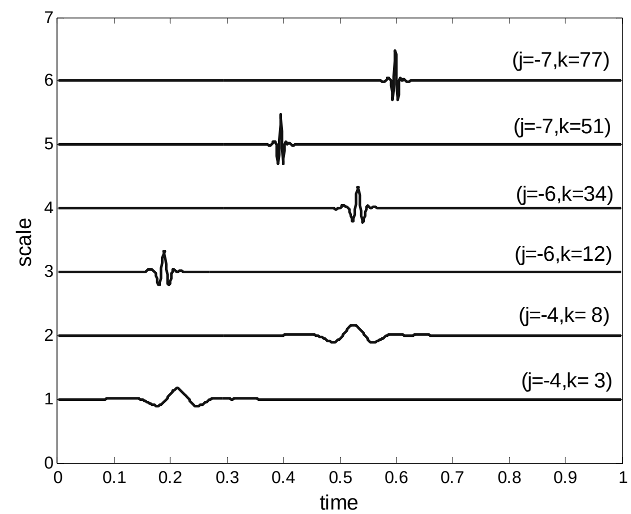

Fig.1: Some Symmlet wavelets at different scales and locations.

If $\psi_{j,k}(t)$ forms an orthonormal, family, we can state that any signal *s(t)* can be written as a weighted sum of wavelet functions (4), the weights being given by the wavelet coefficients ($w_{j,k}$):

$$s(t) = \sum_{j \in Z} \sum_{k \in Z} w_{j,k} \Psi_{j,k}(t) \tag{4}$$

In the context of our research, the signal *s(t)* can be interpreted as being a "WOFDM symbol". In practice, in order to restrict the number of scales j to a finite value, some "scaling functions" must be employed. Thus, any wavelet from a certain scale j can be composed as a weighted sum of scaling functions from the previous scale $j-1$. This, a different form of signal synthesis equation will be obtained:

$$s(t) = \sum_{j<L} \sum_{k} w_{j,k} \Psi_{j,k}(t) + \sum_{k} a_{L,k} \varphi_{L,k} \tag{5}$$

In the equation above, $j=L$ is the coarsest level used for signal composition, and a_L are called approximation coefficients. This relation is close to the computation of an inverse transform, the signal being synthesized from some "decomposition" coefficients. However, in practice, the WOFDM signal *s(t)* is not directly computed using equation (5). One reason is that, for most of the wavelet families, the

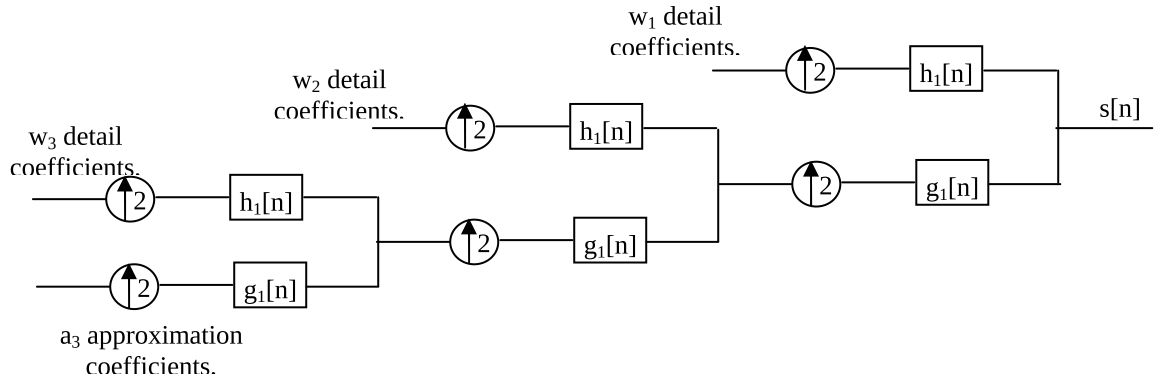

Fig. 2: IDWT implementation using filter banks.

analytical expression of $\Psi(t)$ is not known. Mallat gave an ingenious solution to this problem, by its famous filter bank decomposition algorithm [12]. Using this algorithm, the discrete version of the signal, *s[n]*, is calculated by performing the Inverse DWT (IDWT). At each iteration, an up-sampling operation is performed (with the factor 2), followed by a filtering operation (fig. 2).

Figure 2 describes the IDWT transform for three iterations (decomposition levels). g_1 and h_1 are the impulse responses of the synthesis low-pass and high-pass filter respectively. Their concrete form depends on the wavelets mother which is used. Each iteration of the algorithm illustrated in figure 2, can be described by the equation:

$$x[n] = \sum_k (w[k]h_1[n-2k] + a[k]g_1[n-2k]) \qquad (6)$$

We can consider that the data we have to transmit is a set of approximation and wavelet (detail) coefficients, as follows:

$$data = \{[a_L],[w_L],[w_{L-1}],...,[w_1]\} \qquad (7)$$

This kind of view (input data interpreted as wavelet coefficients) it is only a model for a better understanding. In practice, we simply transmit bipolar values (+1 or -1), which represent the data bits.

The data sequence is brought to the input of the IDWT processor, whose output will be the discrete version of the WOFDM symbol. This process is illustrated in fig. 3.

We will now take a closer look to fig.2, in order to understand the exact meaning of these "coefficients" which compose the input data vector. Consider the "complementary" scheme (the signal processing from right to left): after each iteration the

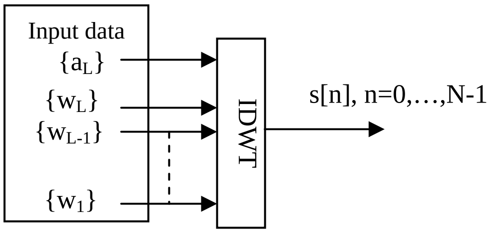

Fig. 3: WOFDM symbol computation by IDWT.

number of wavelet coefficients halves (because each iteration will rely on a down-sampling and filtering couple). Thus, considering that our data vector from (6) has *N* samples, than half of them will be stored in [w_1], and they will be transmitted in the channel using the upper half of the dedicated bandwidth. Next, [w_2] contains one quarter of the symbols to be transmitted. Finally, considering the last iteration performed (the L^{th}, according to fig. 3), we will have 2^{J-L} symbols grouped in the approximations vector and an equal number stored in the details vector (also referred to as wavelet coefficients). From practical implementation reasons, the number of signal samples composing a WOFDM is limited (*N* samples). Consequently, the maximum number of Mallat's algorithm iterations is limited too, to J iterations (eq. 8):

$$J = \log_2 N \qquad (8)$$

The remarks above justify the attribute of "multi-scale transmission" for WOFDM. Indeed, unlike the classical OFDM technique, where each carrier occupies the same bandwidth, in WOFDM we have subcarrier sets, every set covering a different bandwidth (one half of the dedicated bandwidth after the first iteration, one quarter after the second one etc).

The relation between fig. 2 and equation 5 is not straightforward. Note however that the best time (and the poorest frequency) resolution is achieved after the first iteration. At this point, we can figure out that the wavelet coefficients at this scale will modulate the wavelet carriers which have the best time localization within the wavelet family. Note that, higher the number of iterations, less compacted will be the corresponding wavelet carriers in time and more concentrated in frequency. Now, taking into account the time-scale nature of the wavelet transform, we can state that at those scales where the number of symbols to be transmitted is smaller, the duration of each symbol is higher. The same tendency can be highlighted for the wavelet carriers at different scales. These remarks are of particular interest for our study. Since WOFDM relies on an IDWT modulator, it follows that the transmission performance can be influenced by the parameters of this modulator. As stated earlier, we will focus on the number of iterations used in the IDWT computation.

III. THE TRANSMISSION CHAIN

An area of particular interest for the multi-carrier modulation techniques is the transmission in the radio channels. Therefore, our simulation scenario is focused on this type of scenario. In the following, we will consider a transmission in a flat Rayleigh fading channel, the signal being corrupted by additive noise.

The transmission chain used for simulations is shown in figure 4.

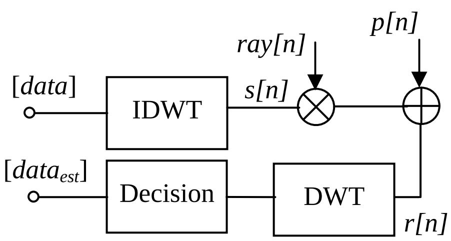

Fig. 4 : Baseband implementation of a WOFDM system.

The blocks which compose this simulation chain will be described in the following subsections.

A. The transmitter

The data source is a sequence of equally likely bipolar symbols (+1 and -1). This data is grouped in blocks of N=1024 samples, before being transferred to the IDWT modulator. The IDWT modulator is the key point of the transmitter. Data is processed as shown in the previous section, and next transmitted in the channel. Because our main goal was to asses the influence of the number of IDWT iterations, we considered two cases with respect to this parameter: $n_{it}=1$ and $n_{it}=4$. In the first case, we can view the data vector at IDWT entry as being a sequence of 512 approximation coefficients and 512 detail coefficients. In the second case, the structure of the data vector will be interpreted by the IDWT modulator as:

$$data = \{a_4(64), d_4(64), d_3(128), d_2(256), d_1(512)\} \quad (9)$$

The subscript values from equation 9 represent the iteration number, whereas in the parenthesis we retrieve the number of symbols transmitted at that scale. The modulator output will be a signal *s[n]* of 1024 samples. If we consider that this signal is a sampled version of an analog signal, the sampling step being T_S, then the duration of the symbols transmitted after j iterations will be:

$$T_j = 2^j T_S \quad (10)$$

This formula will become meaningful when we will try to explain some simulation results, during the next section. It is also important to mention that the variable duration of the transmitted symbol is a direct consequence of the time-scale nature of the wavelet transform. In DWT, each new iteration will halve the number of coefficients, but the total time interval occupied by the samples at each scale remains the same.

B. The channel

The radio channel exhibits small scale fading, which confers to this transmission environment two independent characteristics: time variance and frequency selectivity [14]. The variance in time of the radio channel's behavior can be expressed by the mean of the Doppler shift parameter, which depends on the relative motion between transmitter and receiver (assuming mobile communications) and on the carrier frequency used for transmission. The maximum value of this parameter is expressed by:

$$f_d = \frac{V}{\lambda} = \frac{V}{c} \cdot f_C \quad (11)$$

In eq. 9, V is the relative velocity of the receiver versus the transmitter c is the speed of light, f_c represents the carrier frequency, determined by the wavelength used (denoted by λ). In the baseband simulation, a normalized version of this parameter is used:

$$f_m = f_d \cdot T_S \quad (12)$$

The normalization factor is the sampling step used for simulations (T_S). For simplicity reasons, we may consider that this step equals the symbol time from equation 10, and we use the same notation. In our simulations, we considered two values for f_m: 0.005 and 0.05. Higher the value of this parameter, more rapidly the channel changes in time. A more straightforward parameter which quantifies the variability of the channel is the coherence time, expressed as:

$$T_C = \frac{0.423}{f_d} \quad (13)$$

By extracting f_d from (12) and replacing it in (13), the coherence time can be expressed as:

$$T_C = \frac{0.423 T_S}{f_m} \quad (14)$$

It is of outmost importance for our goals to compare this coherence time with the durations of the symbols transmitted at each scale (after each iteration). This comparison is shown in table 1.

These values will gain a particular interest for the explanation of the simulation results illustrated in the next section. The small scale fading can be modeled using a Rayleigh distribution.

Table 1: Coherence times versus symbol times.

f_m		0.005	0.05
T_C		$84.6T_S$	$8.46T_S$
T_j	$n_{it}=1$	$2T_S$	
	$n_{it}=4$	$16T_S$	

Its impact is given by the multiplicative *ray[n]*. This random sequence is described by its spectral characteristics and by its statistical properties. Thus, the power spectral density (PSD) of the Rayleigh sequence takes into account the Doppler shift. The theoretical formula for this PSD is commonly referred to as Jakes spectrum [14] and is plotted in figure 5, for f_d=138Hz.

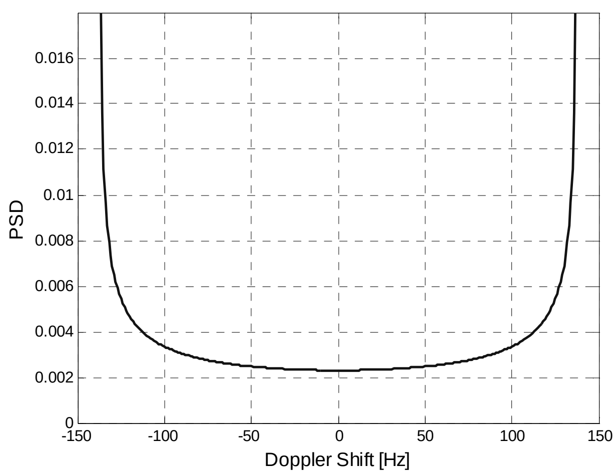

Fig. 5: The theoretical PSD of the fading sequence.

The probability density function the fading sequence is modeled as a Rayleigh distribution:

$$pdf\,(x|\sigma) = \frac{x \cdot e^{-x^2 / 2\sigma^2}}{\sigma^2} \tag{15}$$

A white noise *p[n]* is added to the fading perturbed signal, generating the sequence *r[n]* to be processed by the demodulator:

$$r[n] = s[n] \cdot ray[n] + p[n] \tag{16}$$

All the above conditions for channel simulation outline a flat and time-variant channel scenario.

The white noise variance is gradually changed in order to be able to plot BER versus SNR curves.

C. The receiver

The key point of the receiver is the DWT “demodulator”. The decision on the transmission symbol is made based on a simple zero threshold comparison. The BER is computed for 1 and 4 DWT iterations, corresponding to the value of this parameter used at the transmitter side.

IV. SIMULATION RESULTS

The WOFDM transmission was simulated using Matlab 7. The wavelet related issues of our simulations are implemented in Wavelab, a free Matlab toolbox [14]. The main parameters used in simulations are synthesized in table 2.

Table 2: Simulation parameters global view.

Parameter	**Value**
Number of blocks	10000
Number of samples per block	1024
Wavelets used as carriers	Haar, Daubechies-4, 8 12 and 16 Coifflet-1, 2, 3, 4 and 5 Symlet-4, 6, 8 and 10
SNR range	0 to 30 dB
Channel	AWGN and Flat fading
Normalized Doppler shift, f_m	0.005 and 0.05
DWT iterations (n_{it})	**1 or 4**

Like the multitude of the parameters from table 1 proves it, an extensive set of simulations was conducted. In our study, we will mainly focus on the influence of the last parameter from table 2: the number of DWT iterations used in the modulator and demodulator. The two values of this parameter were chosen such as to provide a relevant view of how this choice can modify the BER performance of our system. In the case of the wavelets used, the numerical index associated with each wavelet represents the number of vanishing moments. Although there is no straightforward analytical equation which links this number and the time support of the wavelets, note that, higher the number of vanishing moments, more dilated will be the wavelets in time. This remark is relevant for some of our simulation results.

Besides the classical BER measure, we will introduce another related parameter, for comparison purposes. This parameter is called Error Increase Ratio (EIR). At a fixed SNR, this measure can be computed as follows:

$$EIR = \frac{BER\_4}{BER\_1} \tag{17}$$

The usage of this parameter is driven by some experimental observations which were made after our simulations. EIR provides a framework which allows to simultaneously take into account both values used for n_{it} . It was noticed [16] that in the considered scenario, increasing the number of DWT iterations leads to a BER degradation, especially when the channel varies rapidly (see table 2). This results lead to an EIR which is generally higher than 1, no meters what is the SNR value. Furthermore, our measurements showed that this parameter is monotonically increasing with SNR. This fact is illustrated in fig. 6, for one wavelet selected from each family, and f_m=0.05.

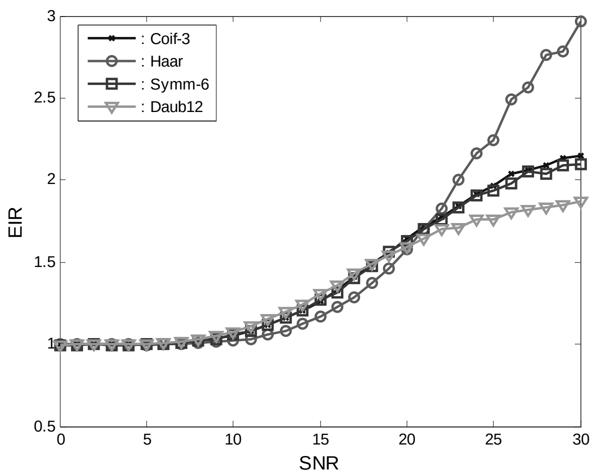

Fig. 6: Error increase ratio for different wavelets, f_m=0.05.

The explanation of this result is that, at high SNRs most of the errors will be caused by the Doppler shift, and not by the noise. As shown in some previous studies ([11], [13]), the amount of errors caused by the variability in time of the channel is related to the time localization of the wavelet carrier. Thus, higher number of iterations means longer duration wavelet carriers and longer symbol time, as illustrated by equation 10. Consequently, after each iteration (at each scale), the coherence time of the channel exhibits a different degree of influence on the WOFDM transmission performance. The most spectacular difference can be observed for the Haar wavelet, where the BER at 30 dB is three times lower when a single iteration is used than in the case with four iterations. The other extreme is Daubechies-12 wavelet, with barely one half BER degradation at four iterations, compared to the Haar case. The other wavelets are somehow at the middle of the two cases. The reason behind this result is that amongst all wavelets, Haar is the more compact in time. Consequently, this wavelet is the most affected by the choice of the number of iterations. As shown in previous studies, the Haar wavelet provides, by far, the best results in the flat fading scenario, especially when a single iteration is used [13]. This explains the spectacular growth of the curve in figure 6, at high SNR. In general, the wavelets with weak time localization will be less sensitive to the number of iterations. Otherwise stated, they provide poorer results even for a single iteration.

The previous conclusion is supported by the plots in figures 7 and 8. In the first case, we choose, from each wavelet family, the two "extreme" wavelets, having the shortest and the longest time support respectively. Wavelets are, in this case, differentiated by the number of vanishing moments. We can clearly identify, in figure 7, two different patterns. Thus, the most influenced by the number of transmission scales are the wavelets with compact support (Coiflet-1, Symmlet-4 and Haar, sometimes referred to as Daubechies-2). Significantly less impacted are the wavelets with longer time support within each family. The conclusion can be highlighted even for wavelets belonging to the same family. In figure 8, we plot the EIR versus SNR curves for all the wavelets from the Daubechies family. By analyzing fig. 8, we may notice that, whereas for Daubechies-20, BER at 30dB is approximately 1.5 times higher when 4 transmission scales are used, for the

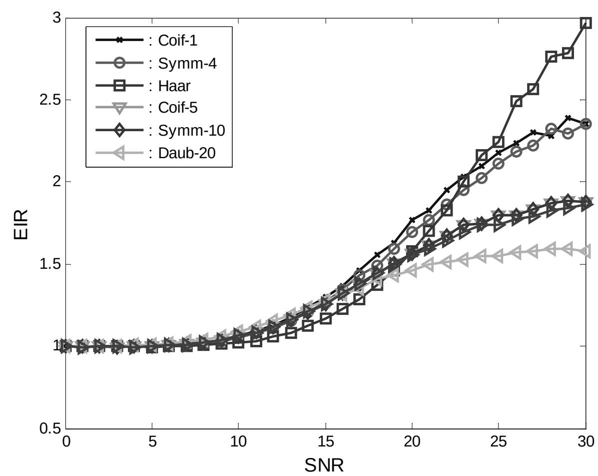

Fig. 7: Error increase ratio for the "extreme" wavelets within each family, f_m=0.05.

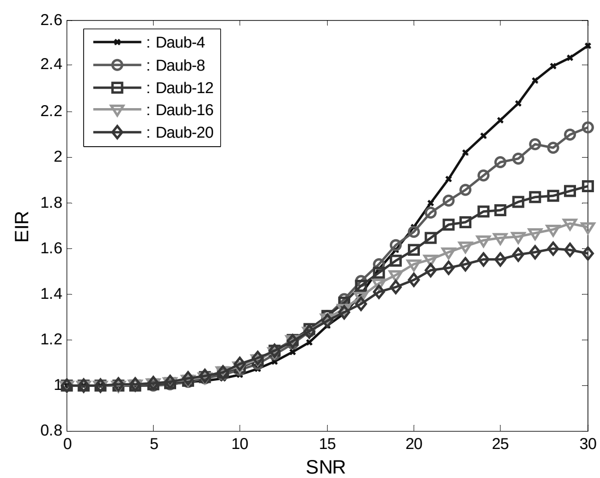

Fig. 8: Error increase ratio for different Daubechies wavelets, f_m=0.05.

Daubechies-4 case the same degree of degradation is already observed at 15dB of SNR.

Note that in all the previous figures, we considered a fast fading scenario, since the coherence time of the channel (see table 1) is barely one half of the symbol duration at the fourth scale. When we simulate the slow fading scenario, the results change spectacularly: there will be no significant BER degradation when increasing the number of transmission scales. The results are shown in figures 9 and 10. In figure 9, we plot the EIR versus SNR for the same wavelets like in figure 6, but for a ten times lower Doppler shift. In this case, there is no noticeable pattern of BER degradation for neither of the considered cases. Furthermore, when if we plot the EIR curves only for those wavelets which, supposedly, are the more impacted by the transmission on four scales, the previous remark is strengthened. Even with a zoomed y axis, there isn't any monotonic EIR versus SNR evolution. The local extremes in the EIR are practically not relevant (between 0.96 and 1.08). A compendium of the results is shown in table 3.

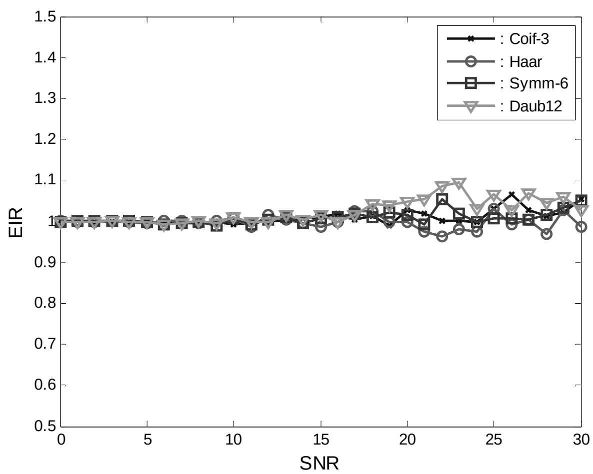

Fig. 9: Error increase ratio for different wavelets, f_m=0.005.

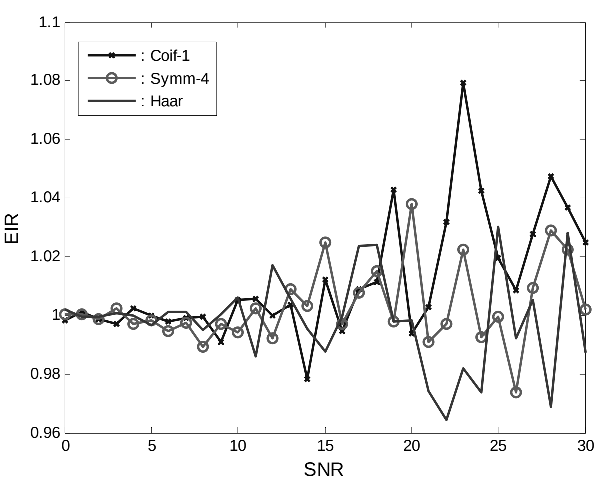

Fig. 10: Error increase ratio for the most time compacted wavelets, at f_m=0.005

Table 3: A compendium of the simulation results

Wavelet type	Min gain	EIR (30 dB)	Min gain	EIR (30dB)
	f_m=0.05		f_m=0.005	
Coif1	12	2.35	3	1.02
Coif2	12	2.40	2	1.02
Coif3	12	2.14	4	1.05
Coif4	12	1.98	2	1.03
Coif5	12	1.87	2	1.03
Daub4	**11**	**2.48**	0	1.00
Daub8	12	2.12	4	1.02
Daub12	13	1.87	3	1.02
Daub16	13	1.69	6	1.08
Daub20	**13**	**1.57**	5	1.09
Symm4	12	2.35	2	1.00
Syym6	12	2.09	2	1.05
Symm8	12	1.93	1	1.02
Symm10	12	1.86	2	1.04
Haar	**10**	**2.96**	0	0.98

This table contains, besides the EIR, the gain brought by the use of a single scale, for all the tested wavelets. Because our SNR resolution was of 1dB, it was not possible to compute an exact value of this parameter. Instead, a minimal gain is defined. For this purpose, the BER at SNR=30 dB when 4 iterations are used is considered as a starting point (denoted BER_i). Next, we identify the lowest SNR (SNR_f) for the one iteration case which leads to better BER results, and we denote this BER value by BER_f. This means that the search is stopped when BER_f becomes higher than BER_i. The minimal gain will be the difference between the two SNRs:

$$SNR\_\min\_gain\,[dB] = 30 - SNR_f \tag{18}$$

Note however that this measure represents a minimal gain only for the highest simulated SNR.

Table 3 confirms that there is a totally different behavior pattern for $f_m=0.05$ and $f_m=0.005$. Remember that, under certain assumptions, the two scenarios can be considered as fast fading and slow fading respectively.

Thus, the number of iterations impact is much higher in the fast fading scenario. In this case, the time variability of the channel has a predominant role in the errors occurrence. This happens because the duration of the transmitted symbol after the third iteration ($8T_S$) is already comparable with the coherence time ($8.46T_S$), becoming much higher at the fourth scale ($16T_S$). The consequence is that, whereas at $f_m=0.05$ spectacular gains are obtained (from 10 to 13 dB), the results for the slow fading case are rather irrelevant. In the later case, the coherence time of the channel is five times longer than the symbol duration, even after the fourth iteration (see table 1).

As already mentioned, the BER performance in time variant channels does not depend only on the duration of the symbols transmitted at each scale: the shape of the carriers must be considered too. More precisely, some of the wavelet carriers are more concentrated in time than others. The impact can be highlighted by table 3. For example, within the Daubechies wavelets family, the most spectacular results (EIR evolution) are obtained for wavelets having a smaller number of vanishing moments, and thus a more compact time support.

Effective BER results are shown in figures 11 and 12, for some of the extreme cases (bolded in table 3). We treated separately the fast and the slow fading case respectively, considering their different behavior. The highest BER improvement at one iteration versus 4 iterations are obtained by the wavelets Haar and Daubechies-4. The later case was chosen for the plot in figure 11. Both these wavelets lead to an EIR of almost three. Previous research has shown [13] that these wavelets provide the best overall BER performance among all the tested wavelets, because they are the most compacted in time. This determines their sensitivity to the choice of n_{it} parameter and explains their relatively poor SNR gain compared to the other wavelets (10dB). Note however that the SNR gain may be a misleading result, as it will be explained later. These remarks support our previous conclusions about the better results of the short duration wavelets. By the other hand, even with an important gain of 13dB, the Daubechies-20 wavelet leads to closed curves in the two cases. Thus, the practical significance of the 13dB gain provided by Daubechies-20 wavelet is rather limited. The meaning of the previous remarks leads to a negative interpretation of the results rather than to a positive one (e.g. independence on the number of iterations). The later result can be explained by the fact that Daubechies-20 is the more dilated in time amongst all the tested wavelets. In our simulations, we noticed that, from the BER point of view, this wavelet provides the worst performance. Thus, with a poor score even for one iteration, the WOFDM transmission based on this wavelet is not significantly impacted by increasing the number of transmission scales.

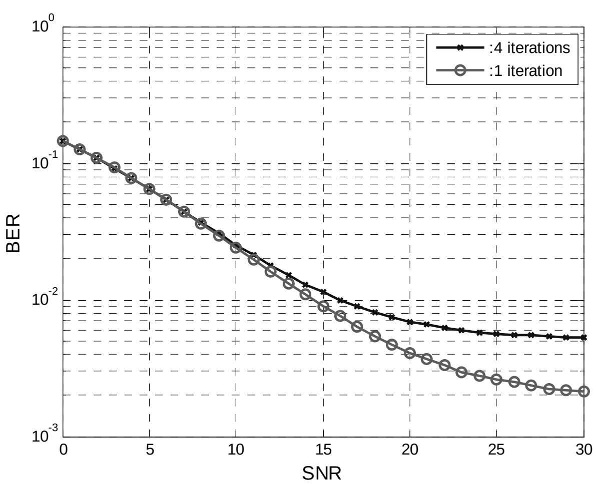

Fig. 11: BER versus SNR, Daubechies-4 case, f_m=0.05.

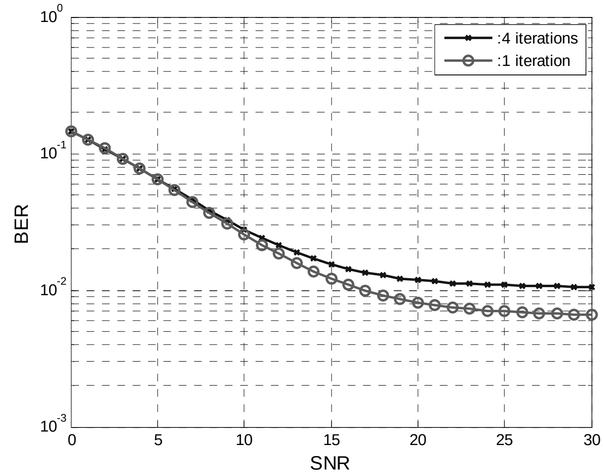

Fig. 12: BER versus SNR, Daubechies-20 case, f_m=0.05.

Further support of our conclusions is given in figure 13. This time we display the BER performance for some time-compacted wavelets belonging to Symmlet and Coiflet families. It is clearly highlighted, once more, the BER degradation which appears when the number of WOFDM transmission scales increases. As an additional remark, there is no noticeable performance difference between the wavelets plotted in this figure.

Another idea is confirmed too, by the plots in figures 11-13: the difference in the BER performance (between one and four transmission scales) occurs starting with high SNR values (above 10 dB). For lower SNR values, it is the AWGN noise which impacts the WOFDM transmission more.

In slow fading conditions, the BER degradation when increasing the number of transmission scales is rather not relevant. This was proved by the previous EIR versus SNR plots. The conclusion is strengthened by figures 14 and 15. We choose from table 3 some wavelets that provide the best SNR

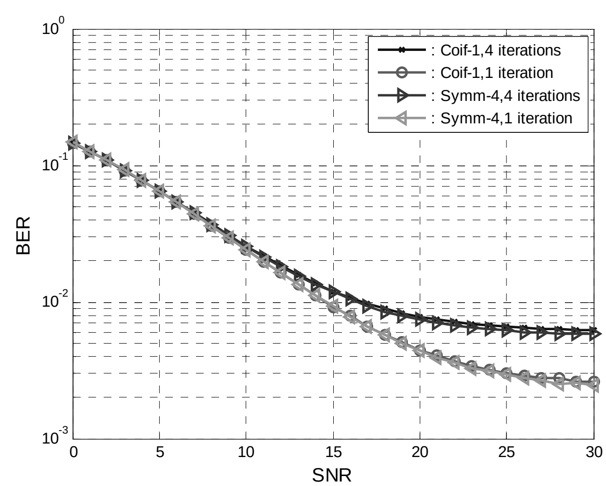

Fig. 13: BER versus SNR for Coiflet-1 and Symmlet-4 wavelets, f_m=0.05

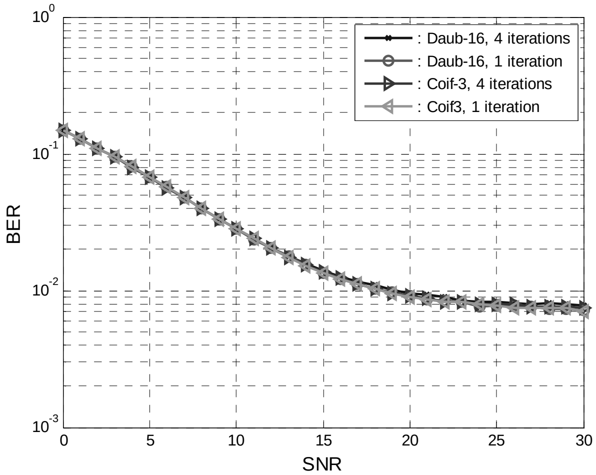

Fig. 14: BER versus SNR for Daubechies-16 and Coiflet -3 wavelets, at f_m=0.005.

minimal gain within their families: Daubechies-16 (6dB of gain) and Coiflet-3 (4dB of gain). The SNR gain brought by the transmission on a single scale is practically unnoticeable on the plot (see fig. 14). The four curves are undistinguishable. This clearly demonstrates how limited is the influence of the number of transmission scales in slow fading conditions.

Repeating the same experience for the wavelets which should be, supposedly, the most affected by the channel variability, the previous conclusion is strengthened. Even for the wavelets with the shortest time support, there is no influence of the number of scales on the WOFDM transmission BER. This happens because the coherence time of the channel is not “attacked” by the duration of the transmitted symbols at neither of the transmission scales. Basically, at the Doppler shift of 0.005 the fading channel tends to an AWGN one, where, as shown by previous studies [3] the BER performance is independent on the WOFDM parameters (wavelets mother, number of DWT iterations / transmission scales).

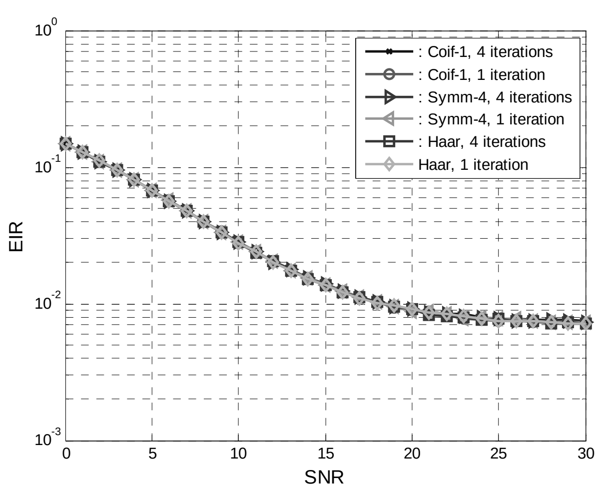

Fig. 15: BER versus SNR for the most compact wavelets, at f_m=0.005.

IV. Conclusions and Further Work

The influence of the number of DWT iterations on the WOFDM transmission BER is studied in this paper. Due to the nature of the DWT algorithm, this parameter can be assimilated with the number of transmission scales. We consider a time variant channel, which exhibits flat Rayleigh fading.

We can clearly differentiate two behavior patterns. In the fast fading scenario (symbols times comparable or lower than the coherence time of the channel), the BER performance is strongly influenced by the number of transmission scales. The best performance is obtained when a single transmission scale is used: SNR gain of at least 10 dB and EIR of at least 1.69. The above values are collected for the maximum SNR used in simulations (30dB). In fact, the BER performance for one iteration exceeds the performance with 4 iterations only in good SNR conditions, when the time variability predominates over the noise, and is the main source of errors in the computed BER.

A deeper look on the results highlight that the most impacted by the number of transmission scales in the fast fading case are those wavelets which have a compact time support (e.g. Haar, Daubechies-4, etc). This happens because, due to their good time localization, these wavelets provide improved BER performance compared to the other ones, especially when a single DWT iteration is used. Practically, we may say that the BER performance depends not only on the ratio between the coherence time and the symbol time at each scale, but on the carriers duration too. Being the most compact in time, these carriers have durations, at the first transmission scale, which, even if longer than the symbol time, remains significantly shorter than the coherence time of the channel. When increasing the number of iterations, the duration of those wavelet carriers increases and becomes comparable or higher than the coherence time of the channel. By the other hand, the other wavelets, having a large time support (e.g., Daubechies-20) have comparable duration with the coherence time of the channel starting even from the first transmission scale. Consequently, increasing the number of scales does not affect the WOFDM transmission so dramatically.

The conclusions are totally different for the slow fading case. For the chosen value of f_m , the coherence time of the channel remains five times longer than the longest symbol duration, obtained at the fourth scale. In this case, the WOFDM performance does not depend on the number of iterations used.

For the continuation of this study, the author intends to support the conclusions above by carrying out an analysis of the way that the errors are distributed across the transmission scales. This allows to better highlight the BER degradation when increasing the number of transmission scales, and to have a more coherent measure of this effect.

`Another interesting direction is to take into account the other major effect introduced by the radio channel, besides the time variability: its frequency selectivity. Indeed, as concluded, in the flat fading channel, the only effect which influences the BER performance is the time variability of the channel. This effect was linked by the author to the time localization of the wavelets used as carriers. It was proved that, in general, the best performance is provided by the carriers having the most compact time support. The pattern might be totally different when the channel is frequency selective too. In this scenario, the wavelet having good frequency localization could be favored. But, given the time-frequency duality, these wavelets are the ones with have a poor time localization, and which provided the worst BER performance for the flat fading channel. What we expect is that, in a channel which is both time variant and frequency selective, the best performance to be given by those wavelets which provide the best compromise between their frequency and their time localization.

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M. Oltean, "On the Wavelet OFDM Performance in Time Variant Channels: Choosing the Number of DWT Iterations", *Proceedings of the WSEAS International Conference DNCOCO'08*, Bucuresti, Romania ISSN: 1790-5109, 2008, pp. 85-90.

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