A New Approach for Reduction of Redundancy Using Optimal Time Domain Sampling and Interpolation

S. Izadpanah , M. M. Ghanbarian , A. Kazerooni

***Abstract*—**In this paper the authors consider a general method, based on time domain samples for spectral manipulation of the time-limited signals. In this direction, all of the needed formulas for practical periodic time-limited interpolation in two cases of low pass and band pass has been derived. The work begins with dividing an arbitrary signal in time domain into the time limited non-overlapping frames, then each frame is processed to find the effective maximum frequency or equivalently the minimum number of samples that satisfy some error criteria. To find the optimum sampling frequency we used periodic interpolation for resampling and reconstruction, and suitable zero finding of the discrete variable nonlinear equation by combination of the time and frequency computation to increase the speed of convergence.

Keywords—Time domain sampling and interpolation, Local bandwidth estimation, Data compression, Spectral analysis.

I. INTRODUCTION

The Shannon's sampling theorem is based on the constant uniform period of sampling. If we have a signal with such slow local frequency variation, in some of its parts, we can expect to re-sample, and reconstruct these certain parts with a less number of data. If this idea and process works successfully, we could remove the redundant data, and consequently achieve to the data compression .

The general view of this process leads to the non-uniform, or irregularly spaced sampling, and reconstruction. Shannon had remarked roughly this complex general type of sampling in the original work on sampling [1].However, the first regular statement about this type of sampling attributed to Caushy[1].

Manuscript received. september 9, 2008: Revised version received november 4, 2008. This work was supported in part by Islamic Azad University.

S. Izadpanah is with the Islamic Azad University , Branch of Dehdasht , Iran (e-mail: conferene.2008@gmail.com).

M. M. Ghanbarian is with the Islamic Azad University , Branch of Kazeroon , Iran (e-mail: cnference.azad@gmail.com).

A. Kazerooni is with the Islamic Azad University , Branch of Kazeroon , Iran (e-mail: conference.1387@gmail.com).

The Yen's theorems are the first analytical work in some special cases on the non-uniform sampling[2]. The first general, and analytical manipulation, could be seen in the Beutler work, that marks a suitable statement, interpretation, and proof of the theorem known as a "folk theorem" [3].

Higgins tried to change the form of Butler's formula similar to the usual cardinal series as in the Shannon reconstruction formula, about ten years after Butler [4]. Others handled, and worked in some aspect on the non-uniform sampling, as in the application on tomography [5], using the transformation method [6], applying the theory of almost periodic functions [7], application in the reconstruction of the image from irregularly spaced samples [8], interpolation from non-uniform samples [9], and so on. However these works generally are limited to the special cases. The other powerfully, applied, and analytical approach to the non-uniform sampling is achieved by Feichtinger, and Grochening [10], which mentions its application in the reconstruction of band-limited image from non-uniform sampling values [11].

The results of the investigation on the above paper, theorems, and applications show that the non-uniform sampling theorem is not planned and designed for removing the redundant data, and data compression. However some engineers were interested in this type of sampling because of some difficulties, and problems were encountered in the nature and practice [5,6,7,10,11].However, our aim is redundancy cancellation by non-uniform sampling, so we considered the piece-wise uniform distribution of data, and in fact we divided an arbitrary signal to the time-limited non-overlapping sections, and each section as a frame. Then with defining, and finding effective maximum frequency for each frame, we tried to sample, and reconstruct the time-limited parts independently from the adjacent frames.

Therefore in the first step we considered the sampling and interpolation of each frame. Then we studied the methods for reducing the time of convergence, and finally we applied the results on the total signal.

II. - EFFECTIVE MAXIMUM FREQUENCY

We assume that the original signal for processing is the digital samples produced by suitable sampling from the analog signal and stored in the vector (X). We denote the sampling frequency by $(f_{s1} = T_{s1}^{-1})$, and the duration of the time limited frame by (T_0), which is contained by (N), uniformly spaced sampling values in the time interval $I = [0, T_0)$, such that:

$$t_n = (n-1)T_{s1} \quad , n = 1, ..., N \tag{1}$$

$$T_0 = N \times T_{s1} \quad , X(n) = X(t_n) \tag{2}$$

The time limited frame suffers from the reciprocal spreading effect [12], therefore we relatively reduced this effect to bound the maximum frequency by spreading each frame in the time domain. Frame spreading is achieved by considering a time limited part as single period of the periodic signal. Now in order to define and compute the effective maximum frequency (f_{me}) for each frame required some error criteria, such as Mean Square Error (MSE), Signal to Noise Ratio (SNR) or Percent Root mean square Difference (PRD). In fact we consider PRD as MSE that is normalized by the signal power. If the vector (p) contained the original samples and the vector (q) denoted the reconstructed signal,. we will have:

$$SSE(p,q) = \sum_{i=1}^{N} \left[p(i) - q(i) \right]^2 \tag{3}$$

$$SS(p) = SSE(p, \vec{0}) \tag{4}$$

$$PRD(p,q) = 100 \times \sqrt{\left[SSE(p,q) / SS(p) \right]} \tag{5}$$

$$MSE(p,q) = SSE(p,q)/N \tag{6}$$

$$SNR(p,q) = 10 \times \log\left[SS(p) / SSE(p,q) \right] \tag{7}$$

$$SNR = 20 \times \left[2 - \log\left(PRD \right) \right] \tag{8}$$

$$PRD = 100 \times 10^{(-SNR/20)} \tag{9}$$

Now we consider the PRD criteria and define the new distribution of data in the interval (I) that is stored in the vector (Y) as below:

$$u_m = (m-1)T_{s2} \quad , m = 1, ..., M \tag{10}$$

$$T_0 = M \times T_{s2} \quad , Y(m) = Y(u_m) \tag{11}$$

Let PRDM be a maximum allowable error in PRD, and (Z_M) denote the reconstructed vector for original N points vector (X) by using the new M points distribution (u_m), and consider the following inequality to define effective maximum frequency for this frame:

$$PRD(X, Z_M) \le PRDM < PRD(X, Z_{M-1})$$

$$M = 2, ..., N \tag{12}$$

$$f_{se} = 2 \times f_{me} = f_{s2} = T_{s2}^{-1} = M/T_0 \tag{13}$$

$$, for \quad M = 1 \Rightarrow f_{s2} = 0 \quad (DC - Level) \tag{14}$$

As mentioned earlier, we must apply the periodic feature to reconstruct (Z_M). In other words, we require the periodic interpolation, discussed in the following section.

III. PERIODIC INTERPOLATION

Let $f(t)$ be a periodic band-limited function with period $(T_0 = f_0^{-1})$, and the largest harmonic (n), so that we use the following interpolation formula [13]:

$$N = 2n + 1 \tag{15}$$

$$T = T_s = T_0/N \tag{16}$$

$$\omega_0 = 2\pi \times f_0 \tag{17}$$

$$f(t) = \sum_{k=0}^{N-1} f(kT) \times p(t - kT) \tag{18}$$

$$p(t) = \frac{Sin\left[N \times \left(\frac{\omega_0 t}{2} \right) \right]}{N \times Sin\left(\frac{\omega_0 t}{2} \right)} \tag{19}$$

But, as we can see from the above equations the number (N) is always odd. This problem is unsuitable for our application, thus we will prove the above formula in the general case, and also will consider some important features of our extracted formula. Remember the conventional Shannon's formula and follow the below equations for $f(t)$:

$$f(t) = \sum_{k=-\infty}^{+\infty} f(kT) \times h(t - kT) \tag{20}$$

$$h(t) = 2 \frac{Sin(\omega_c t)}{\omega_s t} \tag{21}$$

$$f_m < f_c < f_s - f_m \quad OR \quad f_{me} \le f_c \le f_s - f_{me} \tag{22}$$

$$h(t) \xleftrightarrow{FT} H(f) = T \times \Pi(\frac{f}{2 f_c}) \tag{23}$$

Define $p(t)$, and use the periodic characteristic of $f(t)$, results

$$p(t) = \sum_{i=-\infty}^{+\infty} h(t - iT_0) \tag{24}$$

$$f(t) = \sum_{k=0}^{N-1} f(kT) \times p(t - kT) \tag{25}$$

$$p(t) = h(t) * \sum_{i=-\infty}^{+\infty} \delta(t - iT_0) \tag{26}$$

$(*)$ Stand for convolution operation.

$$P(f) = H(f) \times \left[f_0 \times \sum_{i=-\infty}^{+\infty} \delta(f - if_0) \right] \tag{27}$$

$$M = \left\lfloor \frac{f_c}{f_0} \right\rfloor \tag{28}$$

Where the $\lfloor r \rfloor$ is defined as the greatest integer less than or equal to (r).

$$P(f) = N^{-1} \times \sum_{i=-M}^{+M} \delta(f - if_0) \tag{29}$$

$$p(t) = N^{-1} \times \sum_{m=-M}^{+M} e^{jm\omega_0 t} \tag{30}$$

$$\sum_{k=p}^{q} \left(e^{j\theta}\right)^k = e^{j\left(\frac{p+q}{2}\right)\theta} \times \frac{Sin\left[\left(\frac{q-p+1}{2}\right)\theta\right]}{Sin\left(\frac{\theta}{2}\right)} \tag{31}$$

$$p(t) = \frac{Sin\left[(2M+1) \times \left(\frac{\omega_0 t}{2}\right)\right]}{N \times Sin\left(\frac{\omega_0 t}{2}\right)} \tag{32}$$

To have independent samples' value, the two equations below must hold:

$$p(t=0) = 1 \Rightarrow N = 2M+1 \tag{33}$$

$$p(t=kT)\big|_{k\neq 0} = 0 \tag{34}$$

Therefore N must be odd, and we can show that for the odd N the second equation also holds.
Now to find M in terms of N, we select the cut-off frequency (f_c),to be the average of the maximum and minimum allowable values, thus we will have:

$$M = \left\lfloor \frac{f_c}{f_0} \right\rfloor = \left\lfloor \frac{f_s}{2 f_0} \right\rfloor = \left\lfloor \frac{N}{2} \right\rfloor \tag{35}$$

$$p(t) = \frac{Sin\left[\left(2\left\lfloor \frac{N}{2} \right\rfloor + 1\right) \times \left(\frac{\omega_0 t}{2}\right)\right]}{N \times Sin\left(\frac{\omega_0 t}{2}\right)} \tag{36}$$

Some of the characteristics and properties of the above formulae are:

$$p(t=0) = \begin{cases} 1 & N = odd \\ 1 + \frac{1}{N} & N = even \end{cases} \tag{37}$$

$$p(mT)\big|_{m\neq 0} = \begin{cases} 0 & N = odd \\ \frac{(-1)^m}{N} & N = even \end{cases} \tag{38}$$

$$\sum_{k=odd} f(kT) = \sum_{k=even} f(kT) \quad , k = 0,1,\ldots, N-1, N = even \tag{39}$$

$$I_{mn} = \int_{T_0} p(t-nT)p(t-mT)dt \tag{40}$$

$$\Rightarrow I_{mn} = T \times p[(m-n)T] \tag{41}$$

As a result, we deduced that if N is an odd integer the set

$$\{p(t-kT) \quad , k = 0,1,\ldots, N-1\}$$

construct an orthogonal set. Also we can exactly show the following relations:

$$Power = \rangle [f(t)]^2 \langle = E\{[f(kT)]^2\} \tag{42}$$

$$DC = \rangle f(t) \langle = E\{f(kT)\} \tag{43}$$

Where:

$$\rangle \xi(t) \langle = \frac{1}{T_0} \times \int_{T_0} \xi(t)dt \tag{44}$$

$$E\{\xi(kT)\} = \frac{1}{N} \times \sum_{k=0}^{N-1} \xi(kT) \tag{45}$$

IV. BAND-PASS INTERPOLATION

The periodic interpolation can be extended to the band-pass case. We briefly consider two types of band-pass interpolation, first order and second order [14].Let $x_{bp}(t)$ be a band-pass function such that the frequency component is vanished out of the frequency range $(f_l < |f| < f_h)$. These two types of interpolation organize as below:

A. - FIRST ORDER BAND-PASS INTERPOLATION

$$k = Integer, \quad 0 \le k \le k_{\max} = \left\lfloor \frac{f_l}{f_h - f_l} \right\rfloor \tag{46}$$

$$\frac{2 f_h}{k+1} \le f_s \le \frac{2 f_l}{k} \tag{47}$$

$$-f_l + kf_s \le f_{cl} \le f_l \tag{48}$$

$$f_h \le f_{ch} \le -f_h + (k+1) f_s \tag{49}$$

$$x_{bp}(t) = \sum_{n=-\infty}^{+\infty} x_{bp}(nT) h_{bp}(t - nT) \tag{50}$$

$$h_{bp}(t) = \frac{2[Sin(\omega_{ch}t) - Sin(\omega_{cl}t)]}{\omega_s t} \quad (51)$$

$$(f_s)_{\min} = \frac{2f_h}{k_{\max}+1} = \frac{2f_h}{1+\left\lfloor \frac{f_l}{f_h - f_l} \right\rfloor} \quad (52)$$

If the term $\frac{f_l}{f_h - f_l}$ is an integer number then

$$(f_s)_{\min} = 2(f_h - f_l) = 2\Delta f \quad (53)$$

In this type of interpolation, generally we do not have the minimum possible number of independent samples. This fault is removed by the second order or exact band-pass interpolation, that is discussed in the following section.

B. SECOND ORDER BAND-PASS INTERPOLATION

$$T^{-1} = f_s = f_h - f_l \quad (54)$$

(r) satisfies

$$f_l \le f_u < f_h \quad , f_u = -f_l + rf_s \quad (55)$$

(k) Must hold in the three following conditions:

$$k \ne \frac{n}{(r+1)f_s}, |n| = 1, 2, \ldots \quad (56)$$

$$k \ne \frac{n}{rf_s}, |n| = 1, 2, \ldots \quad (57)$$

$$k \ne 0 \quad (58)$$

$$\gamma = k\omega_s / 2 \quad (59)$$

$$\beta = r\gamma \quad (60)$$

$$\alpha = (r+1)\gamma \quad (61)$$

$$\Phi(t) = \frac{Cos(\omega_h t - \alpha) - Cos(\omega_u t - \alpha)}{\omega_s Sin(\alpha) t} \quad (62)$$

$$\Psi(t) = \frac{Cos(\omega_u t - \beta) - Cos(\omega_l t - \beta)}{\omega_s Sin(\beta) t} \quad (63)$$

$$S(t) = \Phi(t) + \Psi(t) \quad (64)$$

$$x_{bp}(t) = \sum_{n=-\infty}^{+\infty} [x_{bp}(nT)S(t-nT) + x_{bp}(nT+k)S(nT+k-t)] \quad (65)$$

In practice data is biased to a DC-Level, so that we remove this DC-Level and then consider the band-pass periodic function as below:

$$x_{bp}(t) = \sum_{n=p}^{q} A_n \times Cos(n \times \omega_0 \times t + \phi_n) \quad (66)$$

Define the minimum and maximum frequency by:

$$f_l = (p - \frac{1}{2}) f_0 \quad (67)$$

$$f_h = (q + \frac{1}{2}) f_0 \quad (68)$$

Now we can extend periodic interpolation to the band-pass case in the two following sections:

C. FIRST ORDER PERIODIC BAND-PASS INTERPOLATION

$$f_s = Nf_0 \quad (69)$$

$$f_{cl} = \frac{k \times f_s}{2} \quad (70)$$

$$f_{ch} = \frac{(k+1) \times f_s}{2} \quad (71)$$

$$L = \left\lfloor \frac{f_{cl}}{f_0} \right\rfloor + 1 \quad (72)$$

$$M = \left\lfloor \frac{f_{ch}}{f_0} \right\rfloor \quad (73)$$

$$\gamma(t) = \left\{ \frac{Sin\left[(2M+1) \times \left(\frac{\omega_0 t}{2}\right)\right]}{N \times Sin\left(\frac{\omega_0 t}{2}\right)} - \frac{Sin\left[(2L-1) \times \left(\frac{\omega_0 t}{2}\right)\right]}{N \times Sin\left(\frac{\omega_0 t}{2}\right)} \right\} \quad (74)$$

$$x_{bp}(t) = \sum_{n=0}^{N-1} x_{bp}(nT) \times \gamma(t - nT) \quad (75)$$

D. SECOND ORDER BAND-PASS PERIODIC INTERPOLATION

First choose the suitable parameter, that is introduced in part B.

$$f_l \le -f_l + rf_s < f_h \Rightarrow \frac{2f_l}{f_s} \le r < 1 + \frac{2f_l}{f_s} \quad (76)$$

$$\xi = \frac{2f_l}{f_s} \Rightarrow \xi \le r < 1 + \xi \quad (77)$$

$$r = \begin{cases} \xi & \xi = Integer \\ \lfloor \xi \rfloor + 1 & \xi \ne Integer \end{cases} \quad (78)$$

$$k = \frac{1}{(r+0.5) \times f_s} \quad (79)$$

$$f_s = f_h - f_l = (q - p + 1) f_0 = Nf_0 \quad (80)$$

$$f_u = -f_l + rf_s = (rN - p + \frac{1}{2}) f_0 \quad (81)$$

$$w = rN - p \Rightarrow f_u = (w + \frac{1}{2}) f_0 \quad (82)$$

$$\theta = \omega_0 t \quad (83)$$

$$\Gamma(t)=\frac{Cos\left[(2q+1)\frac{\theta}{2}-\alpha\right]-Cos\left[(2W+1)\frac{\theta}{2}-\alpha\right]}{2\times N\times Sin(\alpha)\times Sin\left(\frac{\theta}{2}\right)} \quad (84)$$

$$\Lambda(t)=\frac{Cos\left[(2W+1)\frac{\theta}{2}-\beta\right]-Cos\left[(2p-1)\frac{\theta}{2}-\beta\right]}{2\times N\times Sin(\beta)\times Sin\left(\frac{\theta}{2}\right)} \quad (85)$$

$$g(t)=\Gamma(t)+\Lambda(t) \quad (86)$$

$$x_{bp}(t)=\sum_{n=0}^{N-1}\left[x_{bp}(nT)g(t-nT)+x_{bp}(nT+k)g(nT+k-t)\right] \quad (87)$$

V. . PRACTICAL DIFFICULTIES

In this part we focused on the LP-periodic interpolation, which can modify and extend to the BP case. Consider the (FIG.1) as a typical wave form for a basic frame. We reconstructed this frame by the following three important methods: MATLAB SPLINE, SHANNON and PERIODIC, in (FIG.2) through (FIG.4) respectively. All of them suffer from the tail effect that is unsuitably interpolated between the last new sample and the end of the frame. The fitness of the MATLAB SPLINE is nearly insufficient. The SHANNON method likes to vanish rapidly at the two ends of the time-limited part; because it supposes the zero samples at the two ends. The PERIODIC method smoothly varies such that to reach the first sample value at the end of the frame. Therefore the amount of the tail effect depends on the frame behavior. We tried to reduce this dependency of error and thus reduced the amount of error due to the tail effect that is discussed in the following section.

VI. . TAIL EFFECT REDUCTION

Various schemes could be considered to reduce the tail effect. We will select the simplest method which is acceptable and has high quality. If we spread one frame to the two or three frames by considering the mirror image signals at the end or two end sides of the original signals, the volume of computation will increase. Referring to the previous figures, we see that the tail effect considerably occurs between the end of frame $(X(N))$, and the last new sample $(Y(M))$. In other words, the tail effect occurs in the interval (J) :

$$J=(u_M, t_N) \quad (88)$$

We thus decided to store the last sample in each frame, using this sample as a key data during the process of interpolation. In fact, after the conventional desirable interpolation is achieved in the interval $(I-J=[0,u_M])$, we constructed signal by suitable interpolation in the interval (J). Consider Polynomial Interpolation for the end of frame. The auxiliary points for interpolation are the first and last sample in the interval (J), and all the new reconstructed points out of the interval (J), that is produced by conventional desirable interpolation that have good fitness and accuracy.

We claim that the linear interpolation is the best and confident polynomial interpolation. Practically in each frame with any length, the duration of interval (J) is very small with respect to the total interval (I), so that any well-defined curve may be approximated by the line. On the other hand we will show by illustration and numerical values that increasing the order of polynomial generally could not decrease end error. Remember that as the order of polynomial increases, the behavior and shape of the variation becomes more complex. To illustrate the above statement we consider the (FIG.4) through (FIG.6) that was constructed by the periodic method as a desirable conventional interpolation. In figures 7, 8, and 9 we magnified the end of (FIG.4) through (FIG.6) respectively, and showed the end interpolation by three forms of polynomial: first order or Linear (Solid), second order (Dot), and third order(Dash dot), together with the original signal (Solid). The values of error in PRD are collected in the (TABLE.1).

Although in (FIG.8) we saw that the error due to the linear interpolation increased, but this increase is not so critical and important, and remember that the duration of interval (J) against the total interval is $(100/M=4.35\%)$. Thus in general the linear interpolation in addition to the simplicity is a suitable case for our work as it will be seen when we apply our method to an arbitrary total signal at the following sections.

VII. . PRD CURVE

In this section we investigated the general shape and behavior of the typical PRD curve. In section II we saw that the $PRD(X,Z_M)$ was a function of discrete variable (M) so that we denoted $PRD(X,Z_M)$ by the function $PRD(M)$. Note that for computing the $PRD(M)$, first we resampled the original vector (X) to produce vector (Y), and then interpolated the new vector (Y) to extract the constructed vector (Z_M), and in these procedures we applied the tail effect reduction, quantization, and clipping process, if necessary, so that due to the interpolation processes the time of computation was relatively high. Now, consider the (FIG.10) as a typical PRD curve. The results of experimental works showed that the PRD curve generally decreased, except at the beginning when the curve had an oscillatory variation. At the critical value of PRD existed a knee in the curve where we chose the optimum value $(3<PRDM<10 \Leftrightarrow 30.46>SNRM>20)$. In fact finding the optimum value for (M) or, in other words, computing the effective maximum frequency equivalent to solve the following nonlinear discrete type equation:

$$PRD(X,Z_M)=PRD(M)=PRDM \quad (89)$$

At the above equation we named the nearest (M) that produces $PRD(M)\le PRDM$ as a "ZERO" of the nonlinear discrete type equation.

VIII. . INCREASING THE SPEED OF CONVERGENCE

As known, there are some classical methods for zero finding of the continuous nonlinear equation such as: Bisection, False Position, Newton Raphson Slope Search, Secant, etc.[15]. The direct applying of this method to discrete function $PRD(M)$

produces the problem of divergence, oscillation, and instability. Therefore, we first modified the above methods by suitable programming, and then used nonlinear bisection which has extracted by experimental works and the information on the behavior of the PRD curve in the vicinity of the critical region at the knee part of the PRD curve.
Marginal points:

$$LEFT = \{L, PRD(L)\} \quad , RIGHT = \{R, PRD(R)\} \tag{90}$$

Nonlinear function for bisection:

$$PRD(M) - PRD(R) = A \times (M - R)^2 \tag{91}$$

Constant (A) computed by the left point.

A. FREQUENCY DOMAIN COMPUTATION

As for the time PRD, we defined frequency domain PRD ,say FPRD, and used the advantages of the FFT computation to achieve the FPRD curve in the frequency domain, to find near approximation to the effective maximum frequency or equivalently optimum (M).Since the PRD and FPRD curves are different we can not extract optimum value solely by the frequency domain, however the frequency domain is able to produce a good estimation of near marginal point, that could be handled by the time process to reach the optimum value.

B. COMBINATION OF TIME AND FREQUENCY COMPUTATIONS

As mentioned above, we combined the frequency and time domain computation to reach the optimum response as soon as possible. This procedure started with some initialization for time and frequency and then followed by the procedure such as the simplified version that is shown by the flow chart in (FIG.13) to reach the end and then the net time domain analysis continued.

IX. SIMPLE EXAMPLE

We are now in a position to apply our method on the total signal. We considered the (FIG.11) as a simple example, and defined the following parameters:
#Samples= Number of Samples
SPF= #Samples per each Frame
#Frames= Number of Frames
SPLF= # of Samples per Last Frame
CR=Compression Ratio=(#Samples after Removing Redundancy)/(#Samples)
PRDM=PRD-MAX (Let PRDM=7, SNRm=SNR-MIN=23.1)
T=Total Time of Computation in Second
The (FIG.12) show the reconstructed wave-form together with the absolute error signal for (FIG.11), and we see that the ability of method especially in keeping and preserving the edges of each frame and consequently the high performance and ability of applying the method on any arbitrary signal. Some of the important details are collected in the (TABLE.2).

X. OTHER RESULTS

In this section we introduced the results of applying our method to some known signals in the "Windows 3.1" such as Ding, Tada, Chord, and Chimes. The results are collected in the following table (TABLE.3).

XI. CONCLUSION

All of the results show that the method is powerful and works successfully. We guess as well as suggest that the suitable combination of this method by knowledge of the source information could produce new results. We also suggest the generalization of this procedure to the two dimensional spaces and specially for image processing. However all of the above suggestions need huge amounts of experimental work, which we hope will prove to be valuable.

REFERENCES

[1] : A.J. Jerri," The Shannon Sampling Theorems, - Its Various Extensions, and Applications: A Tutorial Review," Proc. IEEE, Vol.65, NO.11, Nov.1977, pp 1565-1596.
[2] : J.L. Yen," On Non-uniform Sampling of Bandwidth-Limited Signals," IRE Trans.on Circuit Theory, Vol.CT-3, Dec.1956, pp 251-257.
[3] : F.J. Beutler," Error Free Recovery of Signals from Irregularly Spaced Samples," SIAM Review, Vol.8, NO.3, July.1966, pp 328-335.
[4] : J.R. Higgins," A Sampling Theorem for Irregularly Spaced Sample Points," IEEE Trans. on Information Theory, Sept.1976, pp 621-622.
[5] : S.X. Pan, and Avinash E. Kak," A Computational Study of Reconstruction Algorithms for Diffraction Tomography: Interpolation Versus Filtered Back-Propagation," IEEE Trans., on ASSP, Vol. ASSP-31, NO.5, Oct.1983, pp 1262-1275.
[6] : J.J Clark, M.R. Palmer, and P.D. Lawrence," A Transformation Method for the Reconstruction of Function from Nou-uniform Spaced Samples," IEEE Trans. ASSP, Vol. ASSP-33, NO.4, Oct.1985, pp 1151-1165.
[7] : A.M. Davis," Almost Periodic Extension of Band-Limited Functions, and Its Application to Non-uniform Sampling," IEEE Trans. on Circuits, and Systems, Vol. CAS-33, NO.10,. Oct. 1980, pp 933-938.
[8] : Ken D. Sauer, and Jan P. Allebach," Iterative Reconstruction of Band-Limited Image from Non-uniformly Spaced Samples," IEEE Trans. on Circuits, and System, Vol. CAS-34, No.12,Dec.1987, pp 1497-1506.
[9] : Mehrdad Soumekh," Band-Limited Interpolation from Un-evenly Spaced Sampled Data," IEEE Trans. on ASSP, Vol.36, No.1, Jan.1988,pp 118-122.
[10] : Hans G. Feichtinger, and Karlheniz Grochening," Iterative Reconstruction of Multivariate Band-Limited Functions from Irregular Sampling Values," SIAM J. Math. Anal., Vol.23, NO.1, Jan. 1992, pp 244-261.
[11] : H.G. Feichtinger, and T. Strohmer," Fast Iterative Reconstruction of Band-Limited Image from Non-uniform Sampling Values," Computer Analysis of Image, and Pattern, 5th International Conference, CAIP'93 Proceedings, Sept.1993, pp 82-89.
[12] : A.B. Carlson, Communication Systems, 3rd Ed., McGraw-Hill, New York 1986.
[13] : D. A. Linden," A Discussion of Sampling Theorems," Proceeding of the IRE, Vol.47, 1959, pp 1219-1226.
[14] : Arthur Kohlenberg," Exact Interpolation of Band-Limited Functions," Journal of Applied Physics, Vol.24, NO.12, Dec.1953, pp 1432-1436.
[15] : Gene H. Hostetter, et al, Analytical, Numerical, and Computational Methods for Science, and Engineering, Prentice-Hall Englewood Cliffs, New Jersey, 1991.

(TABLE.1): The results and values of error in PRD for the figures 4 to 9

The Values of Error in PRD				Solid	Dot	Dash Dot
FIG	End of	(N,M)	Periodic	Linear	Order 2	Order 3
7	FIG.4	191,19	8.7075	3.4632	3.6832	5.3683
8	FIG.5	229,23	8.1310	8.8065	8.1294	8.1150
9	FIG.6	191,15	26.2650	9.3057	11.2997	14.2108

(TABLE.2): The numerical results for different sample values(M) and the error for each frame(PRD) for the Fig.11

PRDM=7, #Samples=1280, SPF=128, SPLF=0, T=12
CR=6.2, PRD=3.4, SNR=29.4										

Frame#	1	2	3	4	5	6	7	8	9	10
M	11	9	1	105	41	7	3	1	2	4
PRD	5.5	3.6	0.7	1.9	0.8	6.4	1.6	0.0	2.3	5.0

(TABLE.3): The numerical results obtained by applying our method to some known audio signals such as Ding, Tada, Cord and Chimes

RESULTS	FIG.11	Ding	Tada	Chord	Chimes
#Samples	1280	11554	27760	24938	15876
SPF	128	128	128	128	128
#Frames	10	91	217	195	125
SPLF	0	34	112	106	4
T	11	16	108	23	25
CR	6.2	14.4	5.3	17.2	12.7
PRD	3.4	3.97	6.1	3.3	29.5
SNR	29.4	28.02	24.34	29.5	26.4

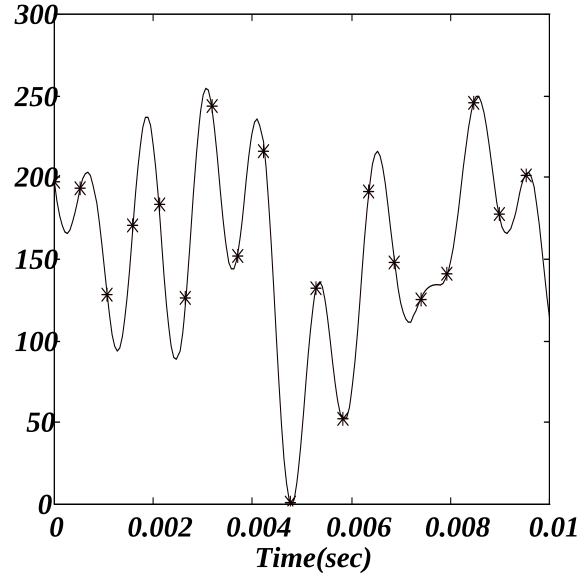

FIG.1 Periodic interpolation with N(Initial Samples)=191 , M(Test Samples)=19

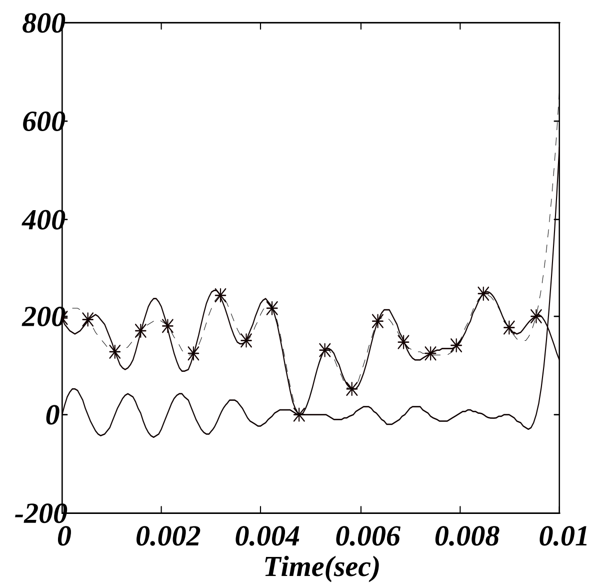

FIG.2 Reconstructed signal by interpolation with spline method(dotted curve) and error curve(solid)

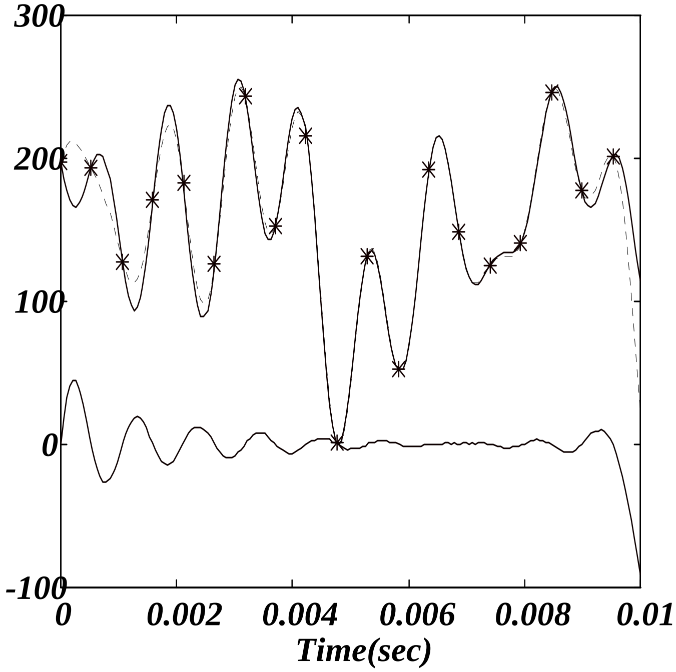

FIG.3 Reconstructed signal by shannon method(dotted curve PRD=9.1) and error curve(solid)

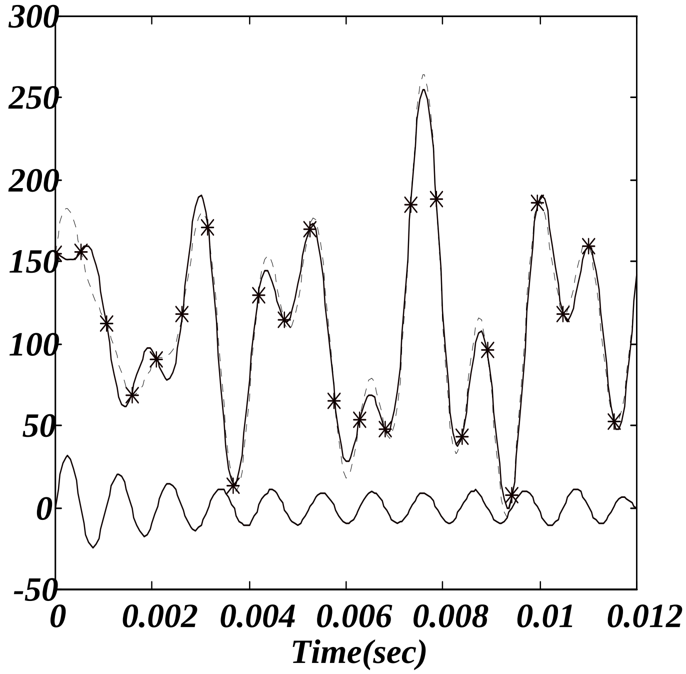

FIG.5 Periodic interpolation method with parameters: B(M,N)=(229,23), PRD=8.1 and the corresponding error curve(dotted)

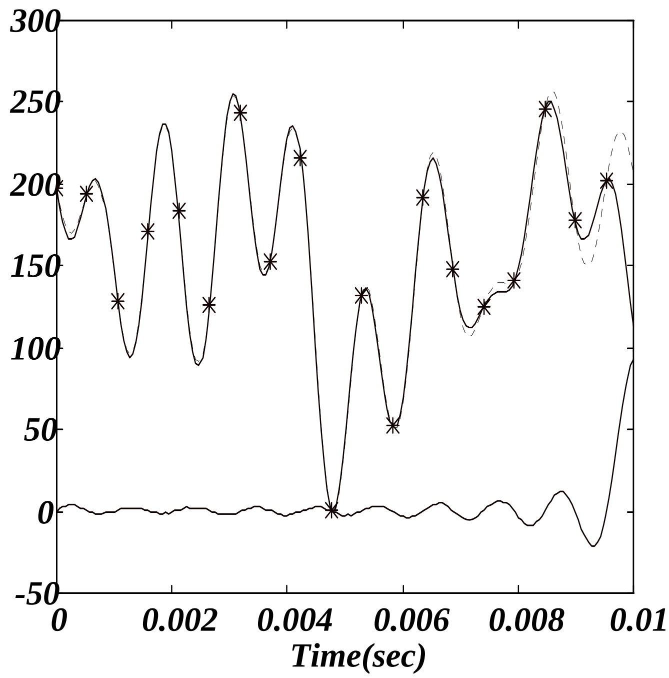

FIG.4 Reconstructed signal by periodic interpolation method(PRD=8.7 , dotted curve) and error curve(solid)

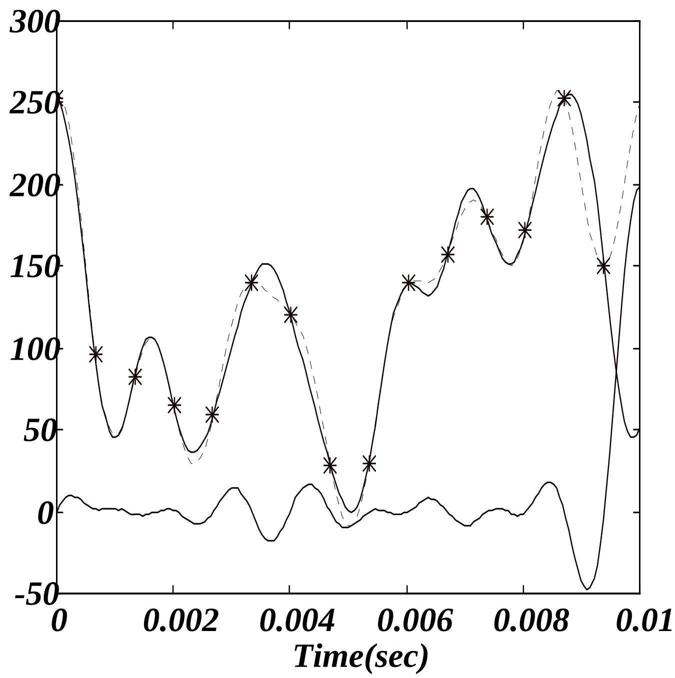

FIG.6 Periodic interpolation method with new Parameters: C(M,N)=(191,15), PRD=26.3 and the corresponding error curve

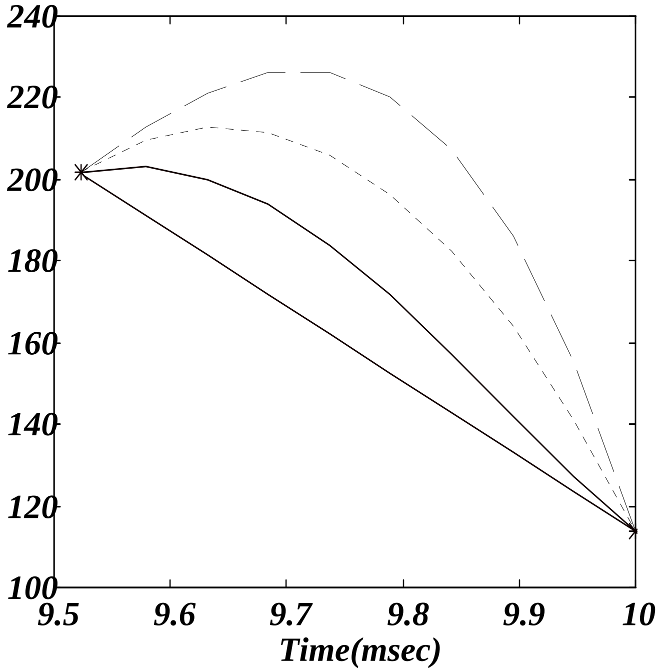

FIG.7 Corresponding to Fig.4: Comparison of different interpolation methods: linear(solid), 2nd order(dotted curve), 3rd order(dashed-dot curve), original (solid)

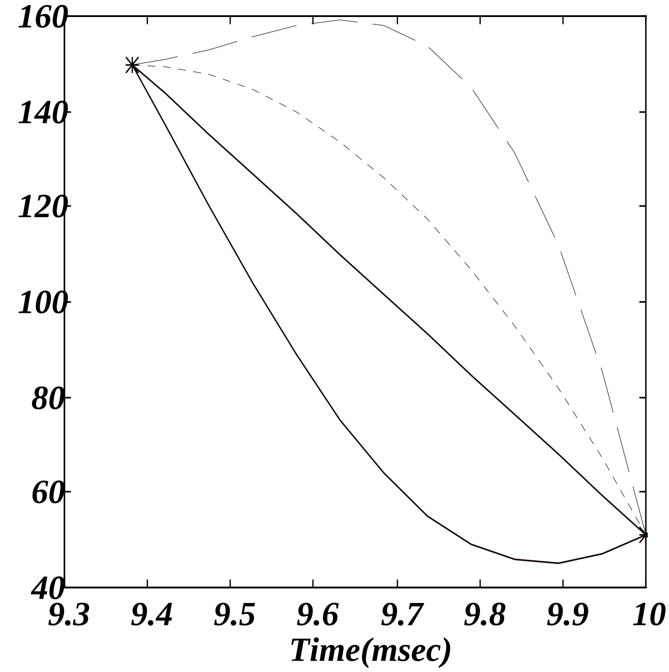

FIG.9 Corresponding to Fig.6: comparison of different interpolation methods in periodic interpolation linear(solid). 2nd order(dotted), 3rd order(dashed-dot), original (solid)

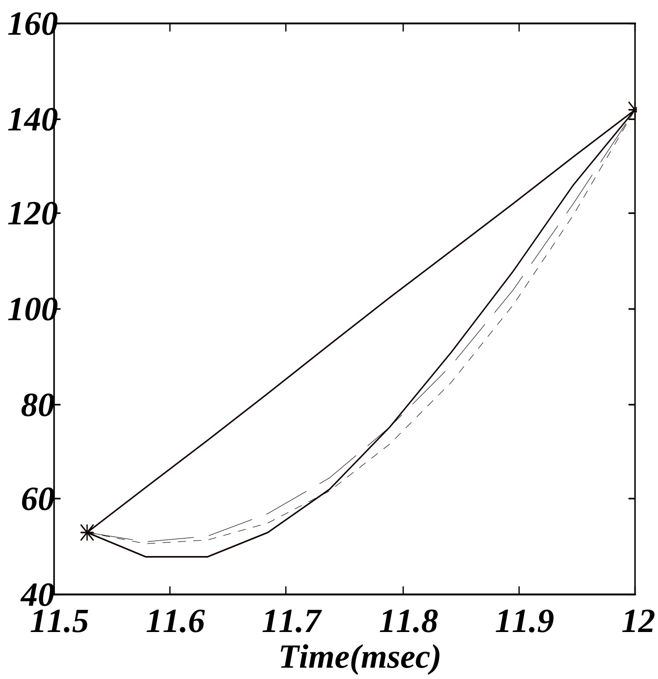

FIG.8 Corresponding to Fig.5: comparison of different interpolation methods in periodic interpolation linear(solid) 2nd order(dotted), 3rd order(dashed-dot), original(solid)

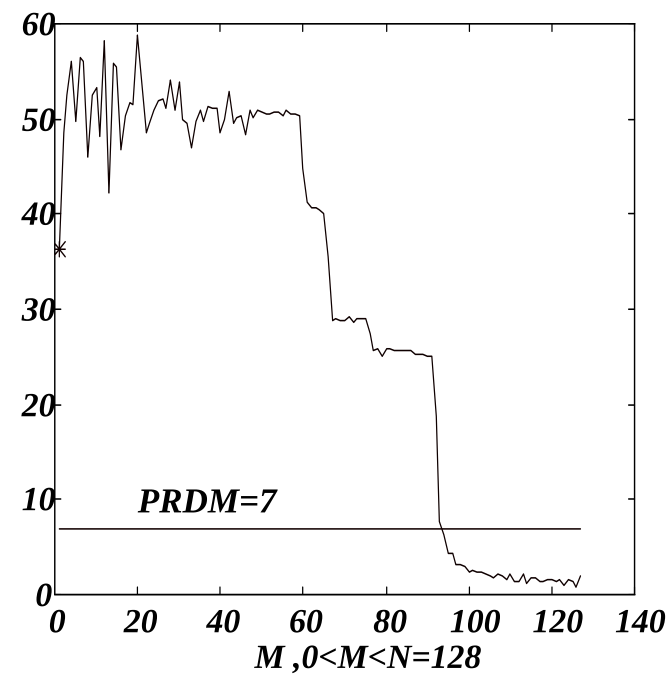

FIG.10 Root mean square difference(PRD)curve

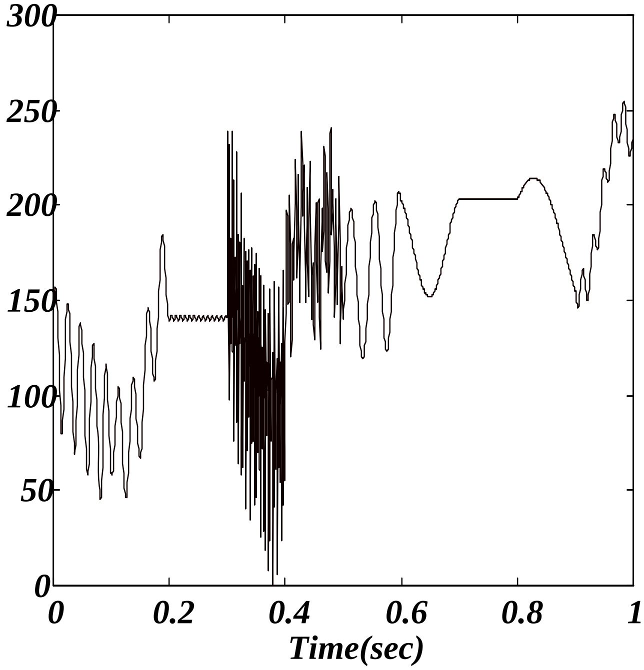

FIG.11 Original(test) audio signal

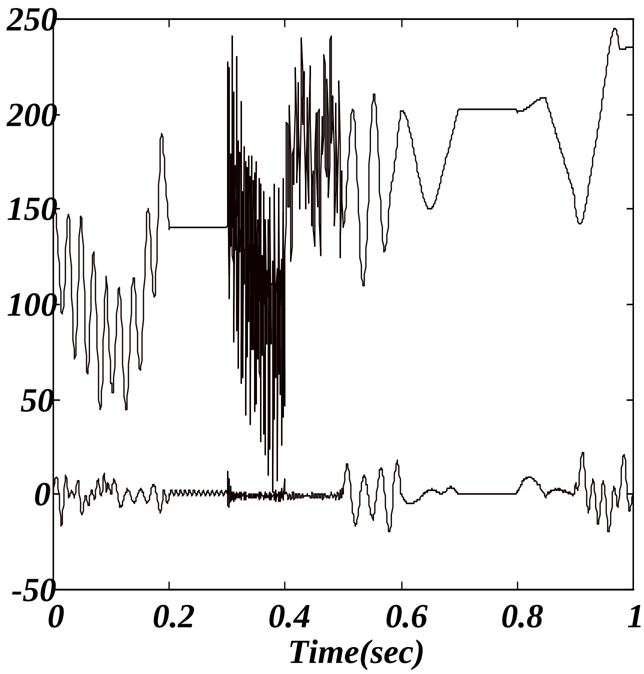

FIG.12 Reconstructed test signal and the corresponding error curve