Path-Bounded Finite Automata on Four-Dimensional Input Tapes

Yasuo Uchida, Takao Ito, Makoto Sakamoto, Ryoju Katamune, Kazuyuki Uchida, Hiroshi Furutani, Michio Kono, Satoshi Ikeda and Tsunehiro Yoshinaga

Abstract— M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities in 1967. Since then, many researchers in this field have been investigating many properties about automata on two- or three-dimensional tapes. By the way, the question of whether processing four-dimensional digital patterns is much difficult than two- or three-dimensional ones is of great interest from the theoretical and practical standpoints. Recently, due to the advances in many application areas such as computer animation, motion image processing, virtual reality systems, and so forth, it has become increasingly apparent that the study of four-dimensional pattern processing has been of crucial importance. Thus, the study of four-dimensional automata, i.e., four-dimensional automata with the time axis as a computational model of four-dimensional pattern processing has also been meaningful. On the other hand, the comparative study of the computational powers of deterministic and nondeterministic computations is one of the central tasks of complexity theory. This paper investigates the computational power of nondeterministic computing devices with restricted nondeterminism. There are only few results measuring the computational power of restricted nondeterminism. In general, there are three possibilities to measure the amount of nondeterminism in computation. In this paper, we consider the possibility to count the number of different nondeterministic computation paths on any input. In particular, we deal with seven-way four-dimensional finite automata with multiple input heads operating on four-dimensional input tapes.

Keywords— computational complexity, finite automaton, four-dimension, multihead, path-bounded

I. Introduction

COMPUTER science has two major components : first, the fundamental mathematics and theories underlying computing, and second, engineering techniques for the design of computer systems — hardware and software. Theoretical computer science falls under the first area of the two major components. It had its beginnings in various field : physics, mathematics, linguistics, electric and electronic engineering, physiology, and so on. Out of these studies came important ideas and models that are central to theoretical computer science. In theoretical computer science, the Turing machine has played a number of important roles in understanding and exploiting basic concepts and mechanisms in computing and information processing. It is a simple mathematical model of computers which was introduced by Alan M.Turing in 1936 to answer fundamental problems of computer science — 'What kind of logical work can we effectively perform — [41] '. If the restrictions in its structure and move are placed on the Turing machine, the restricted Turing machine is less powerful than the original one. However, it has become increasingly apparent that the characterization and classification of powers of the restricted Turing machines should be of great important. Such a study was active in 1950 's and 1960 's. After that, the growth of the processing of pictorial information by computer was rapid in those days. Therefor, the problem of computational complexity was also arisen in the twodimensional information processing. M.Blum and C.Hewitt first proposed two-dimensional automata — two-dimensional finite automata and marker automata, and investigated their pattern recognition abilities in 1967 [1]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two-dimensional tape [10]. Moreover, due to the advances in many application areas such as computer graphics, computer-aided design / manufacturing, computer vision, image processing, robotics, and so on, the study of three-dimensional pattern processing has been of crucial importance. Thus, the study of three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful [12,15-26,38]. By the way question of whether processing four-dimensional digital patterns is much difficult than three-dimensional ones is of great interest from the theoretical and practical standpoints. In recent years, due to the advances in many application areas such as moving image processing, computer animation, and so on, the study of four-dimensional pattern processing has been of crucial importance. Thus, the study of four-dimensional automata as the computational model of four-dimensional pattern processing has been meaningful. For example, in [13,16], a four-dimensional finite automaton was proposed as a natural extension of the three-dimensional finite automaton to four dimensions[8,9,18,20,24-28,34-37,40,42-44]. On the other hand, the comparative study of the computational powers of deterministic computations is one of the central tasks of complexity theory. In this paper, we investigate the computational power of nondeterministic computing devices with restricted nondeterminism. However, there are only few results [3-6] measuring the computational power of restricted nondeterminism. In general, there are three possibilities to measure the amount of nondeterminism in computation. One possibility is to count the number of advice bits (nondeterministic guesses) in particular nondeterministic computations, and the second possibility is to count the number of accepting computation paths. The third possibility is to count the number of different nondeterministic computation paths on any input. This paper considers the third one. In particular, the paper investigates a hierarchy on the degree of nondeterminism of seven-way four-dimensional (simple) multi-head finite automata as a natural extension of the five-way three-dimensional (simple) multi-head finite automata [7,15]. Furthermore, we investigate a relationship between the accepting powers of nondeterminism and self-verifying nondeterminism for seven-way four-dimensional (simple) multihead finite automata with the number of computation paths restricted.

II. Preliminaries

This section summarizes the formal definitions and notations necessary for this thesis.

Definition 2.1. Let Σ be a finite set of symbols. A *four-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all four-dimensional tapes over Σ is denoted by $\Sigma^{(4)}$.

Given a tape $x \in \Sigma^{(4)}$, for each integer j ($1 \leq j \leq 4$), we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\Sigma(n_1, n_2, n_3, n_4)$. When $1 \leq i_j \leq l_j(x)$ for each j ($1 \leq j \leq 4$), let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4), as shown in Fig.1. Furthermore, we define $x[(i_1, i_2, i_3, i_4),(i'_1, i'_2, i'_3, i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for integer j ($1 \leq j \leq 4$), as the four-dimensional tape y satisfying the following conditions:

(i) for each j ($1 \leq j \leq 4$), $l_j(y) = i'_j - i_j + 1$;

(ii) for each r_1, r_2, r_3, r_4($1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$, $1 \leq r_4 \leq l_4(y)$), $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1, r_4 + i_4 - 1)$. (We call $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ the $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$-*segment* of x.)

For each $x \in \sum^{(n_1, n_2, n_3, n_4)}$ and for each $1 \leq i_1 \leq n_1$, $1 \leq i_2 \leq n_2$, $1 \leq i_3 \leq n_3$, $1 \leq i_4 \leq n_4$, $x[(i_1, 1, 1, i_4), (i_1, n_2, n_3, i_4)]$, $x[(1, i_2, 1, i_4), (n_1, i_2, n_3, i_4)]$, $x[(1, 1, i_3, i_4), (n_1, n_2, i_3, i_4)]$, $x[(i_1, 1, i_3, i_4), (i_1, n_2, i_3, i_4)]$, and $x[(1, i_2, i_3, i_4), (n_1, i_2, i_3, i_4)]$ are called the i_1*th* (2-3) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_2*th* (1-3) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_1*th row on the* i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, and the i_2*th column on* the i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, and are denoted by $x(2\text{-}3)_{i_1 \cdot i_4}$, $x(1\text{-}3)_{i_2 \cdot i_4}$, $x(1\text{-}2)_{i_3 \cdot i_4}$, $x[i_1, *, i_3, i_4]$, and $x[*, i_2, i_3, i_4]$, respectively (see Fig.1.).

Definition 2.2. A *seven-way four-dimensional multihead finite automaton* (*SV4-MHFA*) [7,46] is a finite automaton with multiple input heads operating on four-dimensional input tapes surrounded by boundary symbols #'s. These heads can move east, west, south, north,up, down, in the future, but not in the past. A *seven-way four-dimensional simple multihead finite automaton* (*SV4-SMHFA*) is an *SV4-MHFA* which has only one reading head and other counting heads which can only detect whether they are on the boundary symbols or a symbol in the input alphabet.

When a four-dimensional input tape x is presented to a four-dimensional device M, M starts in its initial state with all its heads on $x(1, 1, 1, 1)$. M *accepts* the input tape x if and only

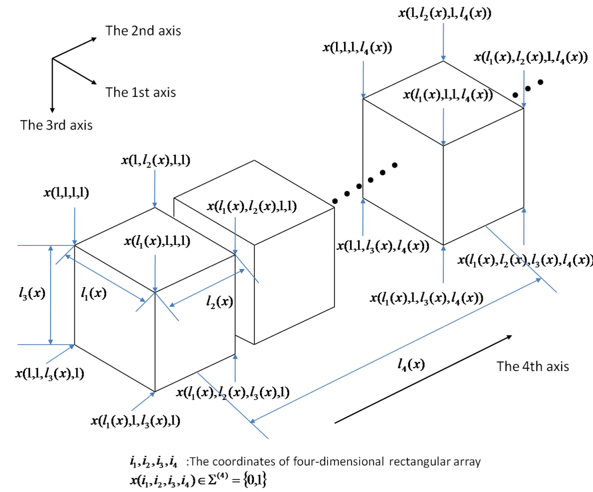

Fig. 1. Four-dimensional input tape.

if it eventually halts in an accepting state with all its heads on the bottom boundary symbols #'s(see Fig.2).

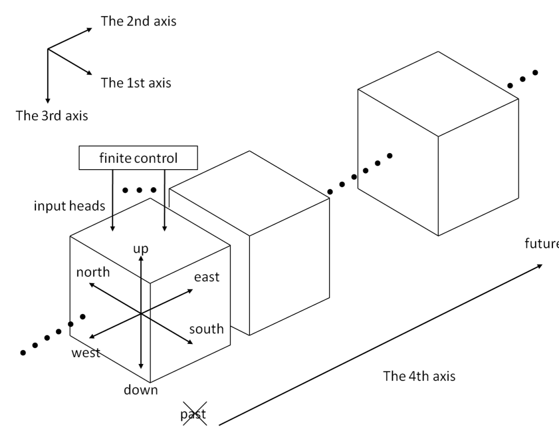

Fig. 2. Seven-way four-dimensional multihead finite automaton.

For a device M, we denote by $T(M)$ the set of all inputs accepted by M. The states of this device are considered to be divided into three disjoint sets of working, accepting, and rejecting states.

Definition 2.3. A *self-verifying nondeterministic* device is a device with four types of states : working, accepting, rejecting, and neutral ones. The self-verifying nondeterministic device M is not allowed to make mistakes. If there is a computation of M on an input x finishing in an accepting (resp., rejecting) state, then x must be in $T(M)$ (resp., x must not be in $T(M)$). For every input y, there is at least one computation of M that finishes either in an accepting state (if $y \in T(M)$) or in a rejecting state (if $y \notin T(M)$).

For each $k \geq 1$, let *SV4-kHFA* denote a *seven-way four-dimensional k-head finite automaton*. In order to represent different kinds of *SV4-kHFA*'s, we use the notation *SV4-XYkHFA*, where

$$(1)\left\{\begin{array}{l} X = N : nondeterministic, \\ X = SVN : self{-}verifying\ nondeterministic\ ; \end{array}\right.$$

$$(2)\left\{\begin{array}{l} Y = SP : simple, \\ there\ is\ no\ Y\ :\ non{-}simple. \end{array}\right.$$

We denote by $\mathcal{L}$[*SV4-XYkHFA*] the class of sets of input tapes accepted by *SV4-XYkHFA*'s.

Let r be a positive integer. A device M described above is *r path-bounded* if for any input x, there are at most r computation paths of M on x[14,33]. We denote an r path-bounded *SV4-XYkHFA* by *SV4-XYkHFA*(r), and denote the class of sets of input tapes accepted by *SV4-XYkHFA*(r)'s by $\mathcal{L}$[*SV4-XYkHFA*(r)].

III. A Review of Three-Dimensional Automata

The question of whether processing three-dimensional digital patterns is much more difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Recently, due to the advances in many application areas such as computer vision, robotics, and so forth, it has become increasingly apparent that the study of three-dimensional pattern processing has been of crucial importance. Thus, the research of three-dimensional automata as a computational model of three-dimensional pattern processing has also been meaningful. This chapter gives the historical review of various properties of three-dimensional automata before beginning the main subject of four-dimensional automata [30-32].

A. Three-Dimensional Turing Machines

This subsection observes the properties of three-dimensional Turing machines [30-32].

Theorem 3.1. *If* $L(m) = o(logm)$, *then*

$$\mathcal{L}[3\text{-}DTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}UTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}ATM^c(L(m))].$$

Corollary 3.1. $\mathcal{L}[3\text{-}DFA^c] \subsetneq \mathcal{L}[3\text{-}NFA^c] \subsetneq \mathcal{L}[3\text{-}AFA^c]$.

Theorem 3.2. *If* (i) $L(m) = o(m^2)$, or (ii) $L(m) \geq logm$ *and* $L(m) = o(m^3)$, then
$\mathcal{L}[FV3\text{-}DTM^c(L(m))] \subsetneq \mathcal{L}[FV3\text{-}UTM^c(L(m))] \subsetneq \mathcal{L}[FV3\text{-}ATM^c(L(m))]$, *and*
$\mathcal{L}[FV3\text{-}UTM^c(L(m))]$ *and* $\mathcal{L}[FV3\text{-}NTM^c(L(m))]$ *are incomparable*.

Corollary 3.2.
(i) $\mathcal{L}[FV3\text{-}UFA^c] \subsetneq \mathcal{L}[FV3\text{-}AFA^c]$.
(ii) $\mathcal{L}[FV3\text{-}UFA^c]$ *is incomparable with* $\mathcal{L}[FV3\text{-}NFA^c]$.
(iii) $\mathcal{L}[FV3\text{-}DFA^c] \subsetneq \mathcal{L}[FV3\text{-}UFA^c]$.

Theorem 3.3. *If* (i) $L(m) = o(m^2)$, *or*
(ii) $L(m) \geq logm$ *and* $L(m) = o(m^3)$, *then*

$$\mathcal{L}[FV3\text{-}UTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}UTM^c(L(m))].$$

Corollary 3.3. $\mathcal{L}[FV3\text{-}UFA^c] \subsetneq \mathcal{L}[3\text{-}UFA^c]$.

Theorem 3.4. $\mathcal{L}[FV3\text{-}UFA^c] \subsetneq \mathcal{L}[FV3\text{-}DTM^c(m^2)]$, *and space* m^2 *is necessary and sufficient for* $FV3\text{-}DTM^c$'s *and* $FV3\text{-}NTM^c$*'s to simulate* $FV3\text{-}UFA^c$*'s.*

Theorem 3.5. $\mathcal{L}[3\text{-}UFA^c] \subsetneq \mathcal{L}[FV3\text{-}DTM^c(m^3)]$, *and space* m^3 *is necessary and sufficient for* $FV3\text{-}DTM^c$*'s to simulate* $3\text{-}UFA^c$*'s.*

Remark 3.1. We conjecture that $\mathcal{L}[3\text{-}UFA^c] \subsetneq \mathcal{L}[FV3\text{-}NTM^c(m^2)]$, but we have not completed the proof of this conjecture yet.

Theorem 3.6. *Space* m^3 *is necessary and sufficient for* $FV3\text{-}DTM^c$*'s to simulate* $FV3\text{-}AFA^c$*'s and* $3\text{-}AFA^c$*'s.*

Open Problems 3.1. (i) Is $\mathcal{L}[3\text{-}NTM^c(L(m))]$ incomparable with $\mathcal{L}[3\text{-}UTM^c(L(m))]$ for any L such that $L(m) = o(logm)$?
(ii) $\mathcal{L}[3\text{-}DTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}NTM^c(L(m))] \subsetneq \mathcal{L}[3\text{-}ATM^c(L(m))]$ for any $L(m) \geq logm$?

B. Three-Dimensionally Space Constructibility and Space Hierarchy

This subsection concerns three-dimensionally space constructible functions and space complexity hierarchy of three-dimensional Turing machines whose input tapes are restricted to cubic ones. First, we summarize two definitions necessary for the following results.

Definition 3.1. A function $L(m)$: **N** $\rightarrow$ **R** is called *three-dimensionally space constructible* if there is a $3\text{-}DTM(L(m))^c$ M such that for each $m \geq 1$, there exists some input tape x with $l_1(x) = l_2(x) = l_3(x) = m$ on which M halts after its storage head has marked off exactly the greatest integer cells which is smaller than or equal to $L(m)$. (In this case, we say that M constructs the function L in the storage tape.)

Definition 3.2. A function $L(m)$: **N** $\rightarrow$ **R** is called *three-dimensionally fully space constructible* if there is a $3\text{-}DTM(L(m))^c$ M which, for each $m \geq 1$ and for each input tape x with $l_1(x) = l_2(x) = l_3(x) = m$, makes use of exactly the greatest integer cells which is smaller than or equal to $L(m)$ and halts.

Next, we show three-dimensionally fully space constructibility and space complexity hierarchies of three-dimensional Turing machines whose input tapes are restricted to cubic ones [30].

Theorem 3.7. *We consider the following three functions* :
$log^{(0)}m = m$,
$log^{(k)}m = log(log^{(k-1)}m)$, *for* $k \geq 1$, *and*
$log^{*}m = min\{x | log^{(k)}m \leq 1\}$.

Then, the functions $log^{(k)}m$ (k: *any natural number*) *and* $log^{*}m$ *are three-dimensionally fully space constructible.*

Theorem 3.8. *For any* $X \in \{D, N, U, A\}$, *for any function* $L(m)$: $\mathbf{N} \rightarrow \mathbf{R}$, *and for any constant* $d > 0$,

$$\mathcal{L}[3\text{-}XTM^c(L(m))] = \mathcal{L}[3\text{-}XTM^c(L(m) + d)].$$

Theorem 3.9. *For any* $X \in \{D, N, U, A\}$, *for any function* $L(m)$: $\mathbf{N} \rightarrow \mathbf{R}$, *and for any constant* $d > 0$,

$$\mathcal{L}[3\text{-}XTM^c(L(m))] = \mathcal{L}[3\text{-}XTM^c(dL(m))].$$

Theorem 3.10. *Let* $L_1(m)$ *and* $L_2(m)$ *be any functions such that*
(i) $L_2(m)$ *is three-dimensionally space constructible,*
(ii) $lim_{i\rightarrow\infty}L_1(m_i)/L_2(m_i) = 0$, *and*
(iii) $L_2(m_i)/logm_i > k$ (i=1, 2, $\cdots$) *for some increasing sequence of natural numbers* m_i *and for some constant* $k > 0$.
Then there exists a set T *in* $\mathcal{L}[3\text{-}XTM^c(L_2(m))]$, *but not in* $\mathcal{L}[3\text{-}XTM^c(L_1(m))]$ *for any* $X \in \{D, N\}$.

Theorem 3.11. *For any functions* $L_1(m)$ *and* $L_2(m)$ *such that* (i) $L_2(m)$ *is three-dimensionally space constructible,* (ii) $L_1(m) = o(L_2(m))$, *there exists a set in* $\mathcal{L}[3\text{-}DTM^c(L_2(m))]$, *but not in* $\mathcal{L}[3\text{-}NTM^c(L_1(m))]$.

Open Problems 3.2. (i) Are the functions $log^{(k)}m$ $(k \geq 3)$ and $log^{*}m$ fully space constructible by one-dimensional deterministic two-head Turing machines or by two-dimensional deterministic Turing machines with square inputs?
(ii) Is there any other unbounded function below $logm$ which is three-dimensionally fully space constructible?
(iii) Is there an infinite tight hierarchy for 3-$ATM^c(L(m))$'s with $L(m) \geq logm$?
(iv) Is there an infinite space hierarchy for 3-$ATM^c(L(m))$'s with $L(m) \leq loglogm$?

C. Recognizability of Connected Tapes

The recognition of the connectedness of digital pictures is one of the most fundamental problems in picture processing. There have been various results related to this problem. Especially, to recognize three-dimensional connectedness seems to be much more difficult than the two-dimensional case, because of intrinsic characteristics of three-dimensional pictures. This subsection mainly shows the recognizability of three-dimensional connected tapes by three-dimensional automata. We use 3-DM_k (3-NM_k, 3-NI_k, 3-UI_k, 3-AI_k) to denote a three-dimensional deterministic k-marker automaton (three-dimensional nondeterministic k-marker automaton, three-dimensional nondeterministic k-inkdot automaton, three-dimensional alternating k-inkdot automaton with only universal states, three-dimensional alternating k-inkdot automaton), where $k \geq 1$ (see [30]). Let T_c be the set of all three-dimensional connected tapes. It is interesting to investigate how much space is required for three-dimensional Turing machines to accept T_c. For this problem, we have

Theorem 3.12. (i) $T_c \in \mathcal{L}[3\text{-}AFA^c]$.
(ii) $logm$ *space is necessary and sufficient for* $FV3\text{-}ATM$*'s to recognize* T_c.

Theorem 3.13. $T_c \in \mathcal{L}[3\text{-}NMA_1]$ [45].

Theorem 3.14. (i) *the necessary and sufficient space for* $FV3$-DTM*'s to simulate* 3-DM_1's (3-NM_1's) *is* $2^{lmloglm}$ (2^k, *where* $k = l^2m^2$). (ii) *the necessary and sufficient space for* $FV3$-NTM*'s to simulate* 3-DM_1*'s* (3-NM_1*'s*) *is* $lmloglm$ (l^2m^2), *where* $l(m)$ *is the number of rows* (*columns*) *on each plane of three-dimensional rectangular input tapes.*

Theorem 3.15. $T_c \notin \mathcal{L}[3\text{-}NI_k]$ [19].

Remark 3.2. $[3\text{-}NI_k] \subsetneq \mathcal{L}[3\text{-}AI_k]$ for any integer k.

Open Problems 3.3. (i) $T_c \notin \mathcal{L}[3\text{-}DTM(L(m))]$ or $T_c \notin \mathcal{L}[3\text{-}NTM(L(m))]$ for $L(m) = o(logm)$?
(ii) $T_c \in \mathcal{L}[3\text{-}UI_1]$?
(iii) Is T_c accepted by a 3-DM_1?

D. Other Topics

In this subsection, we list up other topics and related references about three-dimensional automata.
(i) Properties of special types of three-dimensional Turing machines (leaf-size bounded automata, parallel automata, multicounter automata, etc. on three-dimensional tapes) [17, 29, 30].
(ii) Cellular types of three-dimensional automata [10, 11, 39].
(iii) Closure properties [5, 13, 30].
(iv) Recognizability of topological properties [21-23].
(v) NP-complete problems [5, 16, 30].

E. Concluding Remarks

In this section, we reviewed historical properties of three-dimensional automata. Especially, we dealt with three-dimensional Turing machines, including finite automata, three-dimensionally space constructibility, recognizability of three-dimensional connected pictures, and so on. We believe that there are many problems about three-dimensional automata to solve in the future. We hope that this survey will activate the investigation of three- or four-dimensional automata theory.

IV. Non-Simple Case

We first prove a strong separation between r path-bounded and (r+1) path-bounded for seven-way four-dimensional multihead finite automata.

Theorem 4.1. *For each positive integers* $k{\geq}2$ *and* $r{\geq}1$,

$$\mathcal{L}[SV4\text{-}SVNkHFA(r{+}1)]{-}\mathcal{L}[SV4\text{-}NkHFA(r)]{\neq}\phi.$$

Proof : For each positive integers $k{\geq}2$ and $r{\geq}1$, let $T_1(k,\ r)$=$\{x{\in}\{0,1\}^{(4)} \mid {}^{\exists}n{\geq}2rb(k){+}1$ $[l_1(x){=}l_2(x){=}l_3(x){=}l_4(x){=}n] \wedge {}^{\exists}\ i(0{\leq}i{\leq}r{-}1)$ $[{}^{\forall}j(ib(k){+}1{\leq}\ j\ {\leq}\ (i{+}1)b(k))$ $[x[*,\ *,\ *,\ j]{=}x\ [*,\ *,\ *,\ 2rb(k){-}j{+}1]\ \wedge\ {}^{\exists}z{\in}\{0,1\}^{*}$ $[x[*,\ *,\ 2rb(k){+}1]{=}0^{i}1z$ (the string of the symbols forms a line from the first column to the last column in the ($2rb(k)$+1)th three-dimensional rectangular array of x and from the first row to the last row in a column one after another)]]}, where $b(k)={}_kC_2$ (see Fig.3.). To prove the theorem, it suffices to show that for each $k\geq 2$ and $r\geq 1$, (1) $T_1(k,\ r+1)\in\mathcal{L}$[*SV4-SVNkHFA*($r$+1)], and (2) $T_1(k,\ r+1)\notin\mathcal{L}$[*SV4-NkHFA*($r$)]. First of all we prove Past (1) of the theorem. $T_1(k,\ r+1)$ is accepted by an *SV4-SVNkHFA*(r+1) M which acts as follows. Suppose that an input tape x with $l_1(x)=l_2(x)=l_3(x)=l_4(x)=n$ ($n\geq 2(r+1)b(k)+1$) is presented to M. First, M nondeterministically guesses some i ($0\leq i\leq r$) and checks whether $x[*,\ *,\ *,\ j]$ and $x[*,\ *,\ *,\ 2(r+1)b(k)\text{-}j+1]$ are identical for each j ($ib(k)+1\leq j\leq(i+1)b(k)$). This check can easily be done by using a well-known technique in [12]. If $x[*,\ *,\ *,\ j]\neq x[*,\ *,\ *,\ 2(r+1)b(k)\text{-}j+1]$ for some j ($ib(k)+1\leq j\leq(i+1)b(k)$) and $x[*,\ *,\ *,\ 2(r+1)b(k)+1]=0^i1z$ (the string of the symbols forms a line from the first column to the last column in the $(2(r+1)b(k)+1)$th three-dimensional rectangular array of x and from the first row to the last row in a column one after another) for some $z\in\{0,\ 1\}^*$, then M enters a rejecting state. If $x[*,\ *,\ *,\ 2(r+1)b(k)+1]\neq 0^i1z$ (the string of the symbols forms a line from the first column to the last column in the $(2(r+1)b(k)+1)$th three-dimensional rectangular array of x and from the first row to the last row in a column one after another) for some $z\in\{0,\ 1\}^*$, M enters a neutral state, whether or not $x[*,\ *,\ *,\ j]=x[*,\ *,\ *,\ 2(r+1)b(k)\text{-}j+1]$ for each j ($ib(k)+1\leq j\leq(i+1)b(k)$). It is obvious that M accepts $T_1(k,\ r+1)$. On the other hand, by using a standard technique in [9, 10], we can get Part (2) of the theorem.

□

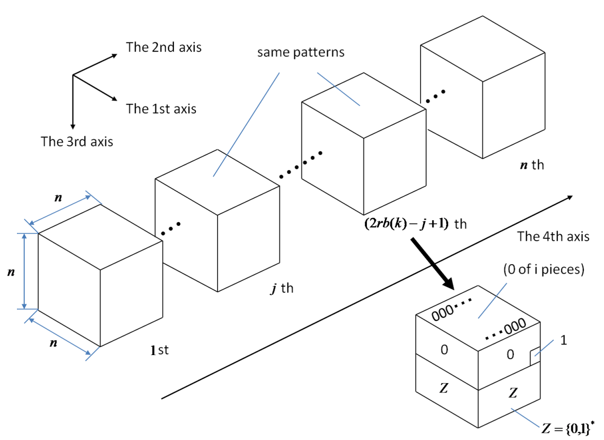

Fig. 3. A tape in $T_1(k,r)$.

From Theorem 4.1, we have the following corollary :

Corollary 4.1. *For each* $X\in\{N,\ SVN\}$, *and for each positive integers* $k\geq 2$ *and* $r\geq 1$,

$$\mathcal{L}[\textit{SV4-XkHFA}(r)]\subsetneq\mathcal{L}[\textit{SV4-XkHFA}(r+1)].$$

We next show a strong separation between self-verifying nondeterminism and nondeterminism.

Theorem 4.2. *For each positive integer* $k\geq 2$.

$$\mathcal{L}[\textit{SV4-NkHFA}(2)]-\mathcal{L}[\textit{SV4-SVNkHFA}]\neq\phi.$$

Proof : For each positive integer $k\geq 2$, let $T_2(k)=\{x\in\{0,1\}^{(4)}\,|\,^\exists n\geq 4b(k)\ [l_1(x)=l_2(x)=l_3(x)=l_4(x)=n]\ \wedge\ ^\exists i\ (0\leq i\leq 1)\ \ ^\exists j\ (ib(k)+1\leq j\leq(i+1)b(k))\ \ [x[*,*,*,j]\neq x[*,*,*,4b(k)-j+1]\}$, where $b(k)={}_kC_2$ (see Fig.4.). Then, we have $T_2(k)\in\mathcal{L}$[*SV4-NkHFA*(2)]–$\mathcal{L}$[*SV4-SVNkHFA*]. Then, by using the same idea as in [11,12], we can get the desired result. □

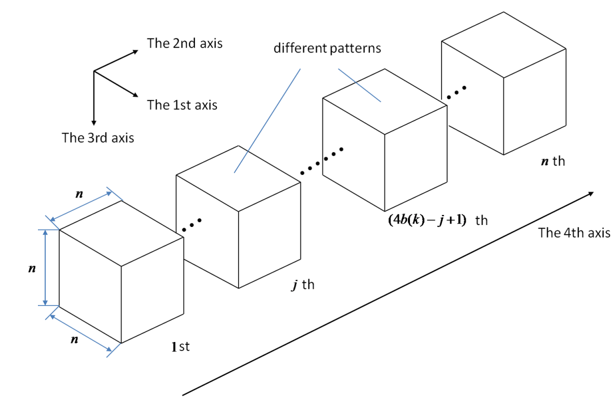

Fig. 4. A tape in $T_2(k)$.

From Theorems 4.1 and 4.2, we have the following corollary:

Corollary 4.2. *For each positive integers* $k\geq 2$ *and* $r\geq 2$,

(1) $\mathcal{L}$[*SV4-SVNkHFA*]$\subsetneq\mathcal{L}$[*SV4-NkHFA*],
(2) $\mathcal{L}$[*SV4-SVNkHFA*(r)]$\subsetneq\mathcal{L}$[*SV4-NkHFA*(r)], *and*
(3) $\mathcal{L}$[*SV4-SVNkHFA*(r+1)] *and* $\mathcal{L}$[*SV4-NkHFA*(r)] *are incomparable.*

V. Simple Case

This section first prove a strong separation between r path-bounded and (r+1) path-bounded machines for the seven-way simple case.

Theorem 5.1. *For each positive integers* $k\geq 2$ *and* $r\geq 1$,

$$\mathcal{L}[\textit{SV4-SVNSPkHFA}(r+1)]-\mathcal{L}[\textit{SV4-NSPkHFA}(r)]=\phi.$$

proof : For each positive integers $k\geq 2$ and $r\geq 1$, let $T_3(k,\ r)=\{x\in\{0,\ 1\}^{(4)}\,|\,^\exists n\geq\max\{2r+1,\ k\}\ [l_1(x)=l_2(x)=l_3(x)=l_4(x)=n]\ \wedge$ [[(the $1st$ three-dimensional rectangular array of x has exactly k 1's) $\wedge\ x[*,\ *,\ *,\ 1]=x[*,\ *,\ *,\ 1+r]\ \wedge\ ^\exists z\in\{0,\ 1\}^*\ [x[*,\ *,\ *,\ 2r+1]=01z$ (the string of the symbols forms a line from the first column to the last column in the $(2r+1)$th three-dimensional rectangular array and from the first row to the last row in a column one after another)]] $\vee$ [(the $2nd$ three-dimensional rectangular array of x has exactly k 1's) $\wedge\ x[*,*,*,2]=x[*,*,*,2+r]\ \wedge\ ^\exists z\in\{0,1\}^*$ $[x[*,\ *,\ *,\ 2r+1]=0^21z$ (the string of the symbols forms a line from the first column to the last column in the $(2r+1)$th three-dimensional rectangular array and from the first row to the last row in a column one after another)]] $\vee\cdots\vee$ [(the rth three-dimensional rectangular array of x has exactly k 1's) $\wedge\ x[*,\ *,\ *,\ r]=x[*,\ *,\ *,\ 2r]\ \wedge\ ^\exists z\in\{0,\ 1\}^*\ [x[*,\ *,\ *,\ 2r+1]=0^r1z$ (the string of the symbols forms a line from the first column to the last column in the ($2r$+1)th three-dimensional rectangular array and from the first row to the last row in a column one after another)]]]}. By using the same technique as in the proof of Theorem 5.1 in [12], we can get the desired result. □

From Theorem 5.1, we have the following corollary :

Corollary 5.1. *For each* $X \in \{N, SVN\}$, *and for each positive integers* $k \geq 2$ and $r \geq 1$,

$$\mathcal{L}[\mathit{SV4\text{-}XSPkHFA}(r)] \subsetneq \mathcal{L}[\mathit{SV4\text{-}XSPkHFA}(r+1)].$$

We next show a strong separation between self-verifying nondeterminism and nondeterminism.

Theorem 5.2. *For each positive integer* $k \geq 2$,

$$\mathcal{L}[\mathit{SV4\text{-}NSPkHFA}(2)] - \mathcal{L}[\mathit{SV4\text{-}SVNSPkHFA}] \neq \phi.$$

Proof : For each positive integer $k \geq 2$, let $T_4(k)=\{x \in \{0, 1\}^{(4)} \mid {}^{\exists} n \geq \max\{4, k\}$ $[l_1(x)= l_2(x)=l_3(x)=l_4(x)= n] \wedge {}^{\exists} i(1 \leq i \leq 2)$ [(the ith three-dimensional rectangular array of x has exactly k '1's) $\wedge$ $x[*, *, *, i] \neq x[*, *, *, i+2]]\}$. Then, by using the standard technique in [11], we can show that

$$T_4(2k\text{-}1) \in \mathcal{L}[\mathit{SV4\text{-}NSPkHFA}(2)] - \mathcal{L}[\mathit{SV4\text{-}SVNSPkHFA}].$$ □

From Theorems 5.1 and 5.2, we have the following corollary.

Corollary 5.2. *For each positive integers* $k \geq 2$ *and* $r \geq 2$,

(1) $\mathcal{L}[\mathit{SV4\text{-}SVNSPkHFA}] \subsetneq \mathcal{L}[\mathit{SV4\text{-}NSPkHFA}]$,
(2) $\mathcal{L}[\mathit{SV4\text{-}SVNSPkHFA}(r)] \subsetneq \mathcal{L}[\mathit{SV4\text{-}NSPkHFA}(r)]$,

and

(3) $\mathcal{L}[\mathit{SV4\text{-}SVNSPkHFA}(r+1)]$ *and* $\mathcal{L}[\mathit{SV4\text{-}NSPkHFA}(r)]$ *are incomparable.*

VI. Conclusion

Recentry, due to the advances in computer animation, motion image processing, robotics, and so on, the study of four-dimensional information processing has been of great importance. For instance, four-dimensional image is now needed in visual communication, such as virtual reality systems. Even in the Internet environment, new protocols have been proposed for virtual reality communication on the *WWW*. In the medical field, we can easily get the precise four-dimensional volumetric image of a human body by excellent equipments such as *X*-ray *CT* scanner and *MRI* scanner. Thus, the study of four-dimensional automata has been meaningful as the computational model of four-dimensional information processing.

In this paper, we dealt with path-bounded seven-way four-dimensional finite automata, and showed some properties about them. We first investigated a hierarchy on the degree of nondeterminism of seven-way four-dimensional (simple) multihead finite automata as a natural extension of the five-way three-dimensional (simple) multihead finite automata. Next, we showed a relationship between the accepting powers of nondeterminism and self-verifying nondeterminism for seven-way four-dimensional (simple) multihead finite automata with the number of computation paths restricted.

It is interesting to investigate a hierarchy based on the degree of nondeterminism for eight-way four-dimensional multihead finite automata which can move east, west, south, north, up, down, future, or past. Moreover, it will be interesting to investigate the case of four-dimensional *alternating* multihead finite automata (see [2] for the concept of *alternation*).
Finally, we would like to hope that some unsolved questions concerning this paper will be explicated in the near future.

References

[1] M.Blum and C.Hewitt, Automata on a two-dimensional tape, *IEEE symposium on Switching and Automata theory*, pp.155-160, 1967.
[2] A.K.Chandra, D.C.Kozen, and L.J.Stockmeyer, Alternation, *Journal of the Association for Computing Machinery*, Vol.28, No.1, pp.114-133, 1981.
[3] J.Diaz and J.Toran, Classes of bounded nondeterminism, *Mathematical Systems Theory*, Vol.23, pp.21–32, 1990.
[4] P.Duris, J.Hromkovič, and K.Inoue, A separation of determinism, Las Vegas and nondeterminism for picture recognition, *To appear in JCSS.*
[5] J. E. Hopcroft and J. D. Ullman , *Introduction to Automata Theory, Languages, and Computation*, Addison-Wesley, Reading, Mass., 1979.
[6] J.Hromkovič, J.Karhumäki, H.Klauck, G.Schnitger, and S.Seibert, Measures of nondeterminism in finite automata, *To appear in JCSS.*
[7] J.Hromkovič, and G.Schnitger, Nondeterministic communication with a limited number of advice bits, *Proc. STOC* '96, *ACM*, pp.551–560, 1996.
[8] K.Iihoshi, M.Sakamoto, T.Ito, M.Saito, H.Furutani, M.Kono and, Katsushi Inoue, Leaf-size bounded computation for four-dimensional alternating Turing machines, *Proceedings of the 11th International Symposium on Artificial Life and Robotics*, *Oita*, *Japan*, 2006 (CD-ROM).
[9] K.Iihoshi, M.Sakamoto, T.Ito, N.Tomozoe, Y.Nagamizu, H.Furutani, M.Kono, S.Ikeda, T.Tamaki, and K.Inoue, Leaf-size hierarchy of four-dimensional alternating Turing machines, *Proceedings of the 12th International Symposium on Artificial Life and Robotics*, *Oita*, *Japan*, 2007 (CD-ROM).
[10] K. Inoue and A. Nakamura, An n-dimensional on-line tessellation accepter (in Japanese), *The Transactions of the IECE*, Vol.J59-D, No.4, pp.229-236, 1976.
[11] K. Inoue and I. Takanami, A note on bottom-up pyramid acceptors, *Information Processing Letters*, Vol.8, No.1, pp.34-37, 1979.
[12] K.Inoue, I.Takanami, and H.Taniguchi, Three-way two-dimensional simple multihead finite automata – Hierarchical properties – *Trans. IECE* '79/2, Vol.62, No.2, pp.65–72, 1979.
[13] K.Inoue and I.Takanami, A survey of two-dimensional automata theory, *Information Sciences*, Vol.55, pp.99-121, 1991.
[14] S.Inoue, A study of Path-bounded multihead finite automata, *Master Graduate School of Yamaguchi University*, 2003.
[15] S.Inoue, K.Inoue, A.Ito, Y.Wang, Path-bounded three-way two-dimensional finite automata, *in*, 2003 *Chugoku-Section Joint Convention Record of Institute of Electrical and Information Engineerings*, *Japan*, pp.262–263, 2003.
[16] A. Ito, K. Inoue, and I. Takanami, Complexity of acceptance problems for two-dimensional automata, *A perspective in Theoretical Computer Science*, *Series in Computer Science*, Vol.16, pp.70-94, 1989.
[17] T. Ito, M. Sakamoto, Y. Nagamizu, K. Iihoshi, N. Tomozoe, H. Furutani, M. Kono, S. Ikeda, T. Tamaki, and K. Inoue, Three-dimensional parallel Turing machines, *Proceedings of the 12th International Symposium on Artificial Life and Robotics*, *Oita*, *Japan*, pp.131-134, 2007.
[18] T.Ito, M.Sakamoto, A.Taniue, T.Matsukawa, Y.Uchida, H.Furutani, and M.Kono, Parallel Turing machines on four-dimensional input tapes, *Proceedings of the 15th International Symposium on Artificial Life and Robotics*, *Oita*, *Japan*, OS5-5,pp.918-921, 2010 (CD-ROM).
[19] T. Makino, H. Okabe, S. Taniguchi, M. Sakamoto, and K. Inoue, Three-dimensional multi-inkdot automata, *International Journal of Artificial Life and Robotics*, Vol.9, No.2, pp.99-101, 2005.
[20] Y.Nakama, M.Sakamoto, M.Saito, S.Taniguchi, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Accepting powers of four-dimensional alternating Turing machines with only universal states, *Proceedings of the 10th International Symposium on Artificial Life and Robotics*, *Oita*, *Japan*, 2005.

[21] A. Nakamura and K. Aizawa, On the recognition of properties of three-dimensional pictures, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol.PAMI-7, pp.708-713, 1985.

[22] A. Nakamura and K. Aizawa, Detection of interlocking components in three-dimensional digital pictures, *Information Sciences*, Vol.40, pp.143-153, 1986.

[23] A. Nakamura and A. Rosenfeld, Three-dimensional connected pictures are not recognizable by finite-state acceptors, *Technical Report TR-566, Computer Science Center, University of Maryland*, 1991.

[24] H. Okabe, M.Sakamoto, and K. Inoue, Computational complexity for the simulation of four-dimensional marker automata by four-dimensional Turing machines, *Proceedings of the 8th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, pp.114-117, 2003.

[25] H. Okabe, S.Taniguchi,T. Makino, S.Nogami, Y. Nakama, M.Sakamoto, and K. Inoue, Computation for motion image processing by 4-dimensional alternating Turing machines, *Proceedings of the 7th World Multiconference on Systemics, Cybernetics and Informatics*, $Orlando$, USA, pp.241-246, 2003.

[26] M.Saito, M.Sakamoto, Y.Nakama, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Some properties of four-dimensional multicounter automata, *Proceedings of the 10th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, 2005.

[27] M.Saito, M.Sakamoto, K.Iihoshi, T.Ito, H.Furutani, M.Kono, and K.Inoue, Hierarchies based on the number of counters or the space allowed by four-dimensional multicounter automata, *Proceedings of the 11th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, 2006 (CD-ROM).

[28] M.Saito, M.Sakamoto, Y.Nakama, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Some properties of four-dimensional multicounter automata, *International Journal of Artificial Life And Robotics, Springer*, Vol.l0,No.2, pp.166-170, 2006.

[29] M. Sakamoto, T. Okazaki, and K. Inoue, Three-dimensional multicounter automata, *Proceedings of the 6th International Workshop on Parallel Image Processing and Analysis−Theory and Applications*, $Madras$, $India$, pp.267-280, 1999.

[30] M. Sakamoto, Three-dimensional alternating Turing machines, *Ph.D. Thesis, Yamaguchi University*, 1999.

[31] M.Sakamoto, T.Ito, N.Tomozoe, and H.Furutani, A survey of three-dimensional automata, *The papers of technical meeting on information systems, IEE, Japan*, IS–07–12, pp.11–17, 2007.

[32] M.Sakamoto, N.Tomozoe, H.Furutani, M.Kono, T.Ito, Y.Uchida, and H.Okabe, A survey of automata on three-dimensional input tapes, *WSEAS Transactions on Computers*, Issue 10, Vol.7, pp.1638-1647, 2008.

[33] M.Sakamoto, M.Fukuda, S.Okatani, T.Ito, H.Furutani, and M.Kono, Path-bounded Three-dimensional finite automata, *International Journal of Artificial life and Robotica*, *Springer*, Vol.13, No.1, pp.54-57, 2008.

[34] M.Sakamoto, H.Okabe, S.Nogami, S.Taniguchi, T.Makino, Y.Nakama, M.Saito, M.Kono, and K.Inoue, A note on four-dimensional finite automata, *WSEAS Transactions on Computers*, Issue 5,Vol.3, pp.1651-1656, 2004.

[35] M.Sakamoto, S.Okatani, M.Fukuda, T.Ito, and H.Furutani, and M.Kono, A relationship between Turing machines and finite automata on four-dimensional input tapes, *International Journal of Artificial Life And Robotics*, *Springer*, Vol.l3, No.1, pp.58-60, 2008.

[36] M.Sakamoto, T.Matsukawa, R.Katamune, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Four-dimensional synchronized alternating Turing machines, *Proceedings of the 10th American Conference on Applied Mathematics, Harvard University, Cambridge, USA*, pp.195-200, 2010 (CD-ROM).

[37] M.Sakamoto, R.Katamune, T.Matsukawa, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Some results about hierarchy and recognizability of four-dimensional synchronized alternating Turing machines, *Proceedings of the 10th American Conference on Applied Mathematics*, *Harvard University*, *Cambridge*, *USA*, pp.201-205, 2010 (CD-ROM).

[38] H.Taniguchi, K.Inoue, and I.Takanami, A note on three-dimensional finite automata, *Information Sciences*, Vol.26, pp.65–85, 1982.

[39] H. Taniguchi, K. Inoue, and I. Takanami, k-neighborhood template $\mathcal{A}$-type 2-dimensional bounded cellular acceptor (in Japanese), *The Transactions of the IEICE*, Vol.J69-D, No.3, pp.291-301, 1986.

[40] N.Tomozoe, M.Sakamoto, K.Iihoshi, Y.Nagamizu, T.Ito, H.Furutani, M.Kono, S.Ikeda, T.Tamaki, and K.Inoue, A relationship between the accepting powers of alternating finite automata and nondeterministic on-line tessellation acceptors on four-dimensional input tapes, *Proceedings of the 12th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, 2007 (CD-ROM).

[41] A.M.Turing, On computable numbers, with an application to the Entscheidungsproblem, *in Proceedings of London Math. Soc.*, Vol.2, No.42, pp.230-265, 1936.

[42] Y.Uchida, T.Ito, H.Okabe, M.Sakamoto, H.Furutani, and Michio Kono, Recognizability of multi-inkdot finite automata on four-dimensional input tapes, *Proceedings of the 12th WSEAS International Conference on Computers*, pp.408-413, Heraklion, Greece, 2008.

[43] Y.Uchida, T.Ito, H.Okabe, M.Sakamoto, H.Furutani, and M.Kono, Four-dimensional multi-inkdot finite automata, *WSEAS Transactions on Computers*, Issue 9, Vol. 7, pp.1437-1446, 2008.

[44] Y.Uchida, M.Sakamoto, A.Taniue, R.Katamune, T.Ito, H.Furutani, and M.Kono, Some properties of four-dimensional parallel Turing machines, *Proceedings of the 15th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, OS5-6, pp.922-925, 2010 (CD-ROM).

[45] Y. Yamamoto, K. Morita, and K. Sugata, Space complexity for recognizing connected patterns in a three-dimensional tape (in Japanese), *Technical Report of the IEICE*, No.AL79-104, pp.91-96, 1979.

[46] A.C.Yao and R.L.Rivest, k+1 heads are better than k, *J.ACM*, Vol.25, No.2, pp.337–340, 1978.

Yasuo Uchida is a professor in the Department of Business Administration at Ube National College of Technology, Japan. His current research interests are in the areas of administration simulation game and e-Learning system. He has published several research papers and books in that area. His articles have appeared in journals such as *WSEAS TRANSACTIONS* on *COMPUTERS*, and proceedings of major conferences such as *ISAGA* and *AROB*.

Takao Ito is a professor of the Department of Business Administration at Ube National College of Technology, Japan. He is serving concurrently as professor of Harbin Institute of Technology (*HIT*) in Weihai, China. He received his Doctor degree of Economics from Kyoto University and Ph.D. degree of Engineering from Miyazaki University, Japan. His current research interests include automaton theory, quantitative analysis of supply chain relationships using graph theory, theoretical analysis of the relationship between IT and corporate strategy, and engineering analysis of organizational structures using complex system theory. He was one of the winners of the Best Paper Award in the International Symposium on Artificial Life and Robotics (*AROB*) in January 2006. He has published in several refereed journals and proceedings, particularly in the area of corporate management, IT strategy, and computer science. Dr. Ito is a member of the international advisory board of Scientific Journals International and the editorial review board of International Journal of Business Data Communications and Networking, International Journal of Data Mining, Modeling and Management, and International Journal of Information and Communication Technologies for the Advanced Enterprise. He sits in the International Program Committee of Artificial Life and Robotics (*AROB*). And he also is a member of the Board of Directors of Asian Symbiosis Association, Japan.

Makoto Sakamoto received the Ph.D. degree in computer science and systems engineering from Yamaguchi University in 1999. He is presently an associate professor in the Faculty of Engineering, University of Miyazaki. His first interests lay in hydrodynamics and time series analysis, especially the directional wave spectrum. His current main research interests are automata theory, languages,and computation. He is also interested in digital geometry, digital image processing, computer vision, computer graphics, complex systems. and so forth. He has published many research papers in that area. His articles have appeared in journals such as Information Sciences, Pattern Recognition and Artificial Intelligence, *WSEAS*, *AROB*, *SJI*, and *IEICE* (Japan), and proceedings of conferences such as Parallel Image Processing and Analysis, *SCI*, *WSEAS*, and *AROB*.

Ryoju Katamune received B.E. degree in computer science from University of Miyazaki in 2009. He is presently a postgraduate student in University of Miyazaki. His main interests are automata theory and digital geometry. He is also interested in image processing, computer graphics, and so on. He has published some research papers in that area. His articles have appeared in journals such as *WSEAS TRANSACTIONS* on *COMPUTERS* and proceedings of conferences such as *WSEAS* and *AROB*.

Kazuyuki Uchida received B.E. degree in computer science from University of Miyazaki in 2009. He is presently a postgraduate student in University of Miyazaki. His main interests are automata theory and computational complexity. He is also interested in moving image processing, computer animation, and so on. He has published some research papers in the area of theoretical computer science. His articles have appeared in proceedings such as *WSEAS* and *AROB*.

Hiroshi Furutani received the Ph.D. degree in science from Kyoto University in 1981. He is presently a professor in the Faculty of Engineering, University of Miyazaki. His intersts were problems in theoretical nuclear phisics. His current main research interests are automata theory, evolutionary computation and bio-informatics. His articles have appeared in journals such as Complex systems, Foundations of Genetic Algorithms, and so on.

Michio Kono received the B. Eng., M. Eng. and D.Eng. Degrees in Aeronautical Engineering from Nagoya University, Japan in 1969, 1971 and 1977, respectively. He joined the Department of Control Engineering, Tokyo University of Mercantile Marine, in 1979, where he became a Professor in 1990. Since 1992, he has been a Professor at the Department of Computer Science and Systems Engineering, University of Miyazaki. His research interests include decoupling control, guaranteed cost control, periodic systems, time-delay systems and networked control systems. His articles have appeared in journals such as *WSEAS*, *AROB*, *SICE*(Japan), *ISCIE*(Japan), and proceedings of conferences such as *WSEAS* and *AROB*.

Satoshi Ikeda received the BSc degree in Mathematics from Waseda University, the MSc degree in Mathematics from Shizuoka University and the Ph.D. degree from Hiroshima University. Though a researcher of Japan Society for the Promotion of Science, he was a research assistant of Tokyo University of Agriculture and Technology from 1996 to 2002. He has been an associate professor in Miyazaki University from 2002. His research interest includes graph theory and measure theory. His articles have appeared in journals such as Theoretical Computer Science, Journal of human ergology, Topology and its Applications, and *IEEE* Transactions on Parallel and Distributed Systems, and proceedings of conferences such as *ICALP*,*ICM*,*VLSI*, and *AROB*.

Tsunehiro Yoshinaga received the Ph.D. degree in computer science and systems engineering from Yamaguchi University in 1996. He is presently a professor in the Department of Computer Science and Electronic Engineering at Tokuyama National College of Technology. His research interests are automata theory, computational complexity, and database. He has published research papers in that area. His articles have appeared in journals such as Thoeretical Computer Science, *WSEAS*, *JCST*, *IEICE*, and *IPSJ*, and in proceedings of conferences such as Information and *WSEAS*.