# A Fuzzy Assessment Model for Evaluating the Rate of Aggregative Risk in Software Development

Lily Lin, and Huey-Ming Lee

**Abstract**—In this study, we present a fuzzy assessment model to tackle the rate of aggregative risk in fuzzy circumstances by fuzzy sets theory during any phase of the software development life cycle. Because the proposed assessment method directly uses the fuzzy numbers rather than the linguistic values to evaluate, it can be executed faster than before. The proposed fuzzy assessment method is easier, closer to evaluator real thinking and more useful than the ones they have presented before.

**Keywords**—Fuzzy assessment, Risk analysis, Rate of aggregative risk.

# I. INTRODUCTION

DURING the past decades, computer technologies have changed so fast that the need of large software system becomes much more intensive. Most of the cost evaluations are characterized by high uncertainty. Thus, there are many problems occur in the software system development life cycle, such as postponed schedule, increased cost, inefficiency and abandonment [10].

Generally, risk is the traditional manner of expressing uncertainty in the systems life cycle. In a quantitative sense, it is the probability at such a given point in a system's life cycle that predicted goals can not be achieved with the available resources. Due to the complexity of risk factors and the compounding uncertainty associated with future sources of risk, risk is normally not treated with mathematical rigor during the early life cycle phases [1].

Risks result in project problems such as schedule and cost overrun, so risk minimization is a very important project management activity [13]. Up to now, there are many papers investigating risk identification, risk analysis, risk priority, and

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risk management planning [1-5, 8]. Conger [7] presented a list of possible software risks.

In evaluating the rate of risk factors, most decision-makers or project-managers, in fact, viewed those factors as linguistic values (terms), e.g., very high, high, middle, low, very low and etc. After fuzzy sets theory was introduced by Zadeh [14] to deal with problem in which vagueness is present, linguistic value can be used for approximate reasoning within the framework of fuzzy sets theory [15] to effectively handle the ambiguity involved in the data evaluation and the vague property of linguistic expression, and normal triangular fuzzy numbers are used to characterize the fuzzy values of quantitative data and linguistic terms used in approximate reasoning.

Therefore, Lee [10] classified the risk factors presented by Boehm [2-4], Charette [5], Conger [7], Gilb [8] into six attributes, divided each attribute into some risk items, and built up the hierarchical structured model of aggregative risk and the evaluating procedure of structured model. Lee [10] ranged the grade of risk for each risk item into eleven ranks, and proposed the procedure to evaluate the rate of aggregative risk using two stages fuzzy assessment method. Chen [6] ranged the grade of risk for each risk item into thirteen ranks and proposed the other arithmetic operations instead of the two stages fuzzy assessment method. He defuzzified the trapezoid or triangular fuzzy numbers by the median. Based on [6, 10], Lee et al. [11] proposed the other algorithm to evaluate the rate of aggregative risk.

In previous studies [6, 10-11], they used eleven or thirteen linguistic values for ranking the grades of risk to each risk item, where the linguistic values were represented by the triangular fuzzy numbers. But, it is very complicated to compute. Also, the evaluator only chooses one grade from grades of risk for each risk item. It has difficulty in reflecting the evaluator's incomplete and uncertain thought. Therefore, if we can use fuzzy sense of assessment to express the degree of evaluator's feelings based on his own concepts, the results will be closer to

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the evaluator's real thought. In this study, we propose a fuzzy assessment method to tackle the rate of aggregative risk in fuzzy circumstances by fuzzy sets theory during any phase of the software development life cycle. Since the proposed method directly uses the fuzzy numbers rather than the linguistic values to evaluate, it can be easier, and meet the evaluator real thinking.

# II. PRELIMINARIES

For the proposed algorithm, all pertinent definitions of fuzzy sets are given below [9, 11, 14-16].

Definition 2.1. Fuzzy Point: Let  $\widetilde{a}$  be a fuzzy set on  $R = (-\infty, \infty)$ . It is called a fuzzy point if its membership function is

$$\mu_{\widetilde{a}}(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases} \tag{1}$$

Definition 2.2. Level  $\alpha$  Fuzzy Interval: Let  $[a,b;\alpha]$  be a fuzzy set on R. It is called a level  $\alpha$  fuzzy interval,  $0 \le \alpha \le 1$ , a < b, if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$
 (2)

If a=b, we call  $[a, b; \alpha]$  a level  $\alpha$  fuzzy point at a.

Definition 2.3. Triangular Fuzzy Numbers: Let  $\widetilde{A}=(p,q,r)$ , p < q < r, be a fuzzy set on R. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-p}{q-p}, & \text{if } p \le x \le q \\ \frac{r-x}{r-q}, & \text{if } q \le x \le r \\ 0, & \text{otherwise} \end{cases}$$
 (3)

If r=q, p=q, then  $\widetilde{A}$  is (q, q, q). We call it the fuzzy point  $\widetilde{q}$  at q.

The centroid of  $\widetilde{A} = (p, q, r)$  is

$$C(\widetilde{A}) = \frac{\int_{-\infty}^{\infty} x \mu_{\widetilde{A}}(x) dx}{\int_{-\infty}^{\infty} \mu_{\widetilde{A}}(x) dx}$$

$$= \frac{1}{3}(p+q+r)$$
 (4)

Proposition 1. Let  $\widetilde{A}_1 = (p_1, q_1, r_1)$  and  $\widetilde{A}_2 = (p_2, q_2, r_2)$  be two triangular fuzzy numbers, and k > 0, then, we have

$$(1^{0}) \widetilde{A}_{1} \oplus \widetilde{A}_{2} = (p_{1} + p_{2}, q_{1} + q_{2}, r_{1} + r_{2})$$

$$(2^{0}) k\widetilde{A}_{1} = (kp_{1}, kq_{1}, kr_{1})$$

We can easily show the Proposition 1 by extension principle.

Proposition 2. Let  $\widetilde{A}_1=(p_1,q_1,r_1)$  and  $\widetilde{A}_2=(p_2,q_2,r_2)$  be two triangular fuzzy numbers, and  $k\in R$ , then we have

$$(1^{0}) C(\widetilde{A}_{1} \oplus \widetilde{A}_{2}, \widetilde{0}) = C(\widetilde{A}_{1}, \widetilde{0}) + C(\widetilde{A}_{2}, \widetilde{0})$$

$$(2^{0}) C(k\widetilde{A}_{1}, \widetilde{0}) = kC(\widetilde{A}_{1}, \widetilde{0})$$

We can easily show the Proposition 2 by Proposition 1.

Definition 2.4. Fuzzy relation:

Let  $X, Y \subseteq R$  be universal sets, then

$$\widetilde{R} = \{((x, y), \mu_{\widetilde{R}}(x, y)) | (x, y) \subseteq X \times Y\}$$
 (5)

# III. THE PROPOSED FUZZY ASSESSMENT MODEL

We present the fuzzy assessment model as follows:

Step 1: Assessment form for the risk items:

The criteria ratings of risk are linguistic variables with linguistic values  $V_1$ ,  $V_2$ , ...,  $V_7$ , where  $V_1$  = extra low,  $V_2$  = very low,  $V_3$  = low,  $V_4$  = middle,  $V_5$  = high,  $V_6$  = very high,  $V_7$  = extra high. These linguistic values are treated as fuzzy numbers with triangular membership functions as follows:

$$\widetilde{V}_{1} = (0, 0, \frac{1}{6}),$$

$$\widetilde{V}_{k} = (\frac{k-2}{6}, \frac{k-1}{6}, \frac{k}{6}), \text{ for } k = 2, 3, ..., 6$$

$$\widetilde{V}_{7} = (\frac{5}{6}, 1, 1)$$
(6)

In previous studies [6, 10-11], the evaluator only chooses one grade from grade of risk for each risk item, it ignores the evaluator's incomplete and uncertain thinking. Therefore, if we use fuzzy numbers of assessment in fuzzy sense to express the degree of evaluator's feelings based on his own concepts, the computing results will be closer to the evaluator's real thought.

The assessment for each risk item with fuzzy number can reduce the degree of subjectivity of the evaluator, express the degree of evaluator's feelings based on his own concepts. The results will be closer to the evaluator's real thought. Based on the structured model of aggregative risk and evaluating form of structured model proposed by Lee [10], we proposed the new assessment method of the structured model as shown in Table 1.

In Table 1,

$$\sum_{i=1}^{6} W_2(i) = 1, \quad 0 \le W_2(i) \le 1 \tag{7}$$

, for each i = 1, 2, ... 6.

$$\sum_{i=1}^{n(k)} W_1(k,i) = 1, \ 0 \le W_1(k,i) \le 1$$
 (8)

for k=1, 2,..., 6; n(1)=1, n(2)=4, n(3)=2, n(4)=4, n(5)=2, n(6)=1; i=1, 2,..., n(k).

$$\sum_{l=1}^{7} m_{ki}^{(l)} = 1, \, 0 \le m_{ki}^{(l)} \le 1 \tag{9}$$

, for l=1, 2, ..., 7; k=1, 2, ..., 6; i=1, 2, ..., n(k).

From Table 1, we directly use the fuzzy numbers  $(m_{ki}^{(I)})$  rather than the linguistic values to evaluate. Also, we may express the risk item  $X_{ki}$  as fuzzy discrete type

$$X_{ki} = \frac{m_{ki}^{(1)}}{V_1} \oplus \frac{m_{ki}^{(2)}}{V_2} \oplus \frac{m_{ki}^{(3)}}{V_3} \oplus \frac{m_{ki}^{(4)}}{V_4} \oplus \frac{m_{ki}^{(5)}}{V_5} \oplus \frac{m_{ki}^{(6)}}{V_6} \oplus \frac{m_{ki}^{(7)}}{V_7}$$
(10)

Step 2: By the first stage aggregative assessment

Based on [10], by the centroid method, we have the  $C(\widetilde{V}_1) = 0.0556$ ,  $C(\widetilde{V}_2) = 0.1667$ ,  $C(\widetilde{V}_3) = 0.3333$ ,  $C(\widetilde{V}_4) = 0.5$ ,  $C(\widetilde{V}_5) = 0.6667$ ,  $C(\widetilde{V}_6) = 0.8333$ ,  $C(\widetilde{V}_7) = 0.9444$  as center of mass of  $V_1$ ,  $V_2$ ,  $V_3$ ,  $V_4$ ,  $V_5$ ,  $V_6$ ,  $V_7$ , respectively. Let  $V = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7\}$  be the set of the criteria rating of risk for each risk item. By fuzzy relation on  $X_i \times V$ , we can form a fuzzy assessment matrix  $M(X_i)$  for  $X_i \times V$  [10, 16] for i=1,  $2, \ldots, 6$ .

Evaluate the first stage aggregative assessment risk for attribute  $X_i$  as follows:

$$(R(i,1), R(i,2), R(i,3), ..., R(i,7))$$

$$= (W_1(i,1), W_1(i,2), ..., W_1(i,n(i))) \times M(X_i)$$
(11)

for i = 1, 2, ..., 6.

We denote  $R_1(i) = (R(i,1), R(i,2), R(i,3), ..., R(i,7))$  the vector of the first stage aggregative assessment for attribute  $X_i$  for i=1, 2, ..., 6.

Step 3: By the Second Stage Assessment

The algorithm of the second stage assessment is

$$(R_2(1), R_2(2), R_2(3), R_2(4), R_2(5), R_2(6), R_2(7))$$

$$= (W_{2}(1), W_{2}(2), ..., W_{2}(6)) \times \begin{pmatrix} R_{1}(1) \\ R_{1}(2) \\ R_{1}(3) \\ R_{1}(4) \\ R_{1}(5) \\ R_{1}(6) \end{pmatrix}$$
(12)

(8) , where 
$$R_2(i) = \sum_{k=1}^{6} W_2(k) \times R(k,i)$$
, for  $i = 1, 2, ..., 7$ .

Step 4: Synthetic Analysis

There are three methods, saying maximal membership grade method, fuzzy probability distribution method, and defuzzified by the centroid method, to synthesize analysis of the aggregative risk of aggregative risk as follows:

(A) By maximal membership grade method: Let

$$R_2^m(L) = M_{2}^{7} \underset{j=1}{\alpha_X} R_2(j)$$
 (13)

, where  $R_2(j)$  is as shown in Eq. (6), L=1, 2, ..., 7. The L corresponds to the sub-index of  $V_L$  (the meaning of  $V_L$  as shown in Step 1), it means that the risk rating in software development is  $V_L$  and the membership grade is  $R_2^m(L)$ .

(B) By the fuzzy probability distribution method:

Normalize  $(R_2(1), R_2(2), R_2(3), R_2(4), R_2(5), R_2(6), R_2(7))$  in Eq. (12), let

$$R_2^{m,*}(k) = \frac{R_2(k)}{\sum_{i=1}^{n} R_2(i)}$$
 (14)

, for k=1, 2, ..., 7.

Then, we have that the probability of the aggregative risk in software development for extra low, very low, ..., extra high are  $R_2^{m,*}(1)$ ,  $R_2^{m,*}(2)$ , ...,  $R_2^{m,*}(7)$ , respectively.

(C) Defuzzified by the centroid method: Defuzzified

$$(R_2(1), R_2(2), R_2(3), R_2(4), R_2(5), R_2(6), R_2(7))$$

in Eq. (12) by the centroid method, we have that the rate of aggregative risk for the evaluator's assessing is as follows:

$$Rate = \frac{\sum_{i=1}^{7} C(\widetilde{V}_{i}) \cdot R_{2}(i)}{\sum_{k=1}^{7} R_{2}(k)} = \sum_{i=1}^{7} C(\widetilde{V}_{i}) \cdot (\frac{R_{2}(i)}{\sum_{k=1}^{7} R_{2}(k)})$$
(15)

The value of Rate is the rate of aggregative risk in software development.

# IV. NUMERICAL EXAMPLE

Example: Assume that we have the following attributes, weights, grade of risk for each risk item as shown in Table 2. By the evaluating process shown in Section IV, we have

$$(R_2(I), R_2(2), R_2(3), R_2(4), R_2(5), R_2(6), R_2(7))$$
= (0.0258, 0.357, 0.5982, 0.019, 0, 0, 0) (16)

(A) By maximal membership grade method, we have  $R_2^m(3) = 0.5982$ 

i.e., the aggregative risk is low.

- (B) By the fuzzy probability distribution method, normalizing Eq. (16), we have the same as Eq. (16). It shows that the aggregative risk are 2.58%, 35.7%, 59.82%, 1.9%, for extra low, very low, low, middle, respectively.
- (C) Defuzzified by the centroid nethod as shown in Eq. (16), we have

Rate = 0.26983

i.e., the rate of aggregative risk is 0.26983.

# V. CONCLUSION

Most general surveys force evaluator to assess one grade from the grades of risk to each risk evaluate item, but it ignores the uncertainty of human thought. When evaluators need to choose the assessment from the survey with the method of multiple choices, it causes the general survey become quiet exclusive. The assessment of evaluation with fuzzy numbers can overcome this demerit and reduce the degree of subjectivity of the evaluator. In this paper, we design a new assessment method to evaluate the rate of aggregative risk in software development and directly use the fuzzy numbers rather than the linguistic values to do the evaluation, it can be executed much fast. Therefore, the evaluator can assess the risk grade by fuzzy numbers to each risk item, which making evaluation process is also easier than the ones presented before [10-11]. Since the presented method directly uses the fuzzy numbers rather than the linguistic values to evaluate, it can be easier, and meet the evaluator real thinking.

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Table 1 Contents of the hierarchical structure model

Attribute	Risk item	Weight-2	Weight-1		Linguistic variables							
				$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$		
$X_1$		$W_2(1)$										
	$X_{11}$		$W_1(1,1)$	$m_{11}^{(1)}$	$m_{11}^{(2)}$	$m_{11}^{(3)}$	$m_{11}^{(4)}$	$m_{11}^{(5)}$	$m_{11}^{(6)}$	$m_{11}^{(7)}$		
$X_2$		$W_2(2)$										
	$X_{21}$		$W_1(2,1)$	$m_{21}^{(1)}$	$m_{21}^{(2)}$	$m_{21}^{(3)}$	$m_{21}^{(4)}$	$m_{21}^{(5)}$	$m_{21}^{(6)}$	$m_{21}^{(7)}$		
	$X_{22}$		$W_1(2,2)$	$m_{22}^{(1)}$	$m_{22}^{(2)}$	$m_{22}^{(3)}$	$m_{22}^{(4)}$	$m_{22}^{(5)}$	$m_{22}^{(6)}$	$m_{22}^{(7)}$		
	X <sub>23</sub>		$W_1(2,3)$	$m_{23}^{(1)}$	$m_{23}^{(2)}$	$m_{23}^{(3)}$	$m_{23}^{(4)}$	$m_{23}^{(5)}$	$m_{23}^{(6)}$	$m_{23}^{(7)}$		
	$X_{24}$		$W_1(2,4)$	$m_{24}^{(1)}$	$m_{24}^{(2)}$	$m_{24}^{(3)}$	$m_{24}^{(4)}$	$m_{24}^{(5)}$	$m_{24}^{(6)}$	$m_{24}^{(7)}$		
$X_3$ ·		$W_2(3)$										
	$X_{31}$		$W_1(3,1)$	$m_{31}^{(1)}$	$m_{31}^{(2)}$	$m_{31}^{(3)}$	$m_{31}^{(4)}$	$m_{31}^{(5)}$	$m_{31}^{(6)}$	$m_{31}^{(7)}$		
	$X_{32}$		$W_1(3,2)$	$m_{32}^{(1)}$	$m_{32}^{(2)}$	$m_{32}^{(3)}$	$m_{32}^{(4)}$	$m_{32}^{(5)}$	$m_{32}^{(6)}$	$m_{32}^{(7)}$		
X <sub>4</sub> ·		$W_2(4)$										
	$X_{41}$		$W_1(4,1)$	$m_{41}^{(1)}$	$m_{41}^{(2)}$	$m_{41}^{(3)}$	$m_{41}^{(4)}$	$m_{41}^{(5)}$	$m_{41}^{(6)}$	$m_{41}^{(7)}$		
	$X_{42}$		$W_1(4,2)$	$m_{42}^{(1)}$	$m_{42}^{(2)}$	$m_{42}^{(3)}$	$m_{42}^{(4)}$	$m_{42}^{(5)}$	$m_{42}^{(6)}$	$m_{42}^{(7)}$		
	$X_{43}$		$W_1(4,3)$	$m_{43}^{(1)}$	$m_{43}^{(2)}$	$m_{43}^{(3)}$	$m_{43}^{(4)}$	$m_{43}^{(5)}$	$m_{43}^{(6)}$	$m_{43}^{(7)}$		
	$X_{44}$		$W_1(4,4)$	$m_{44}^{(1)}$	$m_{44}^{(2)}$	$m_{44}^{(3)}$	$m_{44}^{(4)}$	$m_{44}^{(5)}$	$m_{44}^{(6)}$	$m_{44}^{(7)}$		
$X_5$ ·		$W_2(5)$										
J	$X_{51}$		$W_1(5,1)$	$m_{51}^{(1)}$	$m_{51}^{(2)}$	$m_{51}^{(3)}$	$m_{51}^{(4)}$	$m_{51}^{(5)}$	$m_{51}^{(6)}$	$m_{51}^{(7)}$		
	X <sub>52</sub>		$W_1(5,2)$	$m_{52}^{(1)}$	$m_{52}^{(2)}$	$m_{52}^{(3)}$	$m_{52}^{(4)}$	$m_{52}^{(5)}$	$m_{52}^{(6)}$	$m_{52}^{(7)}$		
$X_6$ ·		W <sub>2</sub> (6)	W. (6.1)	40				(5)				
	$X_{61}$		$W_1(6,1)$	$m_{61}^{(1)}$	$m_{61}^{(2)}$	$m_{61}^{(3)}$	$m_{61}^{(4)}$	$m_{61}^{(5)}$	$m_{61}^{(6)}$	$m_{61}^{(7)}$		

Table 2 Contents of the example

Attribute	Risk item	Weight-2	Weight-1		Linguistic variables							
				$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$		
$X_1$ .		0.3										
•	$X_{11}$		1	0	0.17	0.83	0	0	0	0		
$X_2$	11	0.3										
	$X_{21}$		0.4	0	0.53	0.47	0	0	0	0		
	$X_{22}^{21}$		0.4	0	0.89	0.11	0	0	0	0		
	$X_{23}^{22}$		0.1	0.25	0.75	0	0	0	0	0		
	$X_{24}^{23}$		0.1	0.61	0.39	0	0	0	0	0		
$X_3$ .	24	0.1										
	$X_{31}$		0.5	0	0.17	0.83	0	0	0	0		
	$X_{32}^{31}$		0.5	0	0.53	0.47	0	0	0	0		
$X_4$ .	32	0.1										
4	$X_{41}$		0.3	0	0.89	0.11	0	0	0	0		
	$X_{42}^{41}$		0.1	0	0.17	0.83	0	0	0	0		
	$X_{42} \\ X_{43}$		0.3	0	0.17	0.83	0	0	0	0		
	$X_{44}$		0.3	0	0.53	0.47	0	0	0	0		
$X_5$ .	44	0.1										
3 .	$X_{51}$		0.5	0	0	0.81	0.19	0	0	0		
	X <sub>52</sub>		0.5	0	0	0.81	0.19	0	0	0		
$X_{\epsilon}$	32	0.1										
0.	$X_{c1}$		1	0	0.17	0.83	0	0	0	0		
<i>X</i> <sub>6</sub> .	$X_{52}$ $X_{61}$	0.1	1			0.81						