The Incorporation of Transaction Cost Variable in the Maximin Optimization Model and the Implication on Active Portfolio Management

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***Abstract*—**In real investment practice, transaction cost is recognized to play an essential part in determining investors' true portfolio return. Maximin model was one of the established optimization models introduced by previous scholars which have ignored the cost function in their formulation. The importance to have this transaction cost function as additional variable in the model is more emphasized as investors adopts active portfolio management such as rebalancing process. In this paper, we construct an extended Maximin optimization model while provided its application into Shariah compliant securities listed in Bursa Malaysia. Implementing periodical rebalancing strategies of annual, semi-annual and quarterly basis, results in this paper show that, the extended models of Maximin proved to be effective in illustrating the transaction cost effect on the performance of investors' portfolio return.

***Keywords*—** Portfolio optimization, Rebalancing, Transaction cost.

I. Introduction

MAXIMIZE return and minimize risk has been the pillar behind the construction of optimal portfolio which is pioneered by Harry Markowitz dates back on late 1950's and others afterwards. In quest to establish the optimal portfolio, investors are left with a decision to find the best combination and allocation of assets which could meet with their target in investment. The attention in searching the best optimization models that could generate that optimal allocation is present continuously from time to time which then lead to many development and changes made into the original optimization models. Markowitz quadratic programming is a well-known classical model introduced by Harry Markowitz in 1952 which has been famously regarded as the milestone for modern finance theory [1].

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By taking the expected value as a measure for return while variance as a measure of risk, this model suggested the approaches such that investor could either maximize portfolio's profit that is subject to predetermine level of risk or minimize the risk with a specific required rate of return. Promoting quite a similar approach and formulations, Martin Young in 1998 has established the Minimax Portfolio Selection Rule as a simplified linear model that removes the covariance variable which present in the Markowitz model [2]. Regarded this particular variable as cumbersome and time consuming, Young introduced the linear model which believe can greatly enhanced various practical applications when it commonly stands as standard feature on computer spreadsheet programs besides claimed to be able in avoiding the logical problems of a quadratic implied by Markowitz. There are also other simplified optimization models such as lower partial moment (LPM) model [3], [4], the mean-Gini (MG) model [5], mean absolute deviation (MAD) and mean semi absolute deviation model [6], and the extension of Minimax model [7], [8] which contributed in this line of study of portfolio optimization. However, despite the capability of all these models to meet with investors' preference on both risk and return, the estimated solutions most likely to deviate from the true values as the models ignores the transaction cost function in the formulation.

It has been well documented in the literature that transaction cost is the important constraint which include in the investment process. Without acknowledging the cost involve, investor will overestimated their true performance of portfolio return. In addition, the importance of this transaction cost is greatly concerned investors who actively manage their portfolio with rebalancing strategy. Rebalancing is defined as a process which aligns the portfolio proportion with the original target of each asset as to maintain the investor's risk profile [9]. As the efficient frontier obtained in most optimal portfolio will shift as the elapse of time, thus the need to rebalance is triggered by time the deviation is no longer acceptable [10]. Rebalance based on periodical basis is a strategy which is often implied by investors. However, as far as it gives benefit to investors, it has been criticized due to high transaction cost that could incur in the portfolio. It has also been claimed that the higher the rebalancing frequency, the higher transaction cost incurred [11], [12]. Therefore, in order to capture the cost effectiveness of rebalancing strategy, this paper provided empirical analysis

on the portfolios' performance for three rebalancing strategies of annual, semi-annual and quarterly.

The organization of this paper is as follow. After this introduction, Section II attempts to provide the development of Maximin model with the original formulation and the extended version as transaction cost variable is included. All the variables and the assumptions involved are well-defined and explained. Section III discussed the application of the extended Maximin optimization model to Shariah Compliant Securities listed in Malaysia. The samples of data used together with the justification on the selection criteria are highlighted. Next, empirical results and analysis of finding were presented in section IV while concluding remarks was made in section V.

II. Model Development

A. Original Maximin Model

Minimax Portfolio Selection Rule sets by Young [2] provided formulation that focused on the way to maximize the minimum return required by the investor as well as to minimize the maximum loss given some level of return. In some study, a model which focused on to maximize the minimum return is also called as Maximin model [13] and the equations in given as follows,

$$Objective\ Function = Maximize\ Z \quad \textbf{(1)}$$

where Z is a minimum return in Ringgit Malaysia (RM) for every period.

subject to

$$\sum_{j=1}^{n} r_{jt} x_j \geq Z \quad \textbf{(2)}$$

$$\sum_{j=1}^{n} \bar{r}_j x_j \geq \alpha B \quad \textbf{(3)}$$

$$\sum_{j=1}^{n} x_j = B \quad \textbf{(4)}$$

$$0 \leq x_j \leq v_j \qquad j = 1, \dots\dots\dots n \quad \textbf{(5)}$$

Where,

$j = the\ securities\ involve$
$n = number\ of\ securities\ present\ in\ the\ portfolio$
$r_{jt} = return\ in\ security\ j\ over\ period\ of\ t$
$x_j = portfolio\ allocation\ of\ security\ j\ in\ RM$
$\bar{r}_j = average\ return\ in\ security\ j\ over\ entire\ period, T$
$\alpha = minimum\ expected\ return\ required\ by\ investor$
$B = initial\ budget\ in\ RM\ to\ be\ invested$
$v_j = maximum\ allocation\ allowed\ for\ security\ j$

Refer to "(2)", it is highlighted that this Maximin model's captures the interesting downside risk in portfolio volatility as emphasized not to be below Z which eventually excludes the involvement of securities which yields a negative return. However, investors might come into infeasible solution when the sum for all securities in certain period yield a negative return which then required investor to do some adjustment such as allowing the Z value to be free or in the other words accept the negative value of Z. Equation (3) and (4) represent the portfolio average return and budget constraints respectively while "(5)" is the allocations constraint which set up based on the assumptions that short selling is not allowed as the generated allocations must be greater than 0. The v_j value in the other hand is required in order to ensure that the budget will diversified into more securities and to avoid in putting investors into riskier position as if certain security was allocated with large amount.

B. Extended Maximin Model

As mentioned previously, transaction cost variable will be incorporated in the model as this study aims to the solution of portfolio that not only optimal but also cost-effective. All the equations in the original model are valid for this extended version except for "(3)" and "(4)" which were changed into the following equations

$$\sum_{j=1}^{n} \bar{r}_j x_j - \frac{C}{T} \geq \alpha B \quad \textbf{(6)}$$

$$\sum_{j=1}^{n} x_j = B - C \quad \textbf{(7)}$$

Where
$C = total\ transaction\ cost$

that subject to

$$C = C_1 + C_2 \quad \textbf{(8)}$$

If

$$\left| x_{jt} - x_{j\,t-1} \right| > y$$

$$C_1 = \sum_{j=1}^{n} \left| x_{jt} - x_{j\,t-1} \right| \times \omega \quad \textbf{(9)}$$

Else if

$$0 < \left| x_{jt} - x_{j\,t-1} \right| \leq y$$

$$C_2 = \varepsilon \times k \quad \textbf{(10)}$$

Where,

$C_1 = transaction\ cost\ 1$
$C_2 = transaction\ cost\ 2$
$x_{jt} = portfolio\ allocation\ of\ security\ j\ in\ RM\ over\ period\ of\ t$
$x_{jt-1} = portfolio\ allocation\ of\ security\ j\ in\ RM\ over\ period\ of\ t-1$
$y = transaction\ value\ in\ RM$
$\omega = brokerage\ fees\ in\ percentage, \%$
$k = minimim\ brokerage\ fee\ in\ RM$
$\varepsilon = number\ of\ transaction$

Equation (6) indicates that the net portfolio average return is realized after average transaction cost is deducted while in "(7)" budget for portfolio is set to cover both the allocation of invested securities as well as the total transaction cost incurred for respective period. In order to fulfill optimality condition for the generated solutions, investors might have to change the equality sign in "(7)" into less or equal sign cause the rigid constraint sometimes lead into unfeasible solution.

Regarding the transaction cost variable incorporated in the model, the equations involve were established following rules adhered by Bursa Malaysia for stocks or equity. The calculation of transaction cost is divided into fixed and variable cost. From the guidelines given the minimum brokerage fee, *k* RM 40 is imposed for each transaction as long as the investor's value of transaction didn't exceed the y value as stated in "(10)". The variable cost in the meantime is subject to "(9)" which the cost is calculated based on brokerage fees at rate 0.6% given that investor's transaction value more is than the y value. Thus, the defined approximated value of y is RM 6666.67 as we solved the equation of y = RM 40/0.6%. Recognizing all these equations, the extended version of Maximin model is then developed in the Solver. To generate the solutions, sample portfolios are first established with the data of Shariah Complaint Securities listed in Bursa Malaysia as provided in the following section.

III. Application of Extended Maximin Model to Shariah Compliant Securities in Malaysia

Focusing on the individual investor perspective, the sample portfolio was initiated with 50 numbers of Shariah-Compliant stocks listed in Bursa Malaysia with addition of MyETF Dow Jones Islamic Market Malaysia Titans 25 (MyETF-DJIM25). The selection of these securities was carried out through review of most literature which focusing on the price, risk and return criteria [14], [15]. For this paper, we select the stock which traded on average price of more than RM 1 and less than RM 5 together with the stock which have a consistent return and less volatile behavior. The stocks included was diversified into 9 sectors as in Table I, with the given proportion which is varies due to several factors such as the performance and total of listed stock available. For MyETF-DJIM25, besides recognized to have quite a low volatility, the reason of this security to participate was because it has the similar calculation for transaction cost in above as it traded exactly like stocks.

For these 50 securities, daily price were taken from DataStream 5.1 which provides an access to the world's largest and most respected historical financial numerical database for a total of 3 years period from 2010 until 2012. The price is then used to compute the daily return and the average daily returns which are the required variables in the model. The rest of the variables are given as we assume that investor have the initial budget, *B* worth RM 50, 000 with the minimum daily required return, α is 0.0231% which indicated 6% minimum return per year. Maximum allocation allowed for each security, v_j is restricted at 20% of the available budget or RM 10, 000. Prior to computation of results, we recalled that in the objective of this paper, three different rebalancing strategies of annual, semi-annual and quarterly were intend to be evaluate as to determine which strategy is the most effective or worth to perform as transaction cost was incorporated in the model. Therefore, the period of observation in sample portfolio is then categorized with three different frequency such that annual rebalancing strategy with three sub periods of 2009, 2010 and 2011 while portfolios of semi-annually and quarterly rebalancing strategies were having generated results in six and twelve sub periods respectively. The performances of these optimal portfolios were then being compared with the portfolios of naïve strategy and benchmark index, FTSE Bursa Malaysia Emas Shariah Index (FBMEMAS). For naïve strategy, the portfolio allocation is constructed without using the optimization models and simply by dividing the budget evenly for each security in the sample portfolio. All of the results obtained are tabulated in the next section.

Table I. The Proportion of Selected Shariah Complaint Securities in the Sample Portfolio

Sector	Number of Securities	Proportion (%)
Consumer Product	6	11.8
Construction	3	5.9
Infrastructure	2	3.9
Plantation	3	5.9
Industrial Product	20	39.2
Properties	7	13.7
Trading/Services	7	13.7
Finance	1	2.0
Technology	1	2.0
MyETF-DJIM25	1	2.0
Total	**51**	**100**

IV. Results and Analysis of Findings

A. Portfolio Performances of Annual Rebalancing Strategy

Table II shows the performance of portfolio which rebalance in every one year using Maximin model relative to the portfolios of naïve strategy and FBMEMAS. For Maximin's portfolio, an average of only 8 securities was selected in the portfolio in one particular sub period as

compare to the total of 51 securities present in portfolio of naïve strategy. From the table, we can see that a total of RM 1, 631.48 transaction cost was incurred in 3 years period by Maximin portfolio which rebalanced annually while RM 2,040 cost was incurred in the naïve strategy's portfolio. In overall study period, both portfolios were observed to outperform the return performance of FBMEMAS which generated only 31.69%. Maximin's portfolio however in particular generated higher net return of 120.53% as compared to 62.62% generated by portfolio of naïve strategy. High return portfolio is no doubt will accompany with high risk. Furthermore, with limited number of securities, the portfolio will also tend to be riskier than the others. This is justified with the highest total risk of 1.16% which generated by Maximin's portfolio as compared to portfolios of naïve strategy and FBMEMAS that recorded 0.73% and 0.60% total risk respectively.

Table II. The Performance of Maximin's Portfolio in Annual Rebalancing Strategy Relative to Portfolios of Naïve Strategy and FBMEMAS

Portfolio	**Maximin**			**Naïve Strategy**			**FBMEMAS**	
Performance's indicator/Sub Period	**Net Return, %**	**Risk, %**	**T.Cost, RM**	**Net Return, %**	**Risk, %**	**T.Cost, RM**	**Net Return, %**	**Risk, %**
2010	50.24	1.30	392.17	31.17	0.73	2,040.00	17.09	0.53
2011	28.82	1.19	619.31	11.51	0.88	-	3.12	0.75
2012	41.47	0.95	620.00	19.94	0.55	-	11.48	0.46
TOTAL	**120.53**	**1.16**	**1631.48**	**62.62**	**0.73**	**2,040.00**	**31.69**	**0.60**

B. Portfolio Performance of Semi-Annually Rebalancing Strategy

Maximin's portfolio which rebalanced in every six months were summarized to select an average of 7 securities and the performance of net return, risk and transaction cost was shown as in Table III. With the accumulated amount of transaction cost of RM 3,398.95, Maximin's portfolio under this strategy exceeds the cost incurred in the naïve strategy by RM 1,358.95. However, even with higher transaction cost, Maximin portfolio managed to generate the highest total net return of 197.66% and this result as we can see outperform both portfolios of naïve strategy and FBMEMAS. We mention previously that Maximin's portfolio tends to be more risky than the other and it is proven with the calculated risk of 1.26% which is the highest value among the three observed portfolios.

Table III. The Performance of Maximin's Portfolio in Semi-Annually Rebalancing Strategy Relative to Portfolios of Naïve Strategy and FBMEMAS

Portfolio	**Maximin**			**Naïve Strategy**			**FBMEMAS**	
Performance's indicator /Sub Period	**Net Return, %**	**Risk, %**	**T.Cost, RM**	**Net Return, %**	**Risk, %**	**T.Cost, RM**	**Net Return, %**	**Risk, %**
Jan-June 2010	28.21	1.48	354.43	11.10	0.85	2,040.00	3.18	0.60
July-Dec 2010	45.74	1.29	628.69	20.06	0.60	-	13.91	0.46
Jan-June 2011	35.04	1.38	716.60	12.53	0.74	-	4.67	0.56
July-Dec 2011	11.13	1.24	700.00	-1.02	1.00	-	-1.55	0.91
Jan-June 2012	48.22	1.20	580.00	14.36	0.64	-	6.76	0.49
July-Dec 2012	29.32	0.85	419.23	5.58	0.43	-	4.72	0.44
TOTAL	**197.66**	**1.26**	**3398.95**	**62.62**	**0.73**	**2,040.00**	**31.69**	**0.60**

C. Portfolio Performance of Quarterly Rebalancing Strategy

Under this strategy, the generated optimal portfolio of Maximin's model was consists of an average 6 securities for each sub period. As provided in Table IV, the highest total transaction cost incurred in three years was given by Maximin's portfolio with RM 6,853.95. Besides that, as shown in the table, this Maximin's portfolio was once again managed to generate enormous value of net return with 328.03% received in total of 3 years and significantly outperformed both portfolios of naïve strategy and FBMEMAS. For the risk value, 149% total risk was calculated as whole 3 years period of study was concerned.

Table IV. The Performance of Maximin's Portfolio in Quarterly Rebalancing Strategy Relative to Portfolios of Naïve Strategy and FBMEMAS

Portfolio	Maximin			Naïve Strategy			FBMEMAS	
Performance's indicator/Sub Period	Net Return, %	Risk, %	T.Cost, RM	Net Return, %	Risk, %	T.Cost, RM	Net Return, %	Risk, %
Jan-Mac 2010	53.00	2.27	318.37	11.15	0.81	2,040.00	4.87	0.52
Apr-June 2010	8.83	1.55	638.37	-0.05	0.85	-	-1.69	0.66
July-Sept 2010	26.12	1.05	645.07	11.48	0.56	-	7.82	0.44
Oct-Dec 2010	35.50	1.75	646.90	8.59	0.64	-	6.09	0.47
Jan-Mac 2011	38.90	2.01	580.00	11.25	0.93	-	3.23	0.68
Apr-June 2011	20.04	1.13	643.61	1.28	0.49	-	1.44	0.41
July-Sept 2011	2.400	1.27	574.11	-14.85	1.14	-	-13.88	0.95
Oct-Dec 2011	42.53	1.54	576.66	13.83	0.78	-	12.33	0.83
Jan-Mac 2012	33.20	1.22	560.00	10.54	0.53	-	4.96	0.46
Apr-June 2012	23.75	1.24	520.00	3.82	0.73	-	1.80	0.52
July-Sept 2012	24.02	1.37	620.00	2.19	0.52	-	3.09	0.44
Oct-Dec 2012	19.74	0.91	530.87	3.39	0.31	-	1.63	0.44
TOTAL	**328.03**	**1.49**	**6,853.95**	**62.62**	**0.73**	**2,040.00**	**31.69**	**0.60**

D. Analysis of Findings

In this paper, we emphasized on 4 main factors which play a vital role in helping investors to evaluate which portfolios is the best option to invest. Begin with selection of securities; we can see that, Maximin model tends to select limited securities as only an average of 8, 7 and 6 securities out of 51 total securities were present in annual, semi-annually and quarterly rebalancing strategies respectively. This is certainly different from naïve strategy which decided to include all securities present in the samples portfolios. With limited number of securities in the portfolio, it actually gives benefit in term of monitoring work especially for perspective of individual investors, who will likely spend limited time to monitor and managing their portfolios. However, the portfolio with limited securities unfortunately contributed into higher risk. For investors, risk is another important measure before making decision on investment. As we can see from the results, investor who believed in the Maximin model is subject to bear more risk than the others and this is supported by related study which employed original Maximin model in constructing portfolios [13]. Through three rebalancing strategies, it was observed that the risk was increasing from annual to quarterly which then consequently justified the increasing value of return obtained.

Besides risk, return is also the essential factor in the investment process. In this paper, we can see that investor who decides to establish optimal portfolio with Maximin model is most likely to receive a high value of return as compared to others. In particular, Maximin portfolio which rebalanced most frequently as in every 3 months is viewed to have better return performance than the portfolio which rebalanced less frequent such as in every 6 months or 12 months. However, for Maximin portfolio which rebalanced every year, the net return generated was slightly higher than the 6 months rebalanced portfolio which is most probably due to the amount of transaction cost incurred. Recalled that the transaction cost value was took into account in the calculation of the return, thus it is expected that the return performance is affected following the amount of transaction cost incurred. As we established the extended Maximin model by incorporated the transaction cost variable, it is proved that rebalancing strategies incurred high transaction cost. It is also justified that the more frequent investors rebalance their portfolio, the more amount of transaction cost will be incurred and this claim is supported by previous studies [11], [12]. For naïve strategy portfolio, it is important to highlight that the amount of transaction cost incurred was only for the beginning period as no changes in allocation was made till the end.

Summarizing all arguments, we can see that Maximin portfolio which applied rebalancing strategy managed to give individual investors a better performance than portfolio of naïve strategy. Although incurred with high transaction cost, the value of net return received was still high which imply that it is worth for investor to perform. In addition, the limited amount of securities need to held at one particular time might be favor by most of individual investor. Through actively managing the portfolio by rebalancing, investor also will assure to meet their initial target investment as the allocations were changed based on the current performance of securities relative to the market condition. In term of frequency to rebalance, it was summarized that portfolio which rebalance more frequently is more appropriate for risk taker investor who seek for more return. This investor must be willing to spend extra time to monitor and managing portfolios. In contrast, for risk-averse investor, it is best for

them to generate portfolio with less frequent rebalancing strategy. For naïve strategy in the other hand, although the portfolio manage to generate lower risk due to the high number of securities, investor might probably experience losses and be in more harmful position. This is because the longer investor held the portfolio without any changes it will actually expose them with the uncertainty of the market. Furthermore, by allocating the budget evenly to all securities, it will lead to the tedious monitoring and managing work.

V. Conclusion

In this paper, we highlighted the usability of the extended Maximin model to capture transaction cost while encouraged investor to actively manage their portfolio by rebalancing strategy. With the incorporation of the transaction cost, the results presented in this paper were justified to be more realistic and closer to the real one. However, the fact that historical data were used, it means that only estimated portfolio' performance was determined rather than the true performance. Therefore, given the fact that past data can never accurately predict the future condition of capital market due to the high volatility of market environments [16], different results might be generated when estimation results were confronted with the future value. Therefore, we plan to explore two more routes which first to generate the true portfolio' performance and observe on how the performance was different from the expected. We also plan to improve another optimizations model with similar transaction cost variable and do the comparative analysis of portfolio performance.

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