A new model for multiple criteria decision making with ordinal rankings of criteria and alternatives

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Abstract**—** the aim of the paper is to propose a new model for multiple criteria decision making with ordinal rankings of criteria and alternatives. In this setting a set of alternatives is ranked from the best to the worst by a set of criteria, where also criteria are ranked from the most important to the least important, and no cardinal information is available. The goal is to find the best alternative or the overall ranking of all alternatives respectively. This is a modification of the social choice theory setting, where different voters play the role of different criteria and it is assumed that all voters (hence criteria) have the same weight (power). The winner in the social choice theory is selected by a social choice function (procedure), which must satisfy some desirable properties, such as Pareto efficiency, monotonicity, independence of irrelevant alternatives, etc., though famous Arrow's impossibility theorem rules out a possibility of the existence of a procedure satisfying all the important properties for at least three alternatives. In the paper it is shown which properties are (not) met by the proposed method, and in the last part of the paper the use of the method is illustrated by an example of a real-world decision making problem: a selection of the most appropriate textbook for an elementary science education.

Keywords— multi-criteria decision making, ordinal ranking, Pareto efficiency, social choice function, social choice procedure.

I. Introduction

Multiple criteria decision making (MCDM) is an important part of everyday's life as well as it is successfully applied in many areas of human action such as management, marketing, engineering, social science, politics, environmental protection, etc. With regard to character of information available about criteria and alternatives MCDM can be divided into MCDM with *cardinal* and *ordinal* information respectively.

In the former case criteria and alternatives are assigned numerical values (weights) with respect to a goal or criteria respectively, and the best alternative attains the highest value (when criteria are maximization ones). This is typical for the weighted sum approach (WSA), or analytic hierarchy/network process (AHP/ANP), etc., see e.g. [1] and [2].

In the latter case only ordinal information is available, that is ordering (ranking) of n alternatives from the 1st to the nth place with regard to some criteria is known. This problem dates back to the late 18th century and preferential elections context, see works of Borda [3] or Condorcet [4]. In this type of MCDM ordinal information is usually transformed into cardinal information. An easy way to do so provides Borda-Kendall's method of marks (counts), see [3] or [5]: each alternative is assigned the number of points (marks) corresponding to its rank and an alternative with the lowest total sum (or the lowest average, which is equivalent) of marks is the best. However, this and similar methods treat positions in a ranking as weights, but this is an ad-hoc assumption as real weights are not known. Cook and Kress proposed more sophisticated method to generate (cardinal) weights of criteria by the data envelopment analysis, see [6] or [7]. Also, several methods for group decision making with ordinal information were proposed, see e.g. [8], [9], [10] and [11], but none of these methods is suitable for multiple criteria.

However, at present there is no strictly ordinal MCDM method known to author though ordinal information might be sufficient to compare alternatives in many cases. This is somewhat surprising as ordinal way of comparison of objects is more general and also more natural than cardinal approach. People rank objects in accord with their preferences every time when they are selecting a car, a computer, a drink or a dinner in a restaurant, a holiday destination, a book to read, etc, when criteria such as beauty, style, look, interestingness, design, safety, taste, elegance, prestige, knowledge, etc. are involved.

Hence, the aim of the article is to propose a method for MCDM with strictly ordinal information about criteria and alternatives (MCDM-ORCA). Also, properties of this method as a social choice procedure are discussed thoroughly.

Furthermore, to show a potential of the proposed method its use is illustrated on a real-world problem: a selection of the most appropriate textbook for elementary science education. This is an important task because a right choice of a textbook plays an important role in children's education.

The paper is organized as follows: in section 2 MCDM-ORCA is described, in section 3 its mathematical properties in the context of the social choice theory are examined, and in

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section 4 provides the use of the method for a selection of the most appropriate textbook for elementary science education. Conclusions close the article.

II. Formulation of the problem and the MCDM-ORCA method

Setting of the problem *of multi-criteria decision making with ordinal ranking of criteria and alternatives* considered in this article is as follows:

Let $C = \{C_1, ..., C_k\}$ be the set of criteria and let $AL = \{A, B, C, ...\}$ be the set of alternatives. Let the ordering of all criteria according to their importance be as follows: $C_1 \succ C_2 ... \succ C_k$, so C_1 is the most important criterion and C_k is the least important criterion.

Let all alternatives be ranked from the best to the worst by all criteria C_i (such rankings are nothing else than permutations of all alternatives). The goal of the problem is to rank the alternatives from the best to the worst with regard to all criteria.

Definition 1. *For each pair of alternatives A and B there is a binary index-vector* $I_{(A,B)}$ *such that* $I_{(A,B)} = (i_1, ..., i_k)$ *, where* $i_j = 1$ *if an alternative A is ranked better than B with regard to a criterion j, otherwise* $i_j = 0$.

Example 1. Consider four criteria (from the most important C_1 to the least important C_4) and an alternative *A* is ranked better than *B* by criteria C_2 and C_4. Then $I_{(A,B)} = (0,1,0,1)$. As a consequence of Definition 1 there exists also an index-vector $I_{(A,B)}$ such that $I_{(B,A)} = (1,...,1) - I_{(A,B)}$, so: $I_{(B,A)} = (1,0,1,0)$. Both $I_{(A,B)}$ and $I_{(B,A)}$ are binary and are inverse to each other.

Index-vectors provide information about alternatives' pair-wise comparisons by each and every criterion, and they can be used for determining overall dominance between two alternatives. If an alternative *A* dominates some other alternative *B*, it means that *A* is better (ranked higher) than *B* overall (by all criteria).

Definition 2. Let $H_{(A,B)} = (h_1, ..., h_k)$ *be a cumulative index-vector such that* $H_{(A,B)} = \left(i_1, \sum_{j=1}^{2} i_j, ..., \sum_{j=1}^{k} i_j \right)$. $H_{(B,A)}$ *is defined analogously from the index-vector* $I_{(B,A)}$ *, and let* $H_{(B,A)} = (h_1^*, ..., h_k^*)$.

Definition 3a *(a pair-wise dominance relation). An alternative A dominates an alternative B (* $A \succ B$ *) iff :*

$$h_i \geq h_i^* \quad \forall i \in \{1, ..., k\} , \tag{1}$$

and at least one inequality in (1) is strict (and vice versa).

Definition 3b *(equivalent formulation of a dominance relation). An alternative A dominates an alternative B (* $A \succ B$ *) iff for each criterion by which B is ranked better than A, there is a unique and more important criterion, by which A is ranked better than B.*

Example 2. From Example 1 we have $H_{(A,B)} = (0,1,1,2)$ and $H_{(B,A)} = (1,1,2,2)$. According to Definition 3a) an alternative *B* dominates an alternative *A*, because: $h_i^* \geq h_i \quad \forall i \in \{1, ..., 4\}$, and from these four inequalities two inequalities (for *i* = 1 and *i* = 3) are strict.

Consequence of Definitions 3a-b. If an alternative *A* dominates an alternative *B* then *A* is ranked better than *B* by the same or higher number of criteria than vice versa. And also, an alternative *A* is ranked better than *B* by the most important criterion. On contrary if *A* is ranked better than *B* by the most important criterion, then *A* dominates *B* or both alternatives are incomparable. In Example 1 an alternative *A* is ranked better than *B* by criteria C_2 and C_4, so an alternative *B* is ranked better than *A* by criteria C_1 and C_3. No matter what are the (unknown) weights of all criteria, *B* should be ranked better than *A* overall, because it is ranked better by more important criteria (C_1 is more important than C_2, and C_3 is more important than C_4) than *A*.

Example 3. Consider four criteria (from the most important C_1 to the least important C_4) and an alternative *A* is ranked better than *B* by criteria C_1 and C_4, so $I_{(A,B)} = (1,0,0,1)$ and $I_{(B,A)} = (0,1,1,0)$, $H_{(A,B)} = (1,1,1,2)$ and $H_{(B,A)} = (0,1,2,2)$. As can be seen from both cumulative index-vectors, there is no dominance between *A* and *B*, because $H_{(A,B)}$ has greater value on the first position, while $H_{(B,A)}$ has greater value on the third position. Hence, we cannot conclude, which alternative is ranked better in overall.

In Example 3 for some set of weights an alternative *A* can be evaluated better in overall (typically when a criterion C_1 acquires large weight), while for some other set of weights an alternative *B* might be evaluated better (when C_2 and C_3 acquire almost the same weight as C_1, and C_4 acquires low weight).

The dominance relation from Definition 3 induces only quasi-order on a set of all alternatives. Hence, there might be pairs of alternatives which are incomparable, as in Example 3.

The *final overall ranking* of all alternatives is every ranking consistent with all alternative pair-wise comparisons by Definition 3.

To summarize, the proposed model for MCDM with ordinal information (MCDM-ORCA) proceeds in the following steps:

1. All criteria are ranked from the most important to the least important.

2. All alternatives as ranked from the best to the worst with regard to all criteria.

3. Index-vectors and cumulative index-vectors for all pairs of alternatives are established.

4. All pairs of alternatives are compared by a pair-wise dominance relation (1).

5. All alternatives are ranked into one or more final rankings.

By MCDM-ORCA method there are three possible cases regarding a solution in general:

- The final overall ranking of all alternatives constitutes a complete order on a set of alternatives. Then the best alternative is unique and constitutes the best solution to a given problem.

- The final overall ranking of all alternatives is not unique (some alternatives cannot be compared and are regarded as ‘ties’), but the best alternative is unique and constitutes the best solution to a given problem. The fact that some other (lower) ranked alternatives are incomparable is usually unimportant.

- The final overall ranking of all alternatives is not unique, and there are at least two best alternatives tied for the first place. In such a case the best solution to a given problem cannot be found by the described method. A decision maker might try to repeat the method with only these best alternatives, or he/she can use some other method of MCDM, this time with cardinal information (such as AHP/ANP), which might be more suitable for a given problem.

In the next section mathematical properties of the method are examined.

III. On mathematical properties of the proposed method as the social choice procedure

In the *social choice theory* it is assumed that voters provide ordinal rankings of all alternatives (that is rankings from the first to the last place without ties) from some set of feasible alternatives (usually candidates).

Preferences of one individual are called *individual preference list*, while preferences of all voters are called *preference profile*. The goal is to find the consensus ranking of all alternatives or just the best alternative (often a *winner* of an election).

To find a winner, many *social choice functions* or *procedures* were proposed during the last 230 years, beginning with *Borda count* and *Condorcet majority rule* from the late 18th century, see [3] and [4], though some elementary concepts on voting theory were already anticipated in the works by Ramón Llull in the 13th century. Social choice functions or procedures should satisfy a wide set of ‘reasonable‘ criteria such as (see e. g. [12]):

- *Always a winner condition* (AAW): every possible set of preference lists provides at least one winner (social choice).
- *Condorcet winner condition* (CWC) : if there is a condorcet winner, then it is the social choice
- *Pareto condition (efficiency)*: if A is preferred by B by all voters, then B is not the social choice.
- *Independence of irrelevant alternatives* (IIA): The ranking of alternatives A and B by a social choice function should not depend on ranking (or change in a ranking) of some other alternative C.
- *Monotonicity*: if an alternative A is promoted in one preference list then social choice function should respond by not ranking this alternative worse than before.
- *Non-imposition*: Every possible ranking by a social choice function should be achievable from some set of individual preferences.
- *Non-dictatorship*: There is no single individual who determines the ranking of alternatives.

Some of the mentioned properties are not independent, as shown in [13], as for example monotonicity, non-imposition and independence of irrelevant alternatives imply Pareto efficiency.

Another famous result is the *Arrow’s impossibility theorem*, see [12] or [13]:

If the set of alternatives has more than two elements, then Pareto efficiency, non-dictatorship and independence of irrelevant alternatives cannot be satisfied together.

See also [14] and [15] for less known but more general Gibbard–Satterthwaite theorem, which states that for at least three alternatives there is no single-winner voting method under a given set of conditions including non-dictatorship (these methods are manipulable by a tactical voting).

As for two alternatives well-known May’s theorem states [16]:

If there is odd number of voters and two alternatives (and ties are not allowed), majority rule is the only social choice function that satisfies neutrality, anonymity and monotonicity.

Anonymity means that all voters (or votes) are treated equally, and *neutrality* means that if every voter reverses his/her vote (between two alternatives), then the result of a voting is reversed as well.

The larger is the number of alternatives, the more difficult it is to find a consensus. For example the majority rule can handle only two alternatives. For a given social choice function or procedure the *Nakamura number* can be found, see [17]. If the number of alternatives is smaller than corresponding Nakamura’s number, then the procedure will identify the best alternative. If not, voting paradoxes arise and a procedure fails to find a winner. So for example the majority rule has Nakamura number 3.

Some well-known social choice procedures include:

- *Majority rule*: the winner from two alternatives is an alternative which is on top of at least half of individual preference list
- *Condorcet majority rule*: the winner is an alternative which defeats (it has more votes) every other alternative in one-on-one contest
- *Borda Count*: each alternative is assigned the number of points corresponding to its positions in each and every ranking and the winner is an alternative with the lowest sum.
- *Plurality voting*: the winner is an alternative with the most first-place rankings in individual preference lists.
- *Hare system*: an alternative with the fewest top rankings is deleted from the list of alternatives, and the procedure repeats until one alternative – a winner – remains.
- *Dictatorship*: a single voter determines a winner.

Table 1 provides a list of properties (not)satisfied by these methods.

Table 1. Source: modified from [12].

procedure	AAW	CWC	Pareto	Monot.	IIA
Condorcet majority rule	NO	YES	YES	YES	YES
Borda Count	YES	NO	YES	YES	NO
Plurality voting	YES	NO	YES	YES	NO
Hare system	YES	NO	YES	NO	NO
Dictatorship	YES	NO	YES	YES	YES
MCDM-ORCA	*YES*	*NO*	*YES*	*YES*	*YES*

The proposed MCDM-ORCA method can be considered a social choice procedure if criteria are considered voters with different weight. This is of course different from voting theory, where all voters have the same weight (are treated equally), but in many real-world situations (apart from elections of various kinds) decision makers (especially in management) are endowed by a different weight expressing their social status, age, knowledge, etc. Hence, it is possible to examine ORCA with regard to the social choice theory conceptual framework.

Proposition 1: The MCDM-ORCA method has the following properties:
i) It satisfies AAW, Pareto condition, monotonicity and IIA.
ii) It doesn‘t satisfy CWC and anonymity.

Proof:

ia) AAW: from procedure description it is clear that there is one winner or there is more than one winner if two or more alternatives dominate remaining alternatives, but they are incomparable to each other.
ib) Pareto: if an alternative *A* is ranked higher than *B* in all individual profile list, then index-vector $I_{(A,B)} = (1,1,...,1)$ $I_{(B,A)} = (0,0,...,0)$, $H_{(A,B)} = (1,2,3,...)$ $H_{(B,A)} = (0,0,0,...)$, and from the dominance relation (1) it follows that *A* dominates *B*, so *B* cannot be the social choice (a winner).
ic) Monotonicity: Suppose that A is ranked better than B by a social choice function, and promote A in a ranking by an arbitrary criterion. Then new cumulative index-vector $H^*_{(A,B)}$ has the same or the higher values than previous $H_{(A,B)}$, hence the dominance relation between A and B is preserved.
id) IIA: the proposed method is based on pair-wise comparisons, and also index-vectors and cumulative index-vector are constructed for pairs. Thus, introduction of some new alternative will not change the dominance between any pair of old alternatives.

ii) CWC: Consider the setting with 2 alternatives (A, B) and 3 criteria, where A is ranked better than B by the most important criterion, while B is ranked better by remaining two criteria. Then Condorcet winner should be B (it has majority), but from the dominance relation (1) it follows that A and B are incomparable. The proof for non-anonymity follows from different weights of criteria (voters) □.

Hence, MCDM-ORCA method satisfies the most of desirable properties for social choice procedures with the exception for the Condorcet winner condition and anonymity.

In the next section it is shown how the proposed method can be used to solve a real-world problem.

IV. THE SELECTION OF THE MOST APPROPRIATE TEXTBOOK FOR ELEMENTARY SCIENCE EDUCATION

In the Czech Republic each elementary or secondary school decides which textbook will be used for a given class and a given subject of education. As a supply of textbooks is wide, a selection of the most suitable textbook by a teacher is a typical case of multi-criteria decision making situation where an evaluation of different textbooks on selected criteria is rather ordinal in nature than cardinal: it is not possible to assign textbooks some numerical value with regard to criteria such as content, comprehensibility, adequacy to children’s age and knowledge, etc. (with the exception of textbook’s price), but textbooks can be ranked from the best to the worst by such criteria, and the best textbook can be found by the proposed MCDM-ORCA method

Let’s consider six elementary science textbooks for the 6th grade pupils abbreviated by letters A, B, C, D, E and F (see Figure 1) are going to be evaluated by four criteria in order to find the most suitable textbook.

The criteria are:

- *Content (C)*: a textbook should include a required topic, for example energy and its changes, in a depth and an extent required by a given school (teacher). Because each elementary school in the Czech Republic has its own school educational programme, needs for a suitable textbook differ from school to school.
- *Comprehensibility and adequacy (CA)*: text, explanations, concepts, examples, figures, etc. should be comprehensible and adequate to the children's age and prior knowledge.
- *Graphic design (GD)*: text and pictures should be clear and well arranged to foster understanding.
- *Price (P)*: price of a textbook should be as low as possible.

It is clear that the first three criteria cannot be cardinal as it doesn't make sense to assign a numerical value to content, comprehensibility, adequacy to children age and knowledge or graphic design. The only cardinal criterion is the price, but it can be made ordinal too.

In the first step all criteria were ranked from the most important (1.) to the least important (4.):

1. *C*	2. *CA*	3. *GD*	4. *P*

Somewhat unusually, the price is the least important criterion, because it has no sense to use the cheapest textbook if it is useless (it doesn't include required topic, or it is incomprehensible, etc.). But if there are two textbooks satisfying all more important criteria evenly, then the cheaper textbook is more suitable. Moreover, the price of different textbooks is very similar (around 6 euro).

Next, all textbooks were ranked by an expert (an experienced teacher) with regard to all criteria, see Table 2.

Table 2. Rankings of textbooks with regard to all criteria.

Criterion	1.	2.	3.	4.	5.	6.
C	A	D	C	E	F	B
CA	C	A	B	F	E	D
GD	E	B	A	D	C	F
P	D	A	C	B	F	E

In the following step index-vectors and cumulative index-vectors for each pair of alternatives were evaluated:

- Pair*A*-*B*: $I_{(A,B)} = (1,1,0,1)$, $I_{(B,A)} = (0,0,1,0)$, $H_{(A,B)} = (1,2,2,3)$, $H_{(B,A)} = (0,0,1,1)$, and according to a condition (1) from Definition 3 an alternative *A* dominates an alternative *B*: $A \succ B$.
- Pair*A*-*C*: $I_{(A,C)} = (1,0,1,1)$, $I_{(C,A)} = (0,1,0,0)$, $H_{(A,C)} = (1,1,2,3)$, $H_{(C,A)} = (0,1,1,1)$: $A \succ C$.
- Pair*A*-*D*: $I_{(A,D)} = (1,1,1,0)$, $I_{(D,A)} = (0,0,0,1)$, $H_{(A,D)} = (1,2,3,3)$, $H_{(D,A)} = (0,0,0,1)$: $A \succ D$.
- Pair*A*-*E*: $I_{(A,E)} = (1,1,0,1)$, $I_{(E,A)} = (0,0,1,0)$, $H_{(A,E)} = (1,2,2,3)$, $H_{(E,A)} = (0,0,1,1)$: $A \succ E$.
- Pair*A*-*F*: $I_{(A,F)} = (1,1,1,1)$, $I_{(F,A)} = (0,0,0,0)$, $H_{(A,F)} = (1,2,3,4)$, $H_{(F,A)} = (0,0,0,0)$: $A \succ F$.
- Pair *B*-*C*: $I_{(B,C)} = (0,0,1,0)$, $I_{(B,A)} = (1,1,0,1)$, $H_{(B,C)} = (0,0,1,1)$, $H_{(C,B)} = (1,2,2,3)$: $C \succ B$.
- Pair*B*-*D*: $I_{(B,D)} = (0,1,1,0)$, $I_{(D,B)} = (1,0,0,1)$, $H_{(B,D)} = (0,1,2,2)$, $H_{(D,B)} = (1,1,1,2)$: according to a dominance relation (1) both alternatives are not comparable, as in the first coordinate of cumulative index-vectors *B* is better than *D*, but the third coordinate *D* is evaluated better than *B*.
- Pair *B*-*E*: $I_{(B,E)} = (0,1,0,1)$, $I_{(E,B)} = (1,0,1,0)$, $H_{(B,E)} = (0,1,1,2)$, $H_{(E,B)} = (1,1,2,2)$: $E \succ B$.
- Pair *B*-*F*: $I_{(B,F)} = (0,1,1,1)$, $I_{(F,B)} = (1,0,0,0)$, $H_{(B,F)} = (0,1,2,3)$, $H_{(F,B)} = (1,1,1,1)$: both alternatives are incomparable. Though *B* is ranked better by 3 out of 4 alternatives, it is ranked worse than *F* by the most important criterion, see also previous Consequences of Definitions 3a-b.
- ...

After all 15 pairs were compared; the following 12 dominance relations were obtained:

$$A \succ B,\ A \succ C,\ A \succ D,\ A \succ E,\ A \succ F,\ C \succ B,$$

$$C \succ E,\ C \succ F,\ D \succ C,\ D \succ F,\ E \succ B,\ E \succ F\ ,.$$

The final overall ranking of all alternatives have to be consistent with these 12 relations. Three remaining pairs were incomparable, namely: *B*-*D*, *B*-*F* and *D*-*E*.

The final overall rankings are:
(A,D,C,E,B,F) and (A,D,C,E,F,B).

Hence, the best (the most suitable) textbook is *A*.

As it would be interesting to compare this result with some other method, by Borda-Kendall's method of marks one obtains the final ranking (A,C,D,B,E,F), see Table 3. The main difference between both methods is in ranking of alternatives *B*, *C* and *D*.

Table 3. Rankings of textbooks with regard to Borda-Kendall's method of marks.

Overall ranking	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**
Alternative	A	C	D	B	E	F
Sum of points	8	12	13	15	16	20

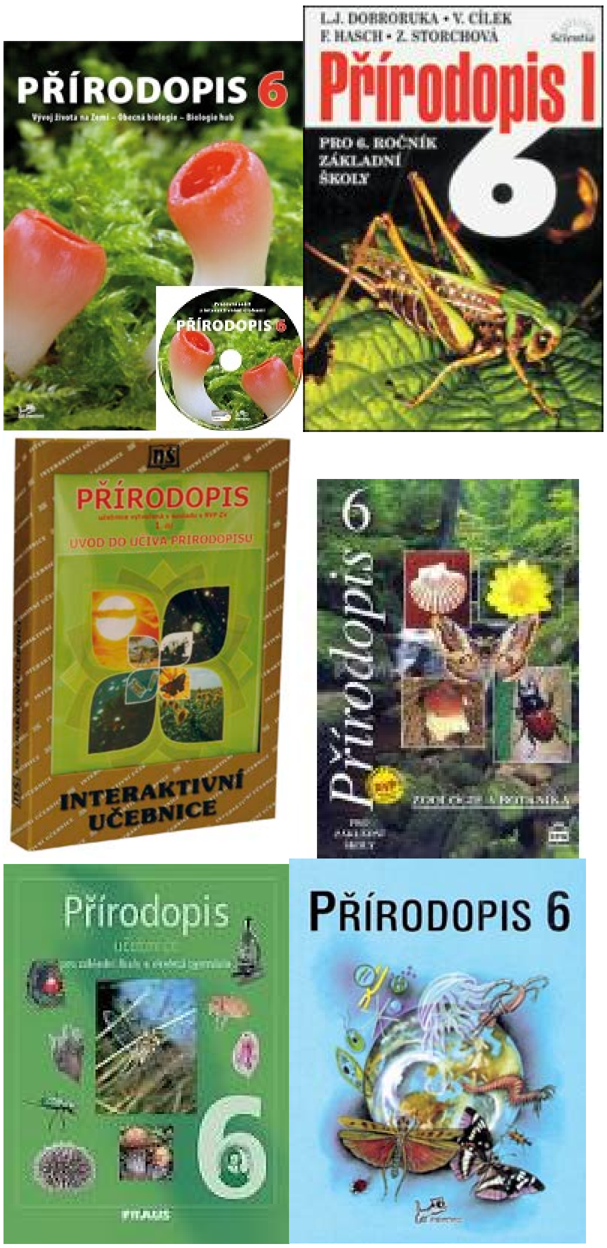

Figure 1. Elementary science textbooks from *A* to *F* for the 6th grade pupils. 'Přírodopis' means 'natural history'. This subject is taught at elementary level before science subjects such as physics, biology or chemistry.

V. Modifications and Extensions of the Proposed Approach

The method described in previous sections can be extended to include ties or uncertainty (*intensity of preference*) between alternatives:

- *Ties among alternatives*: When a decision maker is not sure whether an alternative *A* is better than an alternative *B* or vice versa, than he/she can assign both alternatives 0.5 points. In such a case index-vectors are not binary any more as they can include values 0.5 as well. But the MCDM-ORCA method can be used either. From index-vectors cumulative index-vectors are derived, and the dominance relation (1) is used in the same manner as before to acquire the final ranking of alternatives.

- *Uncertainty of preferences*: The approach above can be further generalized to include uncertainty or intensity of preference. If $p_{j,k} \in [0,1]$ is the intensity of preference of an alternative *J* over an alternative *K* from a set of alternatives *A*, and $p_{k,j} \in [0,1]$ expresses intensity of preference of an alternative *K* over *J*, and $p_{k,j} + p_{j,k} = 1$ is satisfied for all *j*, *k*,, then the relation $p: A \times A \to [0,1]$ is called the *fuzzy preference relation*, see [18] or [19]. Index-vectors now include values from the interval $[0,1]$, and the method can be used as before again, though it wouldn't be ordinal any more.

- *Ties between two criteria*: If two criteria C_j and C_k are tied in terms of their importance, then an alternative *A* dominates an alternative *B* iff *A* dominates *B* for both $C_j \succ C_k$ and $C_k \succ C_j$ cases (the result is not dependent on criteria ranking).

- *Ties among all criteria (the same weights of all criteria)*: if all criteria are considered equal and are not ranked from the most important to the least important, then the method would be converted into classic social choice procedures. However, without ranking of criteria it is not possible to construct index-vectors and cumulative index-vectors from Definitions 1 and 2 respectively required by the dominance relation (1). Nevertheless, when all criteria are of the same importance, it is possible to consider all possible permutations of criteria, and to proceed by ORCA for each and every such permutation to determine a dominance of a given pair of alternatives, if the dominance relation (1) is modified appropriately: an alternative A dominates an alternative B, if and only if the dominance relation (1) holds for every permutation of criteria. But, of course, such a method would be excessively time demanding

Example 4. Consider five criteria (from the most important C_1 to the least important C_5) and two alternatives: *A* and *B*. An alternative *A* is ranked better than *B* by criteria C_1 and C_2, the same as *B* by a criterion C_3, and worse than *B* by remaining criteria. Then:

$$I_{(A,B)} = (1,1,0.5,0,0)\,,\ I_{(B,A)} = (0,0,0.5,1,1)$$

$$H_{(A,B)} = (1,2,2.5,2.5,2.5)\,,\ H_{(B,A)} = (0,0,0.5,1.5,2.5)\,.$$

According to the dominance relation (1) an alternative *A* dominates *B*.

Example 5. Consider four criteria (from the most important C_1 to the least important C_4) and two alternatives: *A* and *B*. Intensities of preferences with regard to given criteria are as follows:

C_1: $p_{A,B} = 0.7$,

C_2: $p_{A,B} = 0.4$,

C_3: $p_{A,B} = 0.8$,

C_4: $p_{A,B} = 0.3$.

Then: $I_{(A,B)} = (0.7,0.4,0.8,0.3)$, $I_{(B,A)} = (0.3,0.6,0.2,0.7)$,

$H_{(A,B)} = (0.7,1.1,1.9,2.2)$, $H_{(B,A)} = (0.3,0.9,1.1,1.8)$.

According to the dominance relation (1) an alternative *A* dominates *B*.

Example 6. Consider four criteria, where C_1 is the most important, C_4 the least important, and C_2 and C_3 are equally important, as a decision maker was not able to decide between them. There are two alternatives *A* and *B* ranked by all criteria as follows:

C_1: (A,B),

C_2: (B,A),

C_3: (A,B),

C_4: (B,A).

Then for importance of criteria $C_1 \succ C_2 \succ C_3 \succ C_4$ we obtain: $I_{(A,B)} = (1,0,1,0)$, $I_{(B,A)} = (0,1,0,1)$, $H_{(A,B)} = (1,1,2,2)$, $H_{(B,A)} = (0,1,1,2)$, so *A* dominates *B*.

But we must examine also the second possible ranking of criteria $C_1 \succ C_3 \succ C_2 \succ C_4$, and in this case we have: $I_{(A,B)} = (1,1,0,0)$, $I_{(B,A)} = (0,0,1,1)$, $H_{(A,B)} = (1,2,2,2)$, $H_{(B,A)} = (0,0,1,2)$.

Hence *A* dominates *B* again. Thus we can conclude that no matter of how criteria C_2 and C_3 are ranked, an alternative *A* dominates an alternative *B*.

VI. Conclusions

The aim of the article was to demonstrate a new method for multi-criteria decision making with ordinal rankings of criteria and alternatives (MCDM-ORCA). The proposed method is computationally simple, natural and intuitive, and can be used for a solution of many real-world problems dealing with ordinal preferences of decision makers. For example it can be used for a selection of the most appropriate employee for a given job (where applicant's salary expectations is the most important criterion, previous work experience as the second one, knowledge and skills as the third one, etc.), the most suitable car (with criteria such as price, consumption, safety, equipment, etc.), a piece of real estate (where a location, price, infrastructure, date of a construction, etc. matters), shares, etc.

In the paper the use of the method was demonstrated on the selection of the most appropriate textbook for elementary science education, which is a common teachers' problem in the Czech Republic.

MCDM-ORCA method can be considered a modification to the social choice procedures, because criteria can be regarded as individual voters, but with a different importance (in the social choice theory it is assumed that votes of all individuals have the same weight). Also, it was demonstrated that the method satisfies some reasonable criteria required for the social choice procedure, such as Pareto efficiency, independence of irrelevant alternatives or monotonicity.

Further research may focus on problems with incomplete rankings or problems with interdependent criteria, also a comparison with cardinal methods or an extension to a group decision making would be interesting.

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References

[1] T. L. Saaty, "*Analytic Hierarchy Process,*". McGraw-Hill, New York, 1980

[2] T. L. Saaty, "*Decision Making with Dependence and Feedback: the Analytic Network Process,*" RWS Publications, Pittsburgh, 1996.

[3] J. C. Borda, "M´emoire sur les ´elections au scrutin," In: *Histoire de l'Acad´emie Royale des Sciences*. Paris, 1781.

[4] M. J. A. N. Condorcet, "*Essai sur L´Application de L´Analyse a la Probabilite des Decisions Rendues a la Pluralite des Voix*," Paris, 1785.

[5] M. Kendall, "*Rank Correlation Methods*," Hafner, New York, NY, USA, 3rd edition, 1962.

[6] W. D. Cook, M. Kress, "A multiple criteria decision model with ordinal preference data," *European Journal of Operational Research*, 1991, Vol. 54, pp. 191-198.

[7] W. D. Cook, J. Doyle, R. Green, M. Kress, "Multiple criteria modelling and ordinal data: Evaluation in terms of subsets of criteria," *European Journal of Operations Research* 1997 (98), pp. 602-609.

[8] W. D. Cook, "Distance-Based and Ad Hoc Consensus Models in Ordinal Preference Ranking," *European Journal of Operational Research* 2006 (172), pp. 369-385.

[9] M. Tavana, F. Lopinto, J.W. Smither, "A Hybrid Distance-Based Ideal-Seeking Consensus Ranking Model," *Journal of Applied Mathematics and Decision Sciences*, 2007, 18 pages.

[10] J. Mazurek, "The Fuzzy Permutation Model for the Ordinal Ranking Problem Solution," In: *Proceedings of the 29th International Conference on Mathematical Methods in Economics 2011*. Janská Dolina, Slovakia, 2011, pp. 462-467.

[11] J. Mazurek, “The ordinal consensus ranking problem with uncertain rankings,“ *Proceedings of the 30th international conference Mathematical methods in Economics 2012*, Karviná: Silesian University in Opava, School of Business Administration in Karvina, 2012, pp. 552-557.

[12] A. D. Taylor, A. M. Pacelli, *“Mathematics and Politics – Strategy, Voting, Power and Proof*, “ second ed. Springer, 2008.

[13] K. Arrow, “Social Choice and Individual Values,“ 1951.

[14] A. Gibbard, "Manipulation of voting schemes: a general result". *Econometrica* **41** (4), 1973, pp. 587–601.

[15] M. A. Satterthwaite, "Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions". *Journal of Economic Theory* 10, 1975, pp. 187–217.

[16] O. K. May, “A set of independent necessary and sufficient conditions for simple majority decisions,“ *Econometrica*, 1952, Vol. 20, Issue 4, pp. 680–684.

[17] K. Nakamura, "The vetoers in a simple game with ordinal preferences". *International Journal of Game Theory* 8, 1979, pp. 55–50.

[18] G. Klir, T. A. Folger, “*Fuzzy Sets, Uncertainty, and Information*, “ State University of New York, 1988.

[19] S. A. Orlovsky, "Decision-making with a fuzzy preference relation“, Fuzzy Sets and Systems,, Vo. 1, No. 3, 1978, pp. 155–167