On the use of contingent claims in portfolio selection problems

M. Cassader, S. Caglio, S. Ortobelli, L., V. Caviezel

***Abstract*—**In this paper we propose some portfolio selection models with contingent claims to invest either in the fixed income market or in the stock option market. Firstly, we describe a possible solution of the portfolio choice problem in the fixed income market taking into account the default risk. With this purpose, we consider CDSs contracts to hedge the default risk of investments in bonds. Secondly, we use European options in two distinct portfolio problems: in a reward-risk portfolio framework, to hedge the underlying portfolio risk of some stock indexes. Since we use a large number of trading European option written on principal international stock indexes, we discuss how to reduce the dimensionality of the large-scale portfolio problems taking into account the liquidity of the options. Finally, we propose an ex post empirical analysis of different portfolio models with contingent claims.

***Keywords*—**contingent claims, credit default swaps, default risk, hedge strategy, performance strategy.

I. Introduction

THE objective of this paper is twofold. We discuss and we evaluate portfolio strategies using contingent claims in two different markets: the fixed income market and the stock option market. In the fixed income market, we describe the classical strategy based on the immunization principles and we proposed strategies that use credit default swaps (CDSs) to hedge the default risk. In the stock option market, we describe portfolio strategies with only options either to hedge the market risk of some stock indexes or for speculative scopes.

In the last decade, a deep process of transformation and innovation in which the financial engineering has introduced and developed new financial instruments and markets has characterized the financial international system. Among new financial contracts, we analyze the CDSs and the options applied in different markets (fixed income market and stock option market). The CDSs allow transferring the risk of insolvency. In particular, a subject that has a credit exposure towards a counterpart considered unreliable transfer the risk of insolvency to another operator willing to assume this risk. Whereas the options have the aim to hedge the downside risk of stocks or indexes, they also used to speculate in volatile markets.

In this paper, when we operate on the fixed income market, we discuss how to avoid the default risk. Recall that Fisher and Weil 1971 [1], Redington 1952 [2] and De Felice 1995 [3] discuss and compare the choices of investors who immunize their portfolio by the price risk without accounting the credit risk. In order to account the default risk in portfolio strategies, we first describe the use of credit default swaps. Secondly, we propose a portfolio empirical analysis where we evaluate the ex post wealth sample path using the classic immunization theory. In particular, we propose the construction of a portfolio by bonds and CDSs. Our aim is to observe how the maximization of the future wealth held by the investor changes when the default risk of the underlying companies is considered. Moreover, we analyze the influence of credit risk measured through the estimation of the implicit probability of default (which is extrapolated from the credit default swaps quotations). Therefore, this methodology does not consider only the price risk in the fixed income portfolio selection, since we also take into account a portfolio of CDSs that hedges the default risk of the portfolio of bonds (see Caglio and Ortobelli [4]). Doing so, the overall portfolio of bonds and CDSs is hedged from both the random additive shifts of the interest rates curve and the possible default of a firm whose bonds are in the portfolio.

Several studies (see, among others, [5]-[8]) deal the portfolio selection problem with contingent claims. Typically, we distinguish three categories of portfolio models in the stock option market: a) portfolio selection among options, b) portfolio selection based on classical option strategies, c) portfolio selection to hedge the global risk exposure.

Blomvall and Lindberg [9], have shown that the efficient market hypothesis (see also [10]) is not generally satisfied when call and put options are used in portfolio problems, in particular when liquidity constraints are not considered. They discuss a scenario generation approach in the Black-Scholes-Merton framework. On the one hand, the classical theory presents Black-Scholes-Merton pricing model and the geometrics Brownian motions as instruments to make choices simulating the future trend of the assets. However, these underlying distributional assumptions are not able to describe correctly the log returns behavior as argued by several papers in the recent literature (see [11], Angelelli et al. [12], Iaquinta et al.[13]). On the other hand, it is well known (see [14]) that

M. Cassader is with the Department SAEMQ, University of Bergamo, Bergamo, 24127 ITA (e-mail: marco.cassader@unibg.it).

S. Caglio is with the Department SAEMQ, University of Bergamo, Bergamo, 24127 ITA (e-mail: s.caglio@studenti.unibg.it).

S. Ortobelli, L., is with the Department SAEMQ, University of Bergamo, Bergamo, 24127 ITA and Department of Finance VŠB-TU of Ostrava, Ostrava, 70121 CZE (corresponding author to provide phone: 0039-0352052564; e-mail: sergio.ortobelli@unibg.it).

V. Caviezel is with the Department SAEMQ, University of Bergamo, Bergamo, 24127 ITA (e-mail: valeria.caviezel@unibg.it).

log returns are not Gaussian distributed (as in Black-Scholes-Merton model). Thus, we could expect better results with more realistic distributional assumptions. Moreover, Topaloglu, et al. 2011 [15] have shown how to select international hedged portfolios using option strategies in a stochastic optimization framework. Furthermore Ahn, et al. 1999 [16] and Annaert, et al. 2007 [17] have shown some methods to elaborate a formula for determining the optimal strike price that minimizes the Value at Risk. Doing so, they prove the inefficiency of a perfect hedging in term of returns.

In this paper, when we operate on the stock option market, we use trading prices of European options with long maturity in portfolio selection problems without assuming a fixed distributional assumption on the underlying (see Cassader et al.[18]). We discuss and apply a principal component analysis (PCA) to reduce the dimensionality of the large-scale portfolio problem. Moreover, we deal and examine the liquidity problem of several options traded in the market and we propose proper constraints in the optimization problem. Finally, we propose an *ex-post* analysis where we compare the *ex-post* wealth obtained maximizing weekly either the Rachev ratio (see [19]) or the classical Sharpe ratio (see [20]).

In Section 2, we describe the credit risk market. In section 3 we introduce the optimization problem and we discuss the methodology used to select optimal hedged portfolios in both markets (fixed income and stock option markets). Section 3 examines, presents and discusses the liquidity problem and the results of the *ex-post* analysis. In the last section we briefly summarize the results.

II. Credit risk market and credit default swaps market

The consequence of the economic crisis of the financial international system is a generalized worsening of the creditworthiness of sovereign and private issuers. Therefore, to manage the credit risk has become the principal goal of risk management institutes. In order to evaluate the possible default loss of a given portfolio of n securities, we should consider the correlation ρ between the default events of the n securities that can be defined as the trend of two issuers to fail at the same time. In the following we use these definition to deal this problem:

1) *Exposure at Default* (EAD) is the estimated value of the loan in the event of default.
2) *Loss Given Default* (LGD) is an estimate of the total loss of the lender in the event of default by the counterpart and it is summarized by the formula:

$$LGD = 1 - Recovery\ Rate\ (RR) \qquad (1)$$

3) *Probability of Default* (PD) is the probability of bankrupt of the underlying firm. In this framework we assume that the probabilities of default among the American companies are independent and then we underestimate the credit risk.

Therefore, if we consider that the Exposure at Default and the Loss Given Default are constant and independent over the entire range of evaluation, the portfolio credit risk is given by the Unexpected Loss:

$$\mathrm{UL}_\rho = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{EAD_i EAD_j LGD_i LGD_j}\rho\sqrt{\mathrm{PD_i}(1-\mathrm{PD_i})\mathrm{PD_j}(1-\mathrm{PD_j})}}$$

The generalized worsening of the creditworthiness has forced the institutional investors to transfer the risk to another subject without selling the underlying asset through the credit default swaps negotiations. CDSs are financial instruments traded over the counter (OTC) markets.

The credit default swap allows the holder of the bond to transfer to a third part the default risk. Technically, the CDSs are bilateral derivative contracts negotiated between the protection buyer and the protection seller. There is a third part in the contract represented by the reference entity that it is the issuer of the bonds. The credit event is characterized by the default of the issuer over a period of time which involves the obligation of the protection seller to pay the notional established. A credit default swap allows the protection buyer, through a periodic premium payment expressed in basis points, to transfer the default risk of reference entity to the protection seller. This operation has the purpose to transfer the risk of incurring in a loss caused by the non-repayment of obligation at maturity on the counterpart.

Since the mid-nineties, these instruments have had an exponential growth in terms of volumes traded and in February 2012 they have reached the amount of 32 000 billion of euro. Thus this is the reason to study and analyze this market with the aim to have some techniques to manage the high level of risk that it is presented in these contracts.

III. The portfolio optimization problems

In this section we discuss different portfolio models for different markets. First we investigate how to invest in a bond portfolio hedged by default risk using a proportional number Credit Default Swaps. Indeed the transaction of every bond involves the negotiation of CDS associated with the reference entity. Secondly we discuss the portfolio selection problem in the stock option market distinguishing two different models: the first to hedge the exposure on some stock indexes, the second for speculation strategies.

A. Portfolio selection in the fixed income market

The goal of the first part is the maximization of future wealth taking into account the default risk. Thus, we also verify whether the trading of credit risk contingent claims is a way to increase the investor's profits.

In this framework we use the daily premium prices (expressed in basis points) paid of 10 Credit Default Swaps written on 20 U.S. companies[1] (presented in the Dow Jones

[1]The bonds refers to the following US companies: 3M, Alcoa, AT&T, Boeing, Caterpillar, Coca-Cola, E.I.duPont de Nemours, General Electric,

Industrial Average Index in February 2012). The deadlines of CDS are the same for all companies (6 months, 1 year, 2 years, 3 years, 4 years, 5 years, 7 years, 10 years, 20 years and 30 years) in order to have a common basis to compare different probabilities of default. As suggested by Berndt et al. 2006 [21] and Collender 2008 [22], we further assume that:

1) The investor has a kind of "premium fund" to pay all the premiums of Credit Default Swaps. These payments are not included in the algorithm for the maximization of wealth.
2) The computation of the probability of default is based on the assumption of absence of a credit curve, i.e. the curve that associates each deadline with a different probability of default.
3) The premium payment is annually postponed.
4) Perfect matches between the time of payment of the premium and the time to default of the reference entity.
5) The CDS premium is calculated as the arithmetic mean of the CDS bid-ask premium.
6) The CDS premiums incorporate information related to the bankruptcy of the reference entity. The contracts were all determined according to the clause "no restructuring" proposed the International Swaps and Derivative Association (ISDA).
7) The recovery rate (RR) is constant for the entire CDS life and it is equal to 40% for all maturities and for all reference entities.
8) The risk-free rate is identified in the annual Treasury bill.
9) There is no counterpart risk since it is assumed that the protection seller is always solvent.

Thus, we have to determine the implied probability of default daily. This probability is associated with each reference entity for any different maturity of CDS contracts. Assuming a CDS with maturity t_n and notional equal to 1 in which the payment is annual and postponed. The premium leg (PL) can be described as the sum of the discounted payments of the protection buyer:

$$PL = \sum_{i=1}^{n}[K \cdot S(t_i) \cdot v(t_i)] \qquad (3)$$

where $S(t_i)$ represents the unconditional probability of survival calculated between 0 and t_i. Thus, $1 - S(t_i)$ is the probability of default, K is the premium payable by the protection buyer and it is considered constant each year and $v(t_i)$ is the discount factor.

Clearly, in a no arbitrage world, the CDS contract value is also equal to PL and the default payment leg, i.e., the payment due by the protection seller, is given by:

$$DP = \sum_{i=0}^{n-1} S(t_i)\lambda(t_i, t_{i+1})(1 - RR)v(t_{i+1}) \qquad (4)$$

IBM, Johnson & Johnson, Kraft Foods, JP Morgan Chase&Co., McDonald's, Merck & C, Pfizer, The Procter & Gamble, The Travelers, United Technologies, Verizon, Wal-Mart stores.

where $\lambda(t_i, t_{i+1})$ is the hazard rate, that is the conditional probability of default.

At the time of conclusion of the contract the CDS has always zero value since the contract must be fair and the premium leg is equal to the default payment leg. Therefore, by applying the equality $PL = DP$ we are able to determine every day the annual unconditional probability of default of each company starting from 01/01/2008 to 02/02/2012.

The evolution of the probability of default and the development of the interest rate risk are estimate by the definition of the contract value. Thus to solve the portfolio selection problem we suggest to optimize monthly three different functionals (namely, F_1, F_2, F_3) subject to the classic constraints of the Fisher and Weil theorem. Therefore we have three different optimization problems:

1) The first functional $F_{1,t}$ is defined as the estimated future value of the bond portfolio at time t plus the average of the CDS portfolio hedging (at the same time t), i.e.:

$$F_{1,t} = V_{B(t)} \cdot \left(1 + TRES_{B(t)}\right) + E\left(Ptf_{CDS(t)}\right) \qquad (5)$$

where $V_{B(t)} = [V_{1,t} \quad \cdots \quad V_{n,t}]$ is the vector of wealths invested in each bond at time t, the vector $TRES_{B(t)} = [TRES_{1,t} \quad \cdots \quad TRES_{n,t}]^T$ is the vector of the yields to maturity of all bonds at time t, while the last part of the formula represents the expected value of the CDS hedging portfolio.

2) In the second functional $F_{2,t}$ we introduce a risk measure defined by the standard deviation of the CDS portfolio computed at time t. Thus the functional $F_{2,t}$ is given by the estimated future value of the bond portfolio at time t plus the average of the CDS subtracted by the standard deviation of the CDS, i.e.:

$$F_2 = Max\left[V_{B(t)} \cdot \left(1 + TRES_{B(t)}\right) + E\left(Ptf_{CDS(t)}\right) - \sigma\left(Ptf_{CDS(t)}\right)\right] \qquad (6)$$

where the mean and the standard deviation are calculated according to the exponentially weighted moving average model (EWMA see [23]). This function reflects the behaviour of a risk-adverse investors. They prefer low volatility of returns in order to limit losses and an high level of certainty about the future evolution of the bond values.

3) With the third functional we consider an alternative risk measure of the CDS portfolio computed at time t. The functional $F_{3,t}$ is defined as the estimated future value of the bond portfolio at time t plus the average of the CDS portfolio hedging minus the Conditional Value at Risk (CVaR) (see [24]) calculated at the 5^{th} percentile, i.e.:

$$F_{3,t} = V_{B(t)} \cdot \left(1 + TRES_{B(t)}\right) + E\left(Ptf_{CDS(t)}\right) - CV@R_{5\%}\left(Ptf_{CDS(t)}\right) \qquad (7)$$

In this formulation we introduce a coherent risk measure to specify the maximum level of losses that the investor is able to support.

Therefore, we maximize monthly the above functionals F_1, F_2, F_3 with the following constraints (which are common for all portfolio problems in the fixed income market):

1) Fixed Macaulay modified duration (MD_{prtf}) of the portfolio of bonds, that is: $MD_{prtf} = k$. We vary the modified duration k (from 6.4 years till 13.05 years with step 0.35 years (see [25] and [26])) to account different levels of price risk.
2) In each recalibration step the initial wealth (W_t) should be equivalent to the total amount invested in bond and CDS portfolio, i.e.: $W_t = V_{B(t)} \cdot 1 + P_{CDS} \cdot 1$ where $P_{CDS(t)} = [P_{1,t} \ \cdots \ P_{n,t}]$ is the value of CDS investment.

This initial wealth can be also expressed as:

$$W_t = \overline{p_{B(t)}} \cdot \overline{w_{B(t)}} + \overline{p_{CDS(t)}} \cdot \overline{w_{CDS(t)}} \tag{8}$$

where p is the price and w is the weight in the portfolio.

Clearly the weights of CDS depend on the quantity invested in bonds of the *k-th* reference entity. The total hedge assumes that each bond is hedged by the CDS associated with the same reference entity. So the following equality holds:

$$\sum_{j \in k-th} w_{B_j} = \sum_{i \in k-th} w_{CDS_i} \tag{9}$$

We assume a starting initial wealth $W_0 = 100\,000$ euro and then we maximize the functionals monthly from January 2008 till January 2012. The methodology to recalibrate wealth periodically is the same discussed by Ortobelli and Angelelli 2009 [27].

B. Portfolio selection in the stock option market

In this section we describe two different portfolio models with contingent claims in the stock option market: in the first portfolio model we use options to hedge the downside risk of some stock indexes, in the second model we use options for speculation.

Typically, investors maximize a performance measure of their portfolio. There exist several performance strategies proposed in literature (see Cogneau and Hübner [28], [29]). In both models we optimize a performance measure applied to the portfolio of returns. In particular, for each optimization problem we use the Sharpe ratio and Rachev ratio as measures of performance.

The Rachev ratio [19] of a portfolio of gross returns X is defined as follows:

$$RR_{\alpha,\beta}(X) = \frac{CV@R_\beta(-X + r_b)}{CV@R_\alpha(X - r_b)} \tag{10}$$

where r_b is a benchmark gross return that we assume equal to 1 when a benchmark is not allowed (as in our empirical analysis),

$$CV@R_\beta(X) = \frac{1}{\beta}\int_0^\beta V@R_u(X)du \tag{11}$$

is the conditional value at risk of random variable X and $V@R_u(X) = -F_X^{-1}(u) = -inf\{x|P(X \le x) \ge u\}$ is the value at risk of the random variable X. The conditional value at risk $CV@R_\beta(X)$ is a coherent risk measure that is the opposite of the mean of the return portfolio losses below the percentile of its distribution. Optimizing this performance could give more local optima, thus we need an heuristic for global optimization to optimize this performance measure.

The Sharpe ratio [20] is defined as the ratio between expected value of gross return and its volatility. This performance measure is able to order properly risk averse investor's choices when the return distribution are Gaussian. The Sharpe ratio is given by:

$$SR(X) = \frac{E(X - r_b)}{\sigma_X} \tag{12}$$

where σ_X is the standard deviation of X.

The reward and risk measures in both performance functionals are computed assuming that the probability of historical observations follows the exponential weighted moving average model (EWMA, see [23]). Thus, we assign exponential probability to them, i.e., for example, considering 125 historical returns, the *k-th* observation (in increasing order time) has the following probability to be realized :

$$p_k = \lambda^{125-k} / \left(\sum_{j=1}^{125} \lambda^{125-k}\right) \tag{13}$$

for some $\lambda \in (0,1]$. In this way we give higher probability to the most recent observations and lower probability to the oldest ones (in our empirical analysis we assume $\lambda = 0.95$).

Clearly, the two portfolio models (for hedging purpose and for speculation purpose) are structured in a completely different way.

Portfolio selection for hedging purpose

In this section, we introduce a methodology for the optimization process with the objective to optimize the performance of an hedged portfolio of index. Let $x = [x_1, \dots, x_n]$ be the vector of percentages invested in each asset. Let us consider m_i put options on the *i-th* index for a total of $m = \sum_{i=1}^n m_i$ options. Clearly, the m_i put options written on the *i-th* index change for their exercise price. Suppose we invest the same percentage $y_{(i)}$ of wealth in each put option written on the *i-th* index (for $i = 1, \dots, n$), then the global percentage invested in put options written on the *i-th* index is $m_i y_{(i)}$.

When we use options for hedging strategies the number of bought options should be related to the portfolio components. Thus, we have to introduce an option constraint to hedge the market risk of a portfolio strategy. The new constraint is defined as follow. Let W be the wealth at a given time, then the amount invested in the *i-th* asset is $x_i W$. Therefore, the number of underlying contract is the following:

$$NC_i = \frac{x_i W}{s_i} \tag{14}$$

where S_i is the price of the underlying index. Since the number of index contracts must be equal to the number of contingent claims that we have bought, then to realize an hedging strategy the following equation holds:

$$\frac{x_i W}{s_i} = \frac{y_{(i)} W m_i}{\sum_{j=1}^{m_i} p_{i,j}} \tag{15}$$

where $p_{i,j}$ is the price of the *j-th* put option (among m_i options) written on *i-th* asset (i=1,...,n).. Therefore the weight $y_{(i)}$ of each put option on the *i-th* asset is:

$$y_{(i)} = \frac{x_i \sum_{j=1}^{m_i} p_{i,j}}{s_i m_i} \tag{16}$$

In order to find the optimal composition $x = [x_1, \dots, x_n]$ of percentages invested in each asset and the related percentages $y_{(i)}$ invested in options we solve the following portfolio problem:

$$\begin{cases} max_{x_1,\dots,x_n} f(x) \\ subject\ to \\ y_{(i)} = \frac{x_i \sum_{j=1}^{n_i} p_{i,j}}{s_i n_i} \\ \sum_{i=1}^{n} (x_i + m_i y_{(i)}) = 1 \\ 0 \le x_i \le 0.2 \end{cases} \tag{17}$$

where $f(x)$ is a performance measure applied to the whole portfolio $x'z + \sum_{i=1}^{n} \sum_{j=1}^{m_i} y_{(i)} s_{i,j}$ of option gross returns $s_{i,j}$ (gross return on the *j-th* option written on the *i-th* index) and stock indexes gross returns z_i (where $z = [z_1, \dots, z_n]$ is the vector of indexes gross returns). Clearly the whole portfolio $x'z + \sum_{i=1}^{n} \sum_{j=1}^{m_i} y_{(i)} s_{i,j}$ uniquely depends on the vector of weights x. The observation at time k (of option and indexes gross returns) are weighted with probability p_k given by (13). In this paper we suppose that is not allowed short selling ($0 \le x_i$) and that is not possible to invest more than 20% of the global wealth in a specific index $x_i \le 0.2$.

The implementation of the theoretical work explained before is developed with MatLab software and time series data were download from the Thompson Reuters Datastream. We use the *fmincon* function of Matlab for optimizing the Sharpe ratio and an heuristic for global optimization for optimizing the Rachev ratio (see [27]) .

Speculative portfolio selection in the stock option market

In this subsection, we deal the portfolio problem in the stock option market for speculative aims. Let $y = [y_1, \dots, y_m]$ be the vector of the single percentages invested in European options. Then we want to find the optimal portfolio of weights y invested in each asset that solve the portfolio problem:

$$\begin{cases} max_{y_1,\dots,y_m} f(y) \\ subject\ to \\ \sum_{i=1}^{m} y_i = 1 \\ 0 \le y_i \le 0.2 \end{cases} \tag{18}$$

where $f(y)$ is a proper performance functional applied to the portfolio of options $y's$ (here $s = [s_1, \dots, s_m]$ is the vector of option gross returns). Also in this section we suppose that short selling is not allowed ($0 \le y_i$) and that is not possible to invest more than 20% of the wealth in a specific option $y_i \le 0.2$.

The dimensionality problem is an important issue to manage in a huge portfolio of contingent claims. When the number of assets exceeds the number of observations, to get a good approximation of the portfolio input statistical measures, it is necessary to find the right trade-off between the number of observations and a statistical approximation of the historical series. In particular, we use two techniques to reduce the dimensionality of large scale portfolio problems: pre-selection and PCA. With pre-selection, only a limited number of assets is chosen before optimizing the portfolio.

Since in the optimization problem (18) we use either the Sharpe ratio or the Rachev ratio as performance measures for each optimization problem, we preselect the first γ (say 20 or 200) options that present the highest performance (Sharpe ratio or Rachev ratio).

When we preselect a large number of asset (say 200 as in our empirical analysis) we perform a PCA of the returns of the preselected returns in order to identify the few factors f_i with the highest variability. Therefore, for each optimization problem and at each recalibration time, we apply a PCA to the correlation matrix of the preselected assets to identify the first 12 components that explain the majority of the global variance. Subsequently, each series s_i ($i = 1, \dots, 200$) can be represented as a linear combination of 12 factors plus a small uncorrelated noise. Using a factor model, we approximate the preselected returns s_i as follows:

$$\begin{gathered} s_{i,k} = \alpha_i + \sum_{j=1}^{12} \beta_{i,j} f_{j,k} + \epsilon_{i,k} \\ i = 1, \dots, 200 \end{gathered} \tag{19}$$

where $s_{i,k}$ and $\epsilon_{i,k}$ are, respectively, gross returns and errors in the approximation of *i-th* asset at time t_k and $\alpha_i, \beta_{i,j}$ are the coefficients of the factor model. The randomness of the portfolio problem depends now on only 12 factors.

IV. AN EX-POST EMPIRICAL ANALYSIS WITH CONTINGENT CLAIM

In this section we show and discuss the portfolio selection results in the fixed income market and in the stock option market. In particular, we first analyze the ex post wealth obtained with the three different functionals introduced in section III applied to portfolio of bonds and CDSs. Secondly we propose an ex-post empirical analysis for two different portfolio models in the stock option market.

A. Practical portfolio selection in the fixed income market

We elaborate an ex-post comparison based on portfolio selection among 88 bonds issued by 20 U.S. companies included in the Dow Jones Industrial Average Index in February 2012. The portfolio selection considers also a portfolio of credit default swaps (10 for each US company) that hedge the default risk. The empirical analysis covers the credit risk crisis period (from January 2008 till February 2012).

We report the *ex-post* final wealth obtained maximizing functionals F_1, F_2 and F_3 in figures 1, 2, 3.

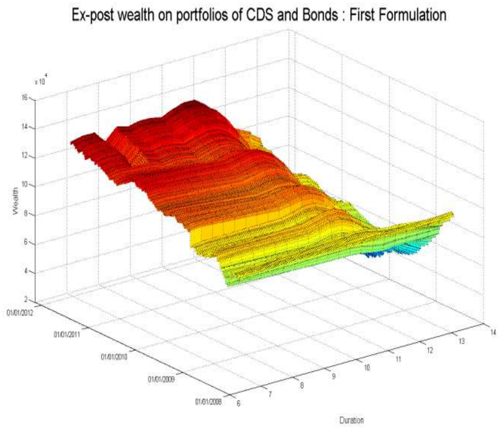

Fig. 1

Fig.1 reports the results obtained maximizing the first functional F_1. We observe that the wealth grows almost uniformly during the subprime crises for duration lower than 10 years. In particular for these duration levels in February 2012 we get an extra wealth of about 25% the initial wealth. For modified durations higher than 12 years we could loss more than 20% of the initial wealth.

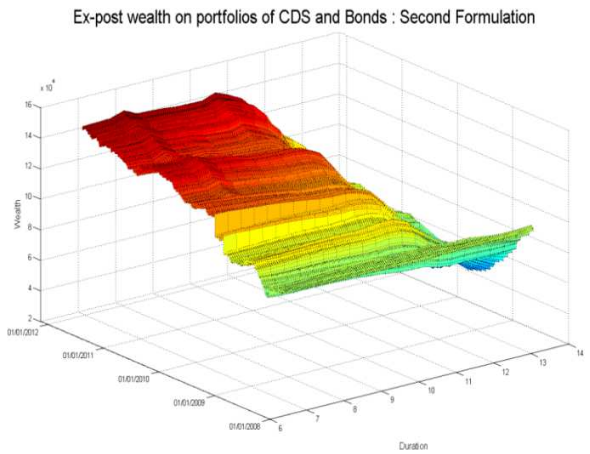

Fig. 2

Fig.2 reports the results obtained maximizing the second functional F_2. Even in this case we observe that the wealth grows for duration levels lower than 11 years. However, in February 2012 we get an extra wealth of about 40% the initial wealth. This improvement suggests that it makes sense to account the variability of CDSs prices in the portfolio problem. Moreover, even in this case for modified durations higher than 12 years we could loss more than 20% of the initial wealth.

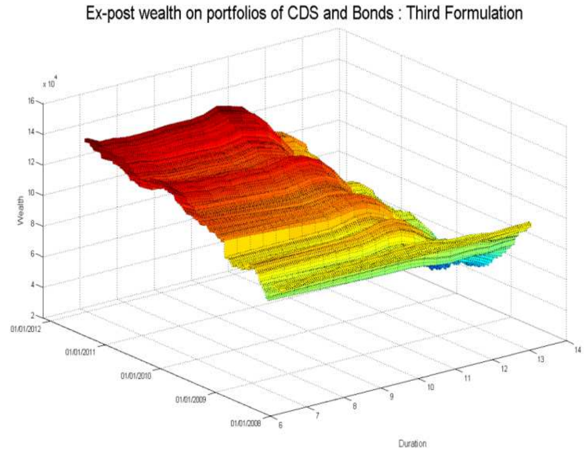

Fig. 3

Fig.3 reports the results obtained maximizing the functional F_3. This figure shows an improvement with respect to functional F_1 that confirms the importance to measure the risk of CDSs prices in portfolio selection problems. However, we do not get the same extra wealth of functional F_2 which the highest (we get only an extra wealth of about 30% the initial wealth). Moreover, the tendency to lose wealth for high levels of modified durations is still confirmed.

B. Practical portfolio selection in the stock option market

In this section we compare the *ex-post* wealth obtained maximizing two different portfolio performance measures (Sharpe ratio and Rachev ratio) in the stock option market. For Rachev ratio we use the parameters $\alpha = \beta = 2\%$. According to Section III we examine the portfolio problem by the point of view of different investors: hedgers and speculators.

An ex-post empirical analysis with hedging strategies

When we apply hedging strategies in the stock option we use only put options written on stock indexes during the period between June 2010 and December 2011. Using software Thompson Reuters Datastream we create a dataset composed by 87 European put options that cover a loss between 5% and 25%. The number of contingent claims to hedge a specific index is different from each other and it has a relevant consequence in term of efficiency. The 16 international stock indexes are: Austrian Traded Index, Cac 40 Index, Dax Index, Dow Jones Industrial Average Index, Euro Stoxx 50 Index, Euro Stoxx Banks, Euro Stoxx Media, Ftse

100 Index, Ftse Mib Index, Hang Seng Index, Ibex 35 Index, Nikkei 225 Index, S&P 500 Index, Stoxx Europe 50 Index, Stoxx Europe 600 Banks, Swiss Market Index.

We optimize the portfolio weekly (every 5 trading days) from 24th June 2010. Moreover, we recalibrate the portfolio daily maintaining constant the optimal portfolio composition computed weekly. The pricing does not depend on the limits of Black-Scholes-Merton formula since future choices are not affected by simulation with the geometric Brownian motion. At each recalibration time we consider a time series data of 125 observations of returns. Optimization process is repeated every 5 trading days for 74 times. During the analysis we used gross return and we start with an initial wealth equal to one (i.e., $W_{t_0} = \sum_{i=1}^{n}(x_i + m_i y_{(i)}) = 1$). At the *k-th* optimization ($k = 0, 1, 2, \dots, 74$), the following two steps are performed to compute the *ex-post* final wealth:

1) Determine the optimal portfolio composition x_k^* and $y_{(k)}^*$ that have the proportions invested in each asset during the period $[t_k; t_{k+1} = t_k + 5]$.
2) we recalibrate daily the portfolio in order to maintain constant the proportions invested in each asset. The *ex-post* final wealth is given by:

$$W_{t_{k+1}} = W_{t_k}\Big(\prod_{s=1}^{5} x_k^{*\prime} z_{t_k+s}^{(ex-post)} + \sum_{i=1}^{n}\sum_{j=1}^{m_i} y_{(i)}^{*} s_{i,j,t_k+s}^{(ex-post)}\Big) \quad (20)$$

where:
$s_{i,j,t_k+s}^{(ex-post)}$ and $z_{t_k+s}^{(ex-post)} = \left[z_{1,t_k+s}^{(ex-post)}, \dots, z_{n,t_k+s}^{(ex-post)}\right]'$ are respectively the gross return on the *j-th* option written on the *i-th* index and the vector of observed daily gross returns both valued during the period $[t_k + s - 1;\ t_k + s]$.

The two steps are repeated for all the optimization problems and for all available observations.

The results of a hedge portfolio strategies are shown in the Fig.4. We observe that the wealth sample path obtained investing only in the stock indexes dominates the analogous wealth sample paths obtained investing in hedging strategies. Thus the selection using Sharpe ratio and Rachev ratio perform better without introducing contingent claims. This is a consequence of the static dataset composed by options with a strike price that it could be very far from the value of the underlying index. Moreover we are hedging a financial instrument with an high level of diversification, with low volatility and then a hedging strategy is logically underperforming.

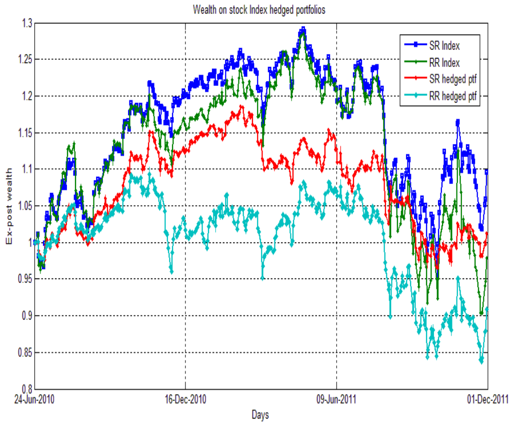

Fig. 4

An ex-post empirical analysis with speculative strategies

When we consider a large scale portfolio problem with put and call options, we have to account of the following two problems: the liquidity of the instruments traded and the dimensionality of the portfolio randomness.

To deal these problems we first introduce liquidity constraints to create a model based on real transactions reducing possibility to invest in illiquid assets. Secondly, we analyze and discuss portfolio strategies based on 20 or 200 preselected options.

In order to limit the liquidity risk we introduce 4 liquidity constraints which are:

1) *Minimum liquidity*. We remove options which do not show historical transactions (zero volume) whose gross returns are constantly equal to one for several days.
2) *Volume*. We require that is possible to purchase and sell only the options that present in the last 10 trading days some volume transactions.
3) *Jumps*. We remove time series that in the past have presented big jumps without any transactions.
4) *Range of strike price.* We use only the options with a strike price in a range of 25% up or down the underlying index

These liquidity constraints reduce the dimensionality of optimization problem and selection but they guarantee the possibility to negotiate the options presented in the investor's portfolio .

Using we create a dataset of 389 European call put options in dollar currency with the same maturity, December 2011, and made up by a time series data of 500 days between 31 December 2009 and 30 November 2011. The data are taken from Thompson Reuters Datastream. We use a moving window of 125 daily observations to evaluate the portfolio performance. As for the previous *ex-post* analysis, we optimize the portfolio every 5 trading days for 74 times. Moreover three decisional steps are performed at each recalibration time starting from 24th June 2010, i.e., :

1) Preselect the first γ assets (where γ is equal to 20 or 200) with the highest performance measure (Sharpe or Rachev). When the number of preselected assets is 200, we apply the PCA component to the correlation matrix of the preselected assets. Then we regress the returns on the first 12 principal components to approximate the variability of the preselected returns.
2) Determine the optimal portfolio composition.
3) During the period $[t_k; t_{k+1}]$ (where $t_{k+1} = t_k + 5$) we have adopted two strategies: the first one recalibrates daily the portfolio maintaining constant the proportions invested in each asset for all period $[t_k; t_{k+1}]$ (as in the previous portfolio model) and the second recalibrating the portfolio at each optimization time t_k. The *ex-post* final wealth is given by:

$$W_{t_{k+1}} = W_{t_k} \prod_{t=1}^{5} y_k^{*'} s_{t_{k+1}}^{(ex-post)} \qquad (21)$$

if we recalibrate daily the portfolio; or by:

$$W_{t_{k+1}} = W_{t_k} y_k^{*'} \prod_{t=1}^{5} s_{t_{k+1}}^{(ex-post)} \qquad (22)$$

if we maintain the same composition of portfolio for 5 days; where:

$$s_{t_{k+1}}^{(ex-post)} = \left[s_{1,t_k+t}^{(ex-post)}, \dots, s_{n,t_k+t}^{(ex-post)} \right]' \qquad (23)$$

it is a vector of observed daily gross returns for the period $[t_k + t - 1;\ t_k + t]$.

The three steps are repeated for all the optimization problems for all available observations. To evaluate the impact of pre-selection and the portfolio strategies, we show and compare the *ex-post* wealth in every different case.

In this *ex-post* empirical comparison we evaluate the portfolio selection based either on 20 preselected options or on 200 preselected options. Fig. 5 reports the *ex-post* wealth obtained maximizing the Sharpe ratio and Rachev ratio strategies with a daily or weekly recalibration of the optimal portfolio's composition when are used 20 preselected asset at each recalibration time. The analysis of wealth with a daily recalibration of portfolio's composition shows many interesting results. We observe that the ex-post final wealth changes essentially during the last period of observation since the options are near to the maturity and the options' volatility enormously increases.

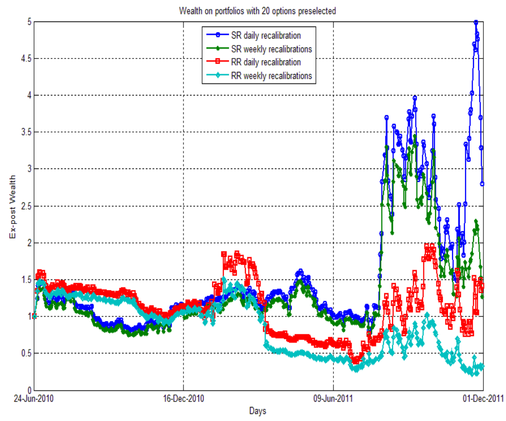

Fig. 5

During the first period of the ex-post analysis (till half of the observed period) the wealth processes are stable and there are no substantial differences among the strategies. During the summer 2010 the Rachev type strategies change completely the portfolio composition and after an increment at the beginning follow a loss greater than 50%. Moreover, in these first two ex-post periods the frequency of recalibration is not an important factor which may influence the final wealth.

During the summer of 2011 the volatility of wealth increases since the maturity of options is closed and also the volatility on the market increases with credit risk crisis. More and more options is been included in the process of pre-selection and we get the best results using strategies which presented a daily recalibration in the portfolio composition.

In the second *ex-post* empirical comparison, we use 200 preselected options and we have to introduce a PCA in the algorithm to obtain a more realistic statistical approximation. Fig. 6 reports the *ex-post* wealth obtained optimizing the Sharpe and Rachev performance measures.

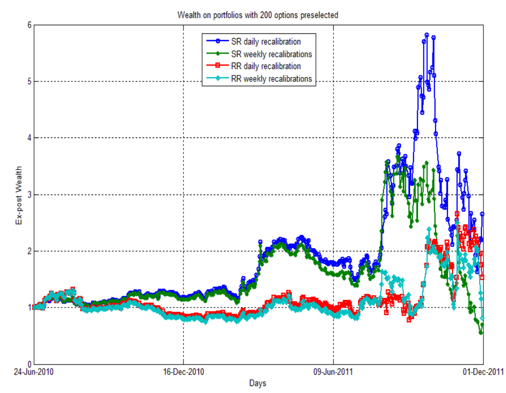

Fig. 6

These results essentially confirm those obtained with 20 preselected options. In particular, during the last 5 months of *ex-post* analysis, we observe that the portfolio volatility increases independently from the number of preselected options.

Even in this case the *ex-post* wealth obtained maximizing the Sharpe ratio dominates the one obtained with the Rachev performance. Moreover, we can justify the differences between Rachev and Sharpe *ex-post* wealth, taking into account that Rachev ratio is consistent with the choices of non-satiable investors who are neither risk averse nor risk lover, differently by the Sharpe ratio that is consistent with the choices of non-satiable risk averse investors. Then, intuitively, having a more prudent approach in the stock option market can be more productive.

Increasing dimensionality of pre-selection both strategies show an improvement of wealth compared to the previous analysis. In particular, Sharpe ratio already double the initial wealth during the spring of 2011, while the strategies with Rachev ratio remain stable in the same period of time but they do not shown losses. Thus, the optimization process appears more efficient with the introduction of a PCA.

Table 1 summarize the final wealth for Sharpe ratio and Rachev ratio in the different pre-selections.

Table 1

Final Wealth	Recalibration	
	Daily	Weekly
Sharpe 20	2,7986	1,2625
Rachev 20	1,4067	0,3288
Sharpe 200	2,6552	0,6990
Rachev 200	1,5245	0,8097

V. Conclusion

This paper studies the portfolio problem when are used different type of contingent claims. In particular, we analyze the *ex-post* wealth obtained with portfolio strategies that account either the default risk of the reference entities issuer of some bonds or the risk of value losses of some stock indexes. Thus, we deal the portfolio problem in two different markets: the fixed income market and the stock option market.

In the fixed income market, we deal the selection problem of a portfolio of bonds hedged by the default risk. According to this aim, we suggest to consider in the portfolio optimization a proportional number of CDSs. In order to better stress the portfolio model we analyze the *ex-post* wealth during the subprime mortgage crisis, characterized by a climate of pessimism and uncertainty about the future course of the securities. In particular, we examine the *ex-post* performance of three different reward-risk strategies applied to portfolio of bonds and CDSs. We observe the worst *ex-post* final wealth when high levels of modified duration hold in the portfolio optimization problem since the price risk has deeper impact in the decisional process. While the *ex-post* wealth is promising for smaller modified durations of all the portfolio strategies.

In the stock option market, the *ex-post* portfolio selection analysis shows the imperfections of contingent claim market between June 2010 and December 2011. In an efficient market we could not have enormous returns. However, the portfolio composed with European options written on principal stock indexes performed too high returns. Thus, we deduce that the volatility of stock option market is too high to support the efficient market assumption. Moreover, when options are used to hedge the portfolio losses we do not get so high levels of the *ex-post* final wealth. We also observe that the daily recalibration of the portfolio's composition gives the highest final wealth reducing also the portfolio losses. The strategy based on the maximization of the Sharpe ratio gives the best performances.

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Marco Cassader is a first year PhD student in Applied Mathematics and Operational Research at University of Bergamo, Italy. He has a Master degree in Management, Finance and International Business at University of Bergamo. His field of research is quantitative finance, in particular portfolio selection problem and contingent claims strategies.

Silvia Caglio has been a student of University of Bergamo. She is applying for a PhD in Mathematical Finance. She has a Master of Science in Management, Finance and International Business at University of Bergamo discussing a thesis entitled: Covered strategy in a bond portfolio selection.

Sergio Ortobelli Lozza is an associate professor in Mathematical Finance at the University of Bergamo. He is also visiting Professor at VSB TU Ostrava Department of Finance, Czech Republic. He holds a Ph.D. in Computational Methods for Financial and Economic Forecasting and Decisions from the University of Bergamo. He taught numerous courses at the Universities of Bergamo, Calabria and Milan, including basic and advanced calculus, measure theory, stochastic processes, portfolio theory, and advanced mathematical finance. His research, published in various academic journals in mathematics and finance, focuses on the application of probability theory and operational research to portfolio theory, risk management, and option theory.

Valeria Caviezel is an assistant professor at the Department of Management, Economics and Quantitative Methods of the University of Bergamo (Italy). She earned a master degree in Economics and Statistics from the University of Milan and then she deepened her studies at the University of Louvain-la-Neuve (Belgium). She is interested in researches about the Item Response Theory and in particular the Rasch analysis, applied to the financial, educational and psychological fields. She teaches probability and statistics.