Analysis and forecast of indicators of industrial production using regression, correlation and cluster analysis

Peter Poór, Gabriela Ižaríková, Jana Halčinová, Michal Šimon

Abstract—This article deals with the indicators of industrial production. Industry and related services have an impact on the development of the whole country and also the development of individual regions, source of jobs. Manufacturing is categorized into classification of economic activities called NACE, which is comparable on international level. In this article, we devote this indicator of industrial production in Slovakia, the number of persons employed, monthly wage labor productivity from revenues from own services and products, revenues from own performances and goods in industrial production. The development of these indicators is described by regression and correlation analysis. To estimate the coefficients of the regression function is the method of least squares. Using appropriate regression functions is performed forecast indicators of industrial production in 2014. The relationship between the various indicators is explained by the correlation matrix. Similarity relations within the indicators examined partially and collectively are expressed using cluster analysis by year and category of industrial production.

Keywords—industrial production, classification of industrial production, economic indicators, prediction.

In conclusion, we would like to express thanks for the support of the projects SGS-2012-063 titled “Integrated design of manufacturing system as metaproduct with a multidisciplinary approach and with using elements of virtual reality” and project NEXLIZ – CZ.1.07/2.3.00/30.0038, which is cofinanced by the European Social Fund and the state budget of the Czech Republic and This article was created by implementation of the grant project VEGA no. 1/0102/11. Methods and techniques of experimental modeling of in-house manufacturing and non-manufacturing processes.

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I. INTRODUCTION

Between the constantly accelerating pace of innovation and technological development the industry must respond flexibly to new requirements. Industry represents an important sector in the world, as well as in the Slovak economy. Its an important part of the industrial production. Manufacturing is part of material production-oriented extraction of minerals and fuels, production and distribution of all kinds of energy, machine processing of extracted materials and derived agricultural products, various repair activities and selected services. Industry and the related services affect the development of the whole country and also the development of individual regions. It is also a source of job opportunities. The impact of economic, social, technical and environmental factors cause in manufacturing various changes. Until 1989 was in every region of at least one supporting industrial plant but after 1989 primarily reflected the transformation of the industry in changing its sector, ownership, size and spatial structure. The regional distribution of industrial production shows that the critical capacity of manufacturing in terms of production, sales and share of employment are concentrated mainly in Western Slovakia.

Table 1 Classification of industrial production by SK NACE

<table>
<thead>
<tr>
<th>C Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA Manufacture of food products, Manufacture of beverages, Manufacture of tobacco products</td>
</tr>
<tr>
<td>CB Manufacture of textiles, Manufacture of wearing apparel, Manufacture of leather and related products</td>
</tr>
<tr>
<td>CC Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials, Manufacture of paper and paper products</td>
</tr>
<tr>
<td>CD Manufacture of coke and refined petroleum products</td>
</tr>
<tr>
<td>CE Manufacture of chemicals and chemical products</td>
</tr>
<tr>
<td>CF Manufacture of basic pharmaceutical products and pharmaceutical preparations</td>
</tr>
<tr>
<td>CG Manufacture of rubber and plastic products, Manufacture of other non-metallic mineral products</td>
</tr>
<tr>
<td>CH Manufacture of fabricated metal products, except machinery and equipment</td>
</tr>
</tbody>
</table>
Industrial production can be divided into categories and special aggregates industry classification of economic activities. For example breakdown by SK NACE in Tab. 1. Classification of economic activities SK NACE is fully harmonized with the European version of NACE. Using this classification is created a statistical binding on all Member States of the European Union. Previous Slovak version of this classification was the Statistical Classification of Economic Activities, the acronym NACE. Reason for revision classification of economic activities was an attempt to take account of a technological and structural changes in the economy and to ensure comparability of economic statistics, not only at European but also at international level. Our statistical office data has been processed according to this classification since 2008, therefore in this paper data are analyzed from 2008 to 2013.

II. RESEARCH METHOD

We use method of least squares (LSM) for the analysis of economic indicators in the industrial production. In describing dynamic phenomena rely on indicators which are grouped into time series. The aim of the analysis time data structure is an appropriate model by which we derived based on data from the past to make predictions for specific periods in the future. Thus created time series model allows us to simulate time series in such a way that the real values and designed a model is not a significant difference. The main task of the analysis of time series is a depiction of the basic tendencies of their development, thus setting the trend.

The principle of least squares method consists in minimizing the sum of squares of empirical values \( y_i \) and theoretical values \( \hat{y}_i = T \) (i.e. \( S = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \rightarrow \min \)). If we consider a linear trend \( y = a_0 + a_1 t \) respectively linear regression function is for \( a_0, a_1 \in R \) valid:

\[
\frac{\partial S(a_0, a_1)}{\partial a_0} = 0, \\
\frac{\partial S(a_0, a_1)}{\partial a_1} = 0 \text{ where } S(a_0, a_1) = \sum_{i=1}^{n} (y_i - a_0 - a_1 t_i)^2 \text{ the parameters are the solution system of equations:}
\]

\[
a_0 n + a_1 \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} y_i,
\]

\[
a_0 \sum_{i=1}^{n} t_i + a_1 \sum_{i=1}^{n} (t_i)^2 = \sum_{i=1}^{n} t_i y_i,
\]

For a polynomial of second and third order it is like in the case of a second order polynomial we get three equations with three unknowns.

\[
a_0 n + a_1 \sum_{i=1}^{n} t_i + a_2 \sum_{i=1}^{n} (t_i)^2 = \sum_{i=1}^{n} y_i
\]

\[
a_0 \sum_{i=1}^{n} t_i + a_1 \sum_{i=1}^{n} (t_i)^2 + a_2 \sum_{i=1}^{n} (t_i)^3 = \sum_{i=1}^{n} t_i y_i,
\]

\[
a_0 \sum_{i=1}^{n} (t_i)^2 + a_1 \sum_{i=1}^{n} (t_i)^3 + a_2 \sum_{i=1}^{n} (t_i)^4 = \sum_{i=1}^{n} (t_i)^2 y_i.
\]

In economic practice, but we also meet with functions that can not be linearized by any transformation. Among them:

- Exponential \( T = a b^t \)
- Modified exponential trend \( T = k + a_0 a_1 t \)
- Logistic \( T = \frac{k}{1 + a_0 a_1 t} \)
- Gompers trend \( T = k a_0 a_1 \)

Selecting the shape of the regression function must respect the logical and factual context of the phenomenon and its laws. Regression function should be as simple and at the same time to guarantee the best possible approximation to the observed values. Selecting the right type of addiction is based on the scatter plot. Choosing the most appropriate model may not always be obvious from the outset, therefore we consider the most appropriate one that is most logical in which the smallest residual variation which has the largest leaks addiction.

The most preferably trend were determined by the value of the correlation coefficient the closer they are to 1, team it is more accurate.

I. INDICATORS IN THE INDUSTRIAL PRODUCTION

In this paper the underlying data were drawn from the database SLOVSTAT and are processed by statistical methods. In assessing the current state of the industry has been used trend analysis of selected indicators in time series and their comparison. Based on the identified knowledge is made prediction of the analyzed indicators of industrial production in 2014.

Based on data from the statistical office of the database can be done by analyzing the development of indicators in the industrial production. In this article are analyzed the following variables:

- the average number of persons employed in manufacturing,
- the average nominal monthly wage in manufacturing,
- the labour productivity from revenues from own services and products in manufacturing,
- receipts for own performances and goods in
manufacturing.

For analysis was used data for the period 2008-2013, because since 2008 are known values of individual indicators by SK NACE classification. This period is referred as the 2009 crisis year, some indicators show a decline or stagnation, but some of this impact is not noticeable.

A. The average number of persons employed

The average number of persons employed includes the average number of employees and self-employed persons. Average number of employees includes permanent and temporary employees who are at work, employment, public servant or a member of an organization, regardless whether they are actually present at work or not. [4] Developments of the average number of persons employed during the review period is shown in Fig.1.

The highest number of persons employed during the period was in 2012 year (452 006 thousand) and the lowest in 2013 year (445 301 thousand). Proportion of employees under each category shown graphically in Fig. 2. From the all employed in manufacturing is the most people employed in manufacture of basic metals and fabricated metal products except machinery and equipment (21%) and the least in the manufacture of coke and refined petroleum products and in the manufacture of basic pharmaceutical products and pharmaceutical preparations (1%).

B. The average nominal monthly wage

The average nominal monthly wage labor costs shall include the amount paid to its own employees as compensation for work or a replacement on the basis of the legal relationship with the employer (work, service, civil service or membership relation). Its gross wage lowered by legal or agreed with the employee deductions. [4] Developments of the average nominal monthly wage during the review period is shown in Fig.3.

The average nominal monthly wage in manufacturing during the period of growth didn't decline even during the crisis. The development trend of the average monthly salary can be described as trend function: (year 2008 - t = 1)

\[
y = 30,98t + 677,21 \quad (r = 0,9972).
\]

Tab. 2 reflects a comparison of the average nominal monthly salary in manufacturing. The monthly wage is higher than the average across manufacturing employees achieved in eight of the thirteen monitored categories. Maximum wage employees are in manufacture of coke and refined petroleum products (224.4%) and the lowest employees in manufacture of textiles, apparel, leather and related products (66.44%). The difference between the highest (CD) and the lowest (CB) average monthly wage is higher than the average monthly wage in the whole manufacturing. The graph in Fig. 4 presents the evolution of the average monthly wage in manufacturing, by categories. It is evident that the trend of development of all
categories is almost identical.

Table 2 Comparison of the average monthly salary by category

<table>
<thead>
<tr>
<th>Category</th>
<th>C</th>
<th>CA</th>
<th>CB</th>
<th>CC</th>
<th>CD</th>
<th>CE</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage (%)</td>
<td>100</td>
<td>89</td>
<td>66.4</td>
<td>83</td>
<td>224</td>
<td>117</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category</th>
<th>CG</th>
<th>CH</th>
<th>CI</th>
<th>CJ</th>
<th>CK</th>
<th>CL</th>
<th>C M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage (%)</td>
<td>104.5</td>
<td>102.2</td>
<td>100.2</td>
<td>99.8</td>
<td>112</td>
<td>118</td>
<td>96</td>
</tr>
</tbody>
</table>

Figure 4 The average nominal monthly wage by category

C. Receipts for own performances and goods in the industrial production

Receipts for own performances and goods sold includes the value of goods and services from their own production and commercial goods destined for domestic and foreign customers. The data are exclusive of value added tax and excise duties. [4] Developments of receipts for own performances and goods during the review period is shown in Fig. 5.

Figure 5 Receipts for own performances and goods in the industrial production

Receipts for own performances and goods in the industrial production in each category increased mostly in the manufacture of transport equipment, which is connected especially with the advent of Kia Motors, Volkswagen and others in Slovakia. Nearly 30% of total sales accounted by sales in the manufacture of transport equipment. The second area, which represents 15% of total sales are metals and metal products, except machinery and equipment. Overview of the percentage rate of sales to total sales for each category is shown in Fig. 6.

D. The labour productivity from revenues from own products and goods

Developments of the labour productivity from revenues from own products and goods in the industrial production in the crisis of 2009 year decreased compared to 2008 year (about 10%) and since then has upward trend (growth factor), trend function has the form: (rok 2008 – t=1)

\[ y = -1317259.98 + 14898920.3t - 45995839.16t^2 + 91458587.37t^3 \]

\( r = 0.9636 \). (9)

Figure 6 Receipts for own performances and goods by category

Figure 7 The labour productivity from revenues from own products and goods in the industrial production

The labour productivity from revenues from own services and products in the industrial production in the crisis of 2009 year decreased compared to 2008 year (about 10%) and since then has upward trend (growth factor), trend function has the form: (rok 2008 – t=1)

\[ y = 139065.4 + 44353.43t + 17675.28t^2 - 1655.11t^3 \]

\( r = 0.9749 \). (10)
Tab. 3 shows a comparison of the labor productivity from revenues from own services and products in the industrial production. The highest labor productivity has manufacture of coke and refined petroleum products (11 times higher than average). Values higher than the overall average is only in three categories (CE, CI, CL). Low labor productivity from revenues from own services and products in the manufacture of textiles, apparel, leather and leather products (CB - 29%), which is the lowest average nominal monthly wage. The graphic display (Fig.8) reflects that labor productivity is different in each category, somewhere stagnant, growing somewhere in the categories CA and CD has a variable character.

<table>
<thead>
<tr>
<th>Category</th>
<th>CG</th>
<th>CH</th>
<th>CI</th>
<th>CJ</th>
<th>CK</th>
<th>CL</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage (%)</td>
<td>67</td>
<td>63</td>
<td>21</td>
<td>7</td>
<td>61</td>
<td>53</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>41</td>
</tr>
</tbody>
</table>

The aim of the present article in this point is the expression of similarity relations by year and by category of industrial production. As a measure of similarity was used squared Euclidean distance, which is one of distance degrees and from hierarchical methods of cluster analysis Ward's method was used. Ward's method differs from previous methods, that are based on the terms of distances between objects, that it is based on criterion of minimizing the increasing error sum of squares of deviation of points from the cluster centroid. The output of the process of hierarchical clustering is following dendrograms. The optimal number of clusters of different dendrograms was expressed by a heuristic approach.

### A. The average number of persons employed

Within the individual parameters were assessed similar mutual relations by year and by category of industrial production. The preliminary data for 2014 point showed their growth. Individual variables interact the growth of one of them will increase the value of another parameter. The most significant correlations are indicated in the correlation matrix in Tab. 4. It is clear that the wage employee turnover and productivity from revenues from own services and products is a significant correlation.

### III. CLUSTER ANALYSIS

Cluster analysis belongs to multivariate statistical methods. It is defined as general logical technique, procedure which allows clustering various objects into groups – clusters on the basis of similarity or dissimilarity. Cluster analysis involves computational procedures, which purpose is to reduce a set of data on several relatively homogeneity groups – clusters, while the condition of reduction is maximal and simultaneously minimal similarity of clusters. Similarity of objects is determined by the degree of similarity (correlation coefficient and association coefficient) or the degree of dissimilarity – degree of distance (distance coefficient). On the basis of clustering, methods of cluster analysis are classified as hierarchical or non-hierarchical methods. Hierarchical methods of cluster analysis are based on the hierarchical systematization of objects and its clusters. The procedures, methods begin from the separate objects which represents clusters. Progressively, the amount of clusters is getting down and in the end all objects, clusters are reduced into the whole. Hierarchical methods lead to hierarchical (tree) structure which is graphically figured as dendrogram (tree diagram). Non-hierarchical methods do not create hierarchical (tree) structure and the objects are categorized into the number of disjunctive clusters specified in advance [5].

The aim of the present article in this point is the expression of similarity relations by year and by category of industrial production. As a measure of similarity was used squared Euclidean distance, which is one of distance degrees and from hierarchical methods of cluster analysis Ward's method was used. Ward's method differs from previous methods, that are based on the terms of distances between objects, that it is based on criterion of minimizing the increasing error sum of squares of deviation of points from the cluster centroid. The output of the process of hierarchical clustering is following dendrograms. The optimal number of clusters of different dendrograms was expressed by a heuristic approach.
production. First analyzed indicator is the average number of persons employed in manufacturing. Similarity of years, respectively category of industrial production in the analysis of indicator is shown in form of a dendrogram, showing clusters according to the distance (similarity) of objects i.e. years.

Based on a heuristic approach for the optimal number of clusters can be considered the following clusters of years:
1) 2008
2) 2009, 2010
The greatest distance, i.e. the slightest similarity with respect to the other year is 2008, i.e. 2008, the average number of persons employed differs most.
The smallest distance, i.e. greatest similarity reported in 2012 and 2013.

Optimal clusters categories of industrial production are:
1) CA, TA, CB, CC, CM, CG, CL
2) CH
3) CD, CF, CE, CI, CJ
The greatest similarity in the assessment of the average number of persons employed is between categories CD and CF.

B. The average nominal monthly wage
Podobným spôsobom boli posudzované aj ostatné ukazovatele využitím zhlukovej analýzy.

Within indicator the average nominal monthly wage for the optimal aggregations years considered:
1) 2008, 2009, 2010
From the dendrogram shown in Figure 11 indicates a relatively large distance between the two clusters, i.e. that the objects (years) within the cluster are similar to each other but the clusters are different.
This is on the smallest distance, i.e. greatest similarity in the average nominal monthly wage between 2011 and 2012.

Optimal clusters categories of industrial production are:
1) CA, CB, CC, CG, CH, CI, CJ, CM
2) CE, CL, CK, CF
3) CD
Between clusters of first, second and third clusters (with only the category of the CD) is the longest distance, ie. Category CD is available under the indicator deviates from others.

C. Receipts for own performances and goods in the industrial production

![Figure 13 Dendrogram of receipts by years](image1)

Optimal tufts years are:
1) 2008
2) 2009, 2010
The smallest similarity expressed the greatest distance in the dendrogram Figure 13 is the 1st, 2nd and 3rd cluster, while the biggest similarity is the years 2011 and 2012.

![Figure 14 Dendrogram of receipts by category of industrial production](image2)

From the dendrogram Figure 14 we determine the optimal manufacturing clusters categories:
1) CA, CC, CB, CM, CF, CG, CH, CJ
2) CK, CE, CI, CL
3) CD

Significantly, we can observe a great distance between the 1st, 2nd and clusters 3rd (CD object).

D. The labour productivity from revenues from own products and goods

![Figure 15 Dendrogram of labour productivity by years](image3)

Long distance, ie. little resemblance (Figure 15) can be observed between clusters each other:
2. 2012, 2013
It follows that a significant change in the context of labor productivity occurs at the turn of 2011-2012.

![Figure 16 Dendrogram of labour productivity by category of industrial production](image4)

Optimal clusters by categories of industrial production for variable labor productivity are:
1) CA, CD, CC, CK, CJ, CM, CG, CB, CE, CF
2) CH, CI
3) CL
Significantly from other objects (categories) deviates only object CL.

Conclusion of cluster analysis
Finally, cluster analysis dendrograms were created taking into account...
account the indicators that were the subject of the present article. The resulting dendrograms consider the influence of individual characteristics and express relationships between the analyzed years (Figure 17) respectively. between the various (Figure 18).

For the evaluation of the results of cluster analysis indicators we have distance matrix (Table 4) and the procedure of cluster analysis (Table 5). Distance matrix contains information about the distances of all pairs of variables. From the dendrogram (Figure 18) and distance matrix (Table 4) shows that the maximum distance ie. the smallest similarity between objects (variables) Labour productivity - Receipts and the longest distance, ie. the smallest similarity between objects (variables) Receipts - Wage. Procedure of cluster analysis contains information about each step in the process of aggregation. The first step was the creation of clusters Receipts - Labour productivity because of their mutual distance (dissimilarity) is the smallest. For the optimal choice of major clusters consider two clusters:

1) Number of employees, Receipts, Labour productivity,
2) Wage, because of their length is the largest.

IV. CONCLUSION

Industry and related services are the core of the Slovak economy, a source of job creation, the driving force of productivity and innovation. Performance of industry and manufacturing is shaping the level of productivity of the Slovak economy in relation to the European Union. The analysis showed that the individual sectors in the industrial production development are highly differentiated. In the category of coke and refined petroleum products are employed but the least people reach there highest salary Receipts for own performances and goods and hence productivity is highest in this category. Recently, there is also an increase in production indicators in the categories of vehicles. The analysis above shows that in order to increase the competitiveness of industrial production and its individual sectors is necessity to look for the optimal way of industry development.
ACKNOWLEDGMENT

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