Metaheuristic algorithms helping to take decisions in investment portfolios

Freddy Baño, Ángel Mena, Fausto Viscaino, Jorge Rubio and Gustavo Rodríguez-Bárcenas

Abstract— Actually, Managers who take decisions need to have a system that provides them information for decision-making in a record time and in the more reliable manner, especially if they have to invest the money of their investors, determining the best investment portfolio with a minimal risk and high return, it consists in actives whose rate is variable at the market, that´s way we have performed an approach to comparing two metaheuristics algorithms, the first using the mean-variance technique directly and the second with a genetic algorithm, both powered by a discrimination algorithm assets less than zero risk and high profitability. Evaluation´s Techniques of every investment portfolio is presented as an aversion risk.

Keywords— Information System, Decision-Making, Investment Portfolio, Metaheuristic Algorithm, Genetic Algorithm.

I. INTRODUCTION

Today is not enough to get information about the different types of investment in a national or international level, having the financial movements at the past and present time, the most important it´s to have an information system tool to provide an algorithms metaheuristics implementation with the best investment portfolio ending with the investigation conclusions.

In Ecuador, based on the new financial political, companies and citizens involved in financial investment need to understand that is possible to achieve great economic returns which require new analytical tools to demonstrate high reliability, you could obtain data through the computer, analyzing and storing them to get the best solutions and then determine which is the most convenient and appropriate investment you have in the financial market.

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The creation and management of investment portfolios, especially of small and medium financial institutions, it´s done using Microsoft Excel and supported especially on the investor´s experience, it´s performed based on the combination of few assets that make up each portfolio. This process is very long and complicated, so the problem becomes similar with the knapsack problem with a limited capacity and it´s each item cost. You can get a good solution with metaheuristics algorithms, which are detailed below concisely providing assessment techniques used to create an investment portfolio using a genetic algorithm and a simplified mean-variance algorithm search, based on then weighting of the *i-th* element subject to the capacity *c*, choosing the elements to maximize the associated gain but subject to the capacity contrains.

A. *Previous work*

Many resources have been used to solve the optimization problem in different fields especially at the investment portfolio´s creation under the principle defined by Harry Markowitz, one of them presented by Pablo Fonseca Arroyo, Manuel Luna Trujillo and Juan Trelles Trabucco of Pontifical Catholic University of Peru in the VIII Congress of the Peruvian Computer Society Proceedings, CSPC-2009 [1, 2], in his summary said "The problem of selection of investment projects to form optimal investment portfolios is an instance of an NP problem -complete known as the 0/1 knapsack problem. In this investigation, we compare two metaheuristics algorithms that solve it. The first is a genetic algorithm and the second one is a taboo algorithm, both powered by an algorithm Greedy Randomized Adaptive Search Procedure (GRASP). The techniques used for capital budgeting located in a context of varying risk aversion and economic environment compared with are also presented."

At the Technological Pereira University [3] investigators have conducted an investment portfolios research in the stock market of Colombia as emerging markets, posing a methodological model that predicts using neural networks and genetic algorithms working with Excel tools and Premium Solver computer tool.

There are investigations where optimization models that enable the implementation of effective strategies, in which effects of transaction costs are illustrated in the performance of investors' portfolios are constructed [4-6].

Recently, many companies have faced difficulties in their technology, capital and labor, and therefore, the respective efforts have been carried out as solutions to these problems. With this, many companies are trying to strengthen their competitiveness by spreading the investment costs and risks through cooperation and sharing of resources and benefits [7].

Algorithmic models are applicable to different contexts, settings where significant results can be glimpsed, they are the example we algorithm genetic, algorithms mean variance, among others that have been studied and applied specifically to investment portfolios, by authors such as Chang, Deb, Fonseca, Fukunaga, Kamaruddin, Ghani, and Ramli [1, 2, 4, 6, 8-14].

B. Definition Problem

Portfolio´s optimization problem involves assigning investments to a number of different goods (assets) to maximize performance and minimize the risk, it´s similar to the knapsack problem which has two sets, $R = \{r_1, r_2, r_3, ..., r_n\}$ and $P = \{p_1, p_2, p_3, ..., p_n\}$, where r_i represents the expected return of the *i-th* element and p_i weight weighting *i-th* element, subject to capacitance *C*, you must select the elements that maximize the gain associated, but subject to capacity constraints.

C. Investment Portfolio

Harry Markowitz was who developed the portfolio theory (investment portfolio), as selected, low risk characteristics and overall return and not just a single value under the expected return. The portfolio selection theory takes into account the long-term return and expected volatility in the short term. Volatility is seen as a risk factor, the portfolio is formed under the risk tolerance of each particular investor, after choosing the highest level of return available for the level of risk chosen [8, 15].

Volatility is just a dispersion statistical measure that could be the variance or standard deviation. In the financial theory this variance, standard deviation or volatility it´s called: "Risk". When two asset portfolios comprise only have to calculate the standard portfolio deviation, however, when portfolios are formed with 3 or *N* assets, plus the standard deviation, correlation (and covariance) should be calculated from active.

It´s necessary understand that an investment portfolio is a combination of financial assets local, national or international (bonds, stocks, land, precious metals, etc.) which combined in such a way that maximizes profitability (profit) and minimize the irrigation, turning forming balanced based on the diversification of investments in different markets and time [15].

Moderate (Balanced): A mix between income and growth based on the financial assets diversification to mitigate the risk. Accept a lower risk degree. Its risk´s degree is reduced and consists mostly of debt´s instruments, some of them equities and another ones are liquid instruments.

II. METHODS

A. Portflio´s Performance

Investment´s yield or return´s investment is measured as the gain or loss of the valued in a period of time; to determine the portfolio´s assets expected return, you can start from the notion of expected return of an individual asset, using the following formula:

$$r_e = \sum_{i=1}^{N} p_i r_i \; ; \; r_e = p_1 r_1 + p_2 r_2 + p_3 r_3 + \ldots + p_N r_N \qquad (1)$$

The expected return (r_e) is a weighted average of all possible returns (r_i), where the weights are the probabilities (p_i) that occurs each of the results.

The possibilities in the above equation must add 1:

$$\sum_{i=1}^{N} p_i = 1 \; ; \quad p_1 + p_2 + p_3 + \ldots + p_N = 1 \qquad (2)$$

Based on the above individual formula and calculating the portfolio´s expected return, the following formula is used:

$$r_{e_P} = \sum_{i=1}^{N} a_i r_{e_i} \; ; \; re_P = a_1 re_1 + a_2 re_2 + \ldots + a_N re_N \qquad (3)$$

Where a, it´s the fraction invested in each one of the *N* active.

At first, it´s necessary to determine the expected return of each one of the N active. These expected returns are denoted *by* re_1, re_2, … , re_N, for assets 1, 2, ..., *N*, respectively.

The portfolio´s expected return (re_P) it´s calculated as the weighted average of the expected returns of the N assets into the portfolio, where the weights correspond to portfolio´s fractions invested in each asset.

In this account, the portfolio´s expected return depends on the same time the expected returns of each one of the portfolio´s active and its money´s fractions belonging to each asset.

B. Portfolio´s Risk

To measure the portfolio´s level risk consists on observing the volatility of its returns. To calculate the variance formula *(σ^2)* is used, which it´s the sum of the squared deviations from the average return. Each deviation is weighted by the probability of its particular return occurs.

The variance is defined:

$$\sigma^2 = \sum_{i=1}^{N} p_i (r_i - r_e)^2 \; ; \qquad \sigma^2 = p_1(r_1 - re)^2 + p_2(r_2 - re)^2 + \ldots + p_N(r_N - re)^2 \qquad (4)$$

Calculating the standard portfolio *(σ_p)* or standard deviation it´s performed with the following formula to a portfolio of assets *n*:

$$\sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} p_i p_j \delta_{ij} \sigma_i \sigma_j} \qquad (5)$$

In δ_{ij} which represents the correlation coefficients, the same that will be calculated as:

$$\delta_{ij} = \frac{\mathrm{Cov}(r_{ei}, r_{ej})}{\sigma_i \sigma_j} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \qquad (6)$$

This correlation measure has some properties that make it preferred to the covariance. For example takes values between 1 and -1 exclusively [16].

If δ_{ij} = -1 it´s said that the returns of the two assets have a perfect negative correlation means that mean while one of the increases, the other one decreases by the same amount.

If δ_{ij} = 1 it has a perfect positive correlation between asset returns, which means that mean while one of them grow the other one does it in the same proportion.

If δ_{ij} = 0 there is not correlations, it means there´s no link between them.

A moderate investment portfolio accept a lower degree of risk

C. Mathematical Model of the investment portfolio

In the mathematical model of the investment portfolio we must calculate the performance of each asset with the equation (1) and the risk of it with the equations (4).

Maximizing the Portfolio´s Perfomance.

a) $Max\ E[R_p] = \sum_{i=1}^{n} w_i E[R_p]$ (8)

Subject to $\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \delta_{ij} \sigma_i \sigma_j = cte$ (9)

$\sum_{i=1}^{n} w_i = 1$ (10)

$w_i \geq 0$ for $i = 1, 2, \ldots, n$

The first constraint expresses the condition to get the level of risk tolerated. The second, known as budget´s limitation requires the total budget invested in the budget´s portfolio. And the last one, known as non-negativity conditions, means that is not allowed short sellings, it means that it´s not allowed to give or borrow money.

Minimizing portfolio´s risk.

b) $Min\ \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \delta_{ij} \sigma_i \sigma_j$ (11)

Subject to $E[R_p] = \sum_{i=1}^{n} w_i E[R_p] = R^*$ (12)

$\sum_{i=1}^{n} w_i = 1$

$w_i \geq 0$ for $i = 1, 2, \ldots, n$

Where w_i = Weighting of the company´s action i

The first constraint expresses the condition to get the expected level of performance. The other restrictions are equal to the model (a) [17].

D. Proposed algorithms

In the program (AGE CREPIN) you should enter the assets to be part of the investment portfolio, it must be an historic value in order to calculate their performance and risk, this with equations (1) and (4), the asset´s data must have the same length as it´s possible. Calculate data until to determine the asset´s covariance allowed to be used by both next algorithms, the distribution of the economic resources is given in percent, taking it more than 1 or 100% (equation (9)), which it must be distributed in each asset making up the investment portfolio:

E. Media metaheuristic algorithm - Variance

This algorithm is based on the Markowitz´s model, where *n* is the number of assets that make up an investment portfolio.

Algorithm 1 Mean-Variance

```
Sort (ListAssets, performance, descending)
Select (ListAssets yield> 0)
Sort (ListAssets, risk descending)
    If Length (ListAssets) <= n
    p = average - n \ 2
    While i <n do
        ListPortfolio (i) = ListAssets (p)
        p = p + 1
        end while
    minrend = minperformance (ListPortfolio)
```

```
            maxrend = maxperformance
            (ListPortfolio)
            While i < 10 do
                  PerformancePortf = Increase
            (minrend, maxrend)
                  Mientras_1 k≠ 1 do
                        Mientras_2 j <n do
                              ListRinde (i, j)
                        = Random
                        (InvestmentRate)
                        End mientras_2
                  End Mientras_1
            End While
      End If
```

F. Proposed Genetic Algorithm

The genetic algorithm proposed, it`s developed to perform an multi-objective optimization creating an investment portfolio, will be the micro genetic algorithm for multi-objective optimization as an elitist type [18] which has a technique to maintain the diversity in the population; the decisions will be taken when the optimization has reached to a set of equally feasible solutions called Pareto optimal set.

In this problem we have at least 2 and maximum 10 optimization variables by the number of assets in the portfolio which are involved in the proposed algorithm:

x_1 = active Name 1; x_2 = Percentage of active participation 1; ...; Name x_n n = active; x_{n+1} = Percentage of active participation *n*.

Individual of the dynamic and static population will be selected randomly between 0 and 1/5 of the maximum combination of *N* assets. After they will be transformed to binary of 8, 17 figures as noted above. The binary codes of all the variables are concatenated to obtain the corresponding chain C_i.

C_i = concatenate $[(x_{i,1})_2 + (x_{i,2})_2 + \cdots + (x_{i,n})_2 + (x_{i,n+1})_2]$

Where $(x_{i,j})_2$ It represents the number *xi, j* written in binary notation.

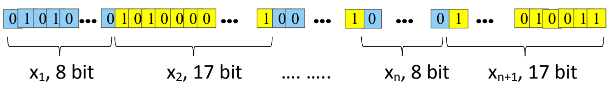


Fig. 1 Genetic Algorithm proposed
Source. Authors

Determining the value of $x_{i,j}$ and dependent variable and its lower and upper limits. The population size is determined is 100 to 3 times the dynamic and static to dynamic, with a number of iterations to 10 as minimum standards for proper optimization.

Restrictions and Evaluation:

Equations (7) as the objective function and (8) as one of restrictions applied. And the other is applied by:

$$G_i = \frac{\sum_{j=1}^{n} \max\left[g_j(x_i),0\right]}{\frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{n} \max\left[g_j(x_i),0\right]} \quad (12)$$

It allows evaluating infeasible individuals.

To evaluated the objective function from each individual of the population it´s performed considering restrictions like:

$$g_1 = \frac{1}{\sum_{i=1}^{N}\sum_{j=1}^{N} p_i p_j \delta_{ij}\sigma_i\sigma_j} - 1 \quad (13)$$

$$g_2 = \frac{\text{Return Portfolio}}{\min(f_i(R),1)} - 1 \quad (14)$$

$$g_3 = \frac{\min(Cov(x_i),1)}{\text{Maximum Covariance}} - 1 \quad (15)$$

Assets whose performance is less than zero are penalized, with an index of 100 infeasibility, in the same order than the assets with higher risk fixed in advance.

Crossing

We must apply a four point´s cross in proportion to the number of assets that form the investment portfolio, starting with the father and after the mother alternately, generating one child in this way and in the second time a child starting with the mother and then applies the father.

Mutation

There is a gene mutation randomly as randomly the individual was selected.

Update from the elite population:

After each iteration, individuals not dominated of P_K resulting population are choosen and added to elite, after each iteration, they are filtering. This is to eliminate all dominated individuals. Individuals with a distance between then less than a certain value δ_{LIM} Euclidean are also deleted.

$$\sqrt{(y_{i,1} - y_{j,1})^2 + (y_{i,2} - y_{j,2})^2 + (y_{i,3} - y_{j,3})^2} \geq \delta_{LIM} \quad (16)$$

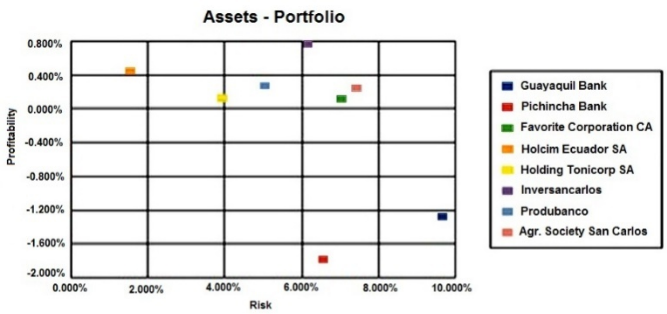
Where: $y_{i,1}$, $y_{i,2}$, and $y_{i,3}$ are the values of the objective functions for the individual i-ésimo; $y_{j,1}$, $y_{j,2}$, $y_{i,3}$,, objective functions for the individual *j- th*.

Genetic Algorithm

Population← GenerateInitialPopulation (n assets)
While Generations <maximum to
EvaluatePopulation (Population)
PopulationBest ←Select (population)
CopyPareto ← Select(BestPopulation)
PopulationCross ←Cross(BestPopulation)
PopulationMutate ←Mutate(BestPopulation)
Population ← BestPopulation ∪ CrossPopulation ∪ PopulationMutate
End while

III. RESULTS

Different experiments were made with the Quito Stock Exchange´s data and they are summarized in the following figure 2:



Name Assets	Profitability (%)	Risk (%)
Guayaquil Bank	-1.27	9.66
Pichincha Bank	-1.78	6.54
Favorite Corporation CA	0.12	7.05
Holcim Ecuador SA	0.44	1.53
Holding Tonicorp SA	0.13	3.93
Inversancarlos	0.77	6.16
Produbanco	0.27	5.05
Agr. Society San Carlos	0.25	7.45

Fig. 2 Profitability and Risk Assets
Source: Quito Stock Exchange.

At the beginning, financial performance and risk were calculated for each asset, after the covariance matrix, then 10 portfolios were constructed for both: the mean-variance algorithm and genetic algorithm, for each investment portfolio with 3 of the total 8 assets; Levene's test was performed to verify whether variances of the results of the two algorithms were equal. For the statistic $F = \sigma_2^2/\sigma_1^2$ with $\alpha = 0.2$. $F = 0.37 < F_{(1-\alpha/2)} = 0.40984$; so that the variances were different.

In the hypothesis´s test about two average and variances unknown and different with $\alpha = 0.5$, *it was* $T = -0.93 < -1.75$; It is the genetic algorithm that gives the best results in fig. 3.

Media - Variance			Genetic Algorithm		
No.	Profitability (%)	Risk (%)	No.	Profitability (%)	Risk (%)
1	0.13000	3.93428	1	0.19675	3.25200
2	0.20237	3.33711	2	0.69222	5.27670
3	0.26974	3.17916	3	0.48595	2.09180
4	0.34044	2.96403	4	0.33475	2.45030
5	0.41224	3.17373	5	0.33475	2.45030
6	0.48312	4.56223	6	0.36884	3.21960
7	0.55813	4.36631	7	0.36551	2.59950
8	0.62564	5.15475	8	0.33475	2.45030
9	0.69604	5.45095	9	0.33475	2.45030
10	0.77000	6.15551	10	0.33475	2.45030

Fig. 3 Report of the Programme AGECREPIN
Source: Authors

IV. DISCUSSION

After the experiments, it was observed that the average obtained with the genetic algorithm are better in 92% certainty than the results of the mean-variance algorithm; however in relation to the runtime uses the genetic algorithm further processing depending on the initial parameters such as number of generations, and mutation rate crossing.

V. CONCLUSIONS

The program created to support decision making for investment portfolios, it´s a good alternative to create investment portfolio because it can handle different assets, processing large data volumes and provide the best results.
It can be shown that the results obtained with the genetic algorithm have relevance in 92% compared to the mean-variance algorithm.
The genetic algorithm´s application has an alternative to create an investment portfolio generating a series of solutions correlated with the optimization problem relatively quickly and effectively.

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