Modified human development index and its weighted alternative

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***Abstract*—**The contribution solves the problem of alternative access towards Modified human development index of the Visegrad Four NUTS 2 regions. The basic aim of the contribution is due to the method of analytic hierarchy process to define the position of NUTS 2 regions in period of 2004 – 2013 years. The sense of applying the method will be setting the order of NUTS 2 regions reflecting their human development index for the year. The analytic hierarchy process (AHP) is a concrete method of multicriteria decision making method which uses the hierarchy of elements and pairwise comparisons. This method is used to derive unknown weights of macroeconomic indicators influencing the Modified human development index.

***Keywords*—**Human development index, analytic hierarchy process, preferences, Visegrad group.

I. Introduction

ATTEMPTS to find an ideal system of evaluation of economic, social and environmental quality often lead us to search for various combinations of already existing aggregates by optimal synthetization of which we are able to create a required indicator.

Nowadays, competitiveness, which is usually expressed by partial microeconomic or macro regional indicators, is generally accepted economic category searching for ways how to define, measure and evaluate economic level of (non-)performance of companies, regions and nations. If the technical side of such an evaluation is disregarded, it is clear that the competitiveness basis is created by regions. Regional competitiveness level offers two views. In a broader sense, competitiveness is understood as part of international labour division, which affects involvement efficiency of diverse conditions of particular regions for estate production, realized on a higher level than a local regional market. However, if we focus on the lower level, the local regional market, we will learn that many economic phenomena which are perceived on national level reflect the regional level, more precisely, they are the manifestation of regional competition [4], affecting directly or indirectly all of the economic entities.

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II. Human development index

One of the indicators which cover a wide level of internal development of regions is Human Development Index (HDI), more precisely; it is its modified degree enabling observation of its level within the level NUTS 2.

HDI that has been used by the United Nations since 1990 [6], clearly brings a different perspective on development issues and should be better able to emphasize the effect of other than just monetary (economic) factors on the economy of a country. The basis of the HDI index is greater explanatory power, which is to follow economic development or sustainable development in general. This index is able to explain better, how two or more countries with the same level of income per capita can end up with different human development outcomes [2].

The article examines the modified version of the index. NHDI – Modified Human Development Index is a modified version of a typical index HDI; nevertheless, NHDI enables a better identification of development in regions NUTS 2. However, the aim of the article is not to present new values of NHDI reflecting situation in regions, but to take into consideration the significant reflection of partial variables of the index in the overall order on regions of Visegrad Four Group. The Visegrad Four Group is represented by 8 cohesion regions of the Czech Republic, 16 voivodships in Poland, 7 counties in Hungary and 4 cohesion regions of Slovakia.

The variables entered into the NHDI are presented in the Table 1.

For purpose of our paper, we adopted the same principle of HDI creating for the national level – the health dimension, knowledge dimension and dimension of living standard. Components of each dimension, however, had to be modified because of the lack of data at the regional level (NUTS II level). Data were used from a regional database of Eurostat and construction of the HDI of V4+ regions (NHDI) was as follows:

1. Health with value of life expectancy at bird that represents, according to Eurostat, the mean number of years that a new born child can expect to live if subjected throughout his life to the current mortality

conditions (age specific probabilities of dying).

2. Knowledge, which has two components:
   a. Tertiary educated people in the age of 25-64, when the indicator is defined as the percentage of the population aged 25-64 who have successfully completed tertiary studies (e.g. university, higher technical institution, etc.). This educational attainment refers to ISCED (International Standard Classification of Education) 1997 level 5-6, that includes the first stage of tertiary education (bachelor and master or equivalent) and second stage of tertiary education (doctoral or equivalent).
   b. Lifelong learning in the form of the participation rate in education and training covers participation in formal and non-formal education and training. The reference period for the participation in education and training is least four weeks. Participation rates in education and training for age group of 25-64 are presented. The data are calculated as annual averages of quarterly EU Labour Force Survey data (EU-LFS).
3. Standard of living measured through GDP per capita in PPS – Purchasing Power Standards (PPS) is a common currency that eliminates the differences in price levels between countries and regions allowing meaningful volume comparisons of GDP between them.

The variables involved in NHDI reflect economic and social characteristics based on regions. Thus it is possible to infer the performances among the regions and diagnose regional disparities more effectively. It is the combination of social and economic characteristics which has a significant role in the overall form of NHDI since the biggest disadvantage of traditional indicators is the orientation on only one particular field. It is absolutely necessary to focus on the combination of wider range of characteristics in the following research. Thus, environmental field comes into consideration which reflects the principles of sustainable development better.

Dimension	**Indicators**	**Index**
health	Life expectancy at birth (years)	Life expectancy index
education	Tertiary education (% of population in 25-64 years)	Tertiary education index
	Lifelong education (% of population in 25-64 years)	Lifelong education index
standard of living	GDP per capita (in PPS)	GDP index

Table 1 Indicators of NHDI

III. Analytic hierarchy process

Analytic hierarchy process is multicriteria decision-making method. The problem is structured in a hierarchy of 3 (or more) levels. The goal of the problem represents the highest level, the second one belongs to criteria, i. e. substantial factors influencing the decision (or evaluation), and alternatives to be assessed are on the last level of hierarchy. The criteria may be quantitative and qualitative, too. Quantitative criteria are of minimizing or maximizing character.

The pairwise comparisons method is used to derive unknown/undetermined weights (priorities) of objects on each hierarchy level. All objects are compared to each other by couples. If there are numerical characteristics of object, these are pair-compared. If the characteristics of objects are qualitative, the nine-point scale is applied to express the difference of preferences in couple of objects. Number one means equality, number nine represents extreme difference between objects [5]. See table 2.

Intensity of importance	*Definition*
1	Equal importance
2	Weak
3	Moderate Importance
4	Moderate plus
5	Strong Importance
6	Strong plus
7	Very strong Importance
8	Very, very strong
9	Extreme importance

Table 2: The nine-point scale

Values of the pairwise comparisons represent estimation of weight ratio of two compared elements of the same hierarchic level:

$$a_{ij} = \frac{w_i}{w_j} \tag{1}$$

where a_{ij} is value of pairwise comparison between the i-th and j-th object, w_i is weight of the i-th object, w_j is weight of the j-th object. The i-th object is equal to itself, corresponding value is 1.

There is multiplicative reciprocity between pair-compared objects:

$$a_{ji} = \frac{1}{a_{ij}} \text{ or } \quad a_{ij} \cdot a_{ji} = 1. \tag{2}$$

Values of pairwise comparisons are inserted in the pairwise

comparison matrix **A**. Maximal eigenvalue $\lambda_{\max}$ and corresponding eigenvector **w** are to be calculated according to the characteristic equation:

$$\mathbf{A}\mathbf{w} = \lambda_{\max}\mathbf{w}. \quad (3)$$

Some special attributes of this matrix ensure relatively simple calculation of its maximal eigenvalue $\lambda_{\max}$ and corresponding eigenvector **w**. When normalized, i.e. $\sum_{i=1}^{n} w_i = 1$, element w_i of vector **w** represents the relative importance of the i-th object.

The pairwise comparison matrix is *square*. All n objects of given hierarchical level are compared to each other and the $n \times n$ matrix is created. It is enough to execute $(n^2 - n)/2$ pairwise comparisons with respect to the reciprocity.

The matrix is *nonnegative*, too. If pairwise comparisons are expressed by the nine-point scale, the possible values are $\{1/9; 1/8; ...; 1/2; 1; 2; ...; 8; 9\}$. If pairwise comparisons are expressed by real number ratio, the value may be negative. Sufficiently large positive number has to be added to all pair-compared entry values to get nonnegative matrix.

The pairwise comparison matrix is *irreducible*. That means it is not possible to rearrange the columns and rows to get zero submatrix. This attribute is ensured when expressing pairwise comparisons by the nine-point scale. If the pairwise comparison value got by the real number ratio is zero, it is necessary to add sufficiently large positive number to all entry values.

The Perron-Frobenius theorem ensures existence of the maximal eigenvalue and corresponding eigenvector including positive components for such matrix [3]. The Wieland theorem is applied to derive the eigenvector:

$$\lim_{k\to\infty} \frac{\mathbf{A}^k\mathbf{e}}{\left\|\mathbf{A}^k\right\|} = c\mathbf{w}, \qquad \left\|\mathbf{A}^k\right\| \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}^k\mathbf{e}, \quad (4)$$

where $\mathbf{A}^k$ is the k-th power of matrix **A**, **e** is vector of ones, i.e. $\mathbf{e}^{\mathrm{T}} = (1; 1; 1; ...; 1)$, c is constant.

Some inconsistency may appear in pairwise comparisons. It means the following consistency condition is not satisfied:

$$a_{ij} \cdot a_{jk} = a_{ik} \text{ for all } i, j, k = 1, 2, ..., n. \quad (5)$$

Inconsistency is measured by inconsistency index I_c. It is calculated for $n \times n$ matrix as follows:

$$I_c = \frac{\lambda_{\max} - n}{n - 1}. \quad (6)$$

The inconsistency index must not exceed the threshold of 10 %. In such a case the matrix is considered to be sufficiently consistent. Otherwise the pairwise comparisons have to be reassessed. [1]

Weighted sum is calculated when weights of all criteria and weights of all alternatives according to all criteria are derived. The result is overall weights of alternatives with regard to the goal. This result gives final ranking of alternatives.

IV. Application

Four economic indicators are used to derive Modified Human Development Index: gross domestic product per capita in PPS, tertiary educated people in age of 25-64 (in 1000), life expectancy (in years) and participation rate in education and training (last 4 weeks) in age of 25-64 (in 1000). Weights of economic indicators are equal in the traditional conception. This index is called traditional modified human development index (TNHDI) in this paper.

Influence of different indicator weights on modified human development index is researched in this paper. Analytic hierarchy process and its eigenvector method are applied to derive still unknown (unexpressed) priorities of economic indicators. This approach is used to rank Visegrad Four NUTS 2 regions in years 2004-2013 according to values of four above mentioned macroeconomic indicators. This new index is called weighted modified human development index (WNHDI) in this paper.

Calculations are made by Microsoft Excel.

The economy expert performed pairwise comparisons of four economic indicators importance according to their significance in the WNHDI. The pairwise comparison matrix is as follows:

	GDP	*TEP*	*LE*	*PRET*
GDP	1	3	2	4
TEP	1/3	1	1/2	3
LE	1/2	2	1	3
PRET	1/4	1/3	1/3	1

where GDP is gross domestic product, TEP are tertiary educated people, LE is life expectancy and PRET is participation rate in education and training.

For example, GDP is weakly more important than LE and PRET is moderate less important than TEP according to the economy expert.

The maximal eigenvalue $\lambda_{\max}$ of this matrix equals 4.087, corresponding (normalized) eigenvector is

$\mathbf{w}^{\mathrm{T}} = (0.462; 0.178; 0.274; 0.086)$. Weight (priority) of GDP is 0.462, weight of TEP is 0.178, weight of LE is 0.274 and weight of PRET is 0.086. The inconsistency index is 2.92 % and it does not exceed the threshold of 10 %. This pairwise comparison matrix is sufficiently consistent.

Indicator values of Visegrad Four NUTS 2 regions in years 2004-2013 are pair-compared, their weights are derived and weighted sums of criteria and criterion values are calculated. Average ranking of regions according to traditional and weighted approach are in the table 3.

Three best and worst average positions are highlighted.

The best average place belongs to region PL12 with regard to both traditional and weighted conceptions. This region is even always the first in traditional approach (see appendix). Region PL22 holds the second best average position according to the traditional approach. Region CZ01 is the second best one in the weighted conception. Regions CZ01 and HU10 have identical average emplacement in traditional approach and they are the third (and fourth) best regions. Region SK01 is the third best region according to the weighted approach with average position of 3.3.

On the contrary, HU23 has the worst emplacement according to the traditional approach and HU31 holds the second worst position with regard to WNHDI. This region is the second worst in the traditional conception, too. Region HU23 occupies the second worst position according to the weighted approach. PL52 holds the third worst average position in the traditional conception and HU32 holds it according to the weighted approach.

Regions HU23 and HU31 are belong to the worst triple in both cases (i.e. traditional and weighted, too). Regions PL12 and CZ01 occur in the best triple according to TNHDI and WNHDI.

V. Conclusion

We can see differences in average places derived by traditional and weighted approaches. This contrast is caused by different priorities (weights) of macroeconomic indicators, of course.

Future research focuses on other possible ways of pairwise comparisons expression and their use in construction of modified human development index.

Czech Republic	**Average – traditional**	**Average – weighted**
CZ01	3.3	1.7
CZ02	15.5	11.8
CZ03	16.8	12.9
CZ04	21.8	19.8
CZ05	12.3	12.6
CZ06	10.2	8.9
CZ07	16.8	16.5
CZ08	18.6	15.9
Hungary		
HU10	3.3	3.7
HU21	29.2	25.3
HU22	29.3	22.3
HU23	34.5	33.7
HU31	33.8	34.5
HU32	28.6	31.5
HU33	31.1	31.1
Poland		
PL11	9.2	10
PL12	1	1.3
PL21	6.3	8.3
PL22	2.4	5
PL31	11.4	17.2
PL32	18.9	23.3
PL33	23.2	25.7
PL34	27.1	30.9
PL41	6.9	6.8
PL42	17.3	20.5
PL43	28.5	27.6
PL51	5.6	6.2
PL52	31.2	30.9
PL61	16.8	20.1
PL62	25.6	29.2
PL63	11.8	12.7
Slovakia		
SK01	7.9	3.3
SK02	19.7	16.6
SK03	23.8	23.4
SK04	30.3	28.8

Table 3: Average ranking of regions by traditional and weighted approach

APPENDIX

Ranking of Visegrad Four NUTS 2 regions in years 2004-2013 derived by the TNHDI (T)/WNHDI (W)

	2004		2005		2006		2007		2008		2009		2010		2011		2012		2013		average	
	T	W	T	W	T	W	T	W	T	W	T	W	T	W	T	W	T	W	T	W	T	W
Czech Republic																						
CZ01	4	1	4	1	4	1	4	2	2	2	3	2	3	2	2	2	3	2	4	2	3.3	1.7
CZ02	17	11	20	13	19	11	17	11	14	11	14	12	15	12	12	11	13	13	14	13	15.5	11.8
CZ03	16	12	18	11	17	12	18	12	16	13	18	13	19	14	16	14	15	14	15	14	16.8	12.9
CZ04	24	20	24	19	23	18	24	20	22	20	20	19	21	21	18	19	21	20	21	22	21.8	19.8
CZ05	15	13	14	12	13	13	13	13	13	12	13	14	13	13	11	12	10	12	8	12	12.3	12.6
CZ06	11	8	10	8	10	9	11	9	10	8	11	10	12	10	9	9	8	9	10	9	10.2	8.9
CZ07	18	16	17	16	15	15	16	17	15	14	21	18	14	17	17	17	17	17	18	18	16.8	16.5
CZ08	19	15	19	15	21	17	21	16	20	17	19	16	20	18	15	15	16	15	16	15	18.6	15.9
Hungary																						
HU10	2	3	3	3	3	3	3	4	3	4	4	4	4	4	4	4	4	4	3	4	3.3	3.7
HU21	25	22	25	23	26	23	30	25	29	25	33	28	31	28	31	27	31	26	31	26	29.2	25.3
HU22	26	18	28	21	28	21	31	22	33	23	31	25	30	24	28	22	28	23	30	24	29.3	22.3
HU23	34	31	34	32	35	35	35	35	35	34	35	34	35	34	34	34	34	34	34	34	34.5	33.7
HU31	32	34	32	33	33	34	34	34	34	35	34	35	34	35	35	35	35	35	35	35	33.8	34.5
HU32	28	29	29	29	25	28	28	33	28	33	28	33	32	32	32	33	29	33	27	32	28.6	31.5
HU33	30	28	30	28	29	29	29	32	30	32	32	32	33	33	33	32	33	32	32	33	31.1	31.1
Poland																						
PL11	7	10	9	10	9	10	9	10	9	10	7	9	8	9	10	10	12	11	12	11	9.2	10
PL12	1	2	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1.3
PL21	8	9	8	9	6	8	6	8	6	9	6	8	7	8	6	8	5	8	5	8	6.3	8.3
PL22	3	5	2	5	2	5	2	5	4	5	2	5	2	5	3	5	2	5	2	5	2.4	5
PL31	10	17	11	18	11	19	10	18	11	18	10	15	10	15	14	18	14	18	13	16	11.4	17.2
PL32	21	25	21	25	20	25	15	24	19	24	16	22	17	22	21	23	19	22	20	21	18.9	23.3
PL33	23	27	23	27	27	27	22	26	23	26	22	24	23	25	23	25	23	25	23	25	23.2	25.7
PL34	29	35	31	35	31	32	27	31	26	31	25	29	25	29	26	28	25	29	26	30	27.1	30.9
PL41	6	6	7	7	8	7	8	7	8	7	9	7	6	7	5	7	6	7	6	6	6.9	6.8
PL42	14	23	15	22	14	20	14	19	17	19	15	20	18	20	22	21	22	21	22	20	17.3	20.5
PL43	27	26	27	26	30	26	26	27	31	30	29	30	27	26	29	29	30	28	29	28	28.5	27.6
PL51	5	7	5	6	5	6	5	6	5	6	5	6	5	6	7	6	7	6	7	7	5.6	6.2
PL52	33	33	35	34	34	33	32	30	27	28	27	27	29	31	30	31	32	31	33	31	31.2	30.9
PL61	13	19	13	20	16	22	20	21	18	21	17	21	16	19	20	20	18	19	17	19	16.8	20.1
PL62	31	32	26	30	24	30	25	29	24	29	24	26	24	27	27	30	26	30	25	29	25.6	29.2
PL63	12	14	12	14	12	14	12	14	12	16	12	11	11	11	13	13	11	10	11	10	11.8	12.7
Slovakia																						
SK01	9	4	6	4	7	4	7	3	7	3	8	3	9	3	8	3	9	3	9	3	7.9	3.3
SK02	20	21	16	17	18	16	19	15	21	15	23	17	22	16	19	16	20	16	19	17	19.7	16.6
SK03	22	24	22	24	22	24	23	23	25	22	26	23	26	23	24	24	24	24	24	23	23.8	23.4
SK04	35	30	33	31	32	31	33	28	32	27	30	31	28	30	25	26	27	27	28	27	30.3	28.8

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