Conceptual Model Mathematical as a Strategy for Learning in Students of Initial Levels of the Careers of Science of Engineering.

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***Abstract*—** In the present investigations the authors show an application scheme of the mathematical models in the resolution of problems of the engineering sciences is presented through research projects that contribute to the development of the mathematical knowledge of the first level students of engineering science. In the study the authors show the application of numerical methods such as linear interpolation, Lagrange polynomial interpolation and regression models through Statgraphics software for electromechanical engineers. These tools are expressed in the resolution of problems of low complexity and allow establishing evaluation criteria to the students on the importance of the study of the mathematics for the professional formation in the area of the engineering. The use of numerical methods tools for solving practical problems in first-level students of engineering science careers develops critical analytical skills and strengthens them in the use of computational sciences for electromechanical engineering

***Keywords*—** Mathematical Model, numerical methods, Lagrange Polynomial.

I. Introduction

The Mathematics is a core subject in the curricular designs of higher education institutions; it is considered one of the basic sciences necessary for the integral educational development of the human being. [1].

There are multiple approaches to mathematics by specialties of the engineering sciences, being their indisputable utility for the professional work of the engineer; because the value of mathematical knowledge for solving problems in the context of science, technology and society is recognized. [2].These aspects make math a key pillar in the education of higher education students.

In universities where engineers are trained, it is vitally important to link mathematics to practice, through the teaching of applied mathematics in the first levels, showing the usefulness of the subject to solve specific problems that arise in professional development.[1,2].

This factor motivates the students as it acquaints them with their future professional work; however there are previous deficiencies resulting from their training at the lower level that limits them in the adequate understanding of mathematical knowledge. One of the alternatives that contribute to the eradication of the deficiencies and the stimulation of the motivation for the mathematics in the students of engineering sciences is the use of mathematical models in the resolution of problems through research projects in the initial levels professional deformation. [3].

II. Procedure for Paper Submission

The mathematical model has been conceptualized by several authors as the quantitative nexus between problem situations and alternative solutions. According to BlomhØj (2003), he refers to "a mathematical model is a relation between certain mathematical objects and their connections on the one hand, and on the other, a non-mathematical situation or phenomenon" [4]

This criterion of BlomhØj evidences the mathematical model as a connection between non-mathematical elements and mathematical elements. In the context of Ecuadorian higher education, the application of mathematical models as a learning strategy is not widely used, it is important to use mathematical models mainly in higher level subjects such as linear algebra and numerical methods, maintaining in most cases as a priority Theory on practice. In the present article the mathematical models are assumed as an essential motivational strategy to achieve the practical linkage of mathematical knowledge in the students of initial levels of higher education.

For the application of the mathematical model effectively it is necessary to maintain a dialectical unity between the problematic situation and the mathematical language [5].

The precise description of the characteristics of the problem allows establishing the proposed solution with the lowest error rate. This implies that there are three key factors for the application of the mathematical model in solving problems:

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1. Description and precision of the characteristics and qualities of the problem of study
2. Application of the mathematical model according to the problem
3. Proposition of solution alternative for the problematic

These factors are manifested in much broader phases that guide the student in the type of model to be used. In the case of the first factor, it is necessary to teach the student the methodology for solving problems, starting with the problem approach in three ways as suggested by Ballester et al (1992):

a) Find the problem related to the determination of certain quantities of magnitudes in the course of the debate with the student.
b) To pose the problematic situation that entails to the mathematical model directly
c) Direct the model and the student to verify the viability of the model according to the problem situation

An essential aspect in the orientation of the problem for mathematical modeling is the way in which the teacher applies the teaching of the problem method according to the effectiveness of the diagnosis of the students; it will be the way to pose the problem. [6].

It is important to add that Ecuadorian education encourages teachers to problem-based learning (PBL), a concept that conforms to mathematical modeling because according to Savery (2006): "PBL is a student-centered instructional (and curricular) approach Which allows the student to conduct research, integrate theory and practice and apply knowledge and skills to reach a viable solution to a given problem "[7].

This type of learning is fundamental to put into practice mathematical models as a way of solving the problems that arise, thus developing the theory of cognitive-constructivism learning, which focuses as a principle that the student builds knowledge itself and that the Teacher is a facilitator supporting the process of knowledge construction. [8].

III. CONCEPTUAL MODEL

For the execution and implementation of the mathematical model it is necessary to establish the premises that make possible its use:

1. The existence of a real problem situation in the context of daily life, which is measurable and descriptable, which generates the need for an alternative solution.
2. The student's observable capacity as modeler must be intuitive and insightful in order to be able to describe the characteristics of the problematic situation and with creativity can identify which model fits the given situation more
3. The formulation of the problem situation using the selected mathematical model, establishing the variables and their relationships to evaluate the feasibility of the model
4. The interpretation of the solution generated by the model and its tentative application to similar situations.

A simple way to show the application of the mathematical model from the operational point of view is through the following scheme:

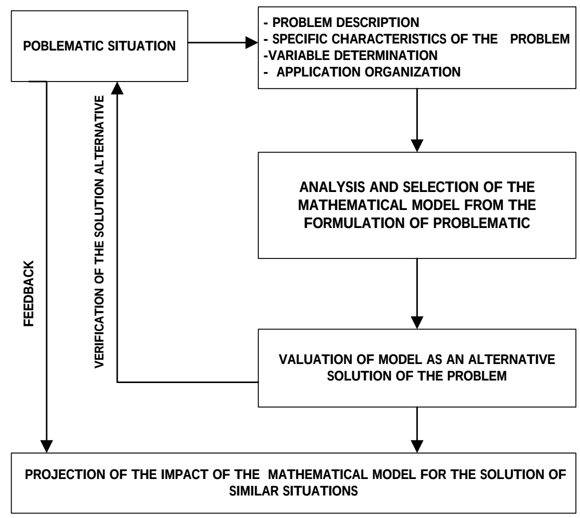

Fig.1: Conceptual Model and her application.

The model begins with the problematic situation that is derived mainly from a question as a starting point, after the student describe and determine the characteristics of the problem, identifying the variables that are going to be related and establish the organization of the information to analyze and select the appropriate mathematical model based on the formulation of the problem. When giving the solution with the model to the problem, the results are critically evaluated to consider whether the model is feasible as an alternative solution.

The impact of the model is then reviewed as a possible solution for similar situations. It is necessary to emphasize that in the evaluation phase of the problem, if the difficulties are found, we proceed with a feedback from the beginning to achieve the level of impact of the model.

IV. APPLICATION OF CONCEPTUAL MODEL

Example of problem for students of Automotive Engineering**:** Relation of the fuel consumption in the vehicles and the air pressure of the tires.

The purpose of the next problem is to exemplify how mathematical modeling is applied in practical problem situations of students of engineering sciences. The situation described below is part of the question: Does the tire pressure affect the fuel consumption of vehicles? With the question arises the problematic situation where the student must analyze the problem specifying the specific characteristics of the same, determining the variables involved, for the case study the student has as variables the following: fuel consumption and tire pressure. Given the focus of the question it is perceptible that the independent variable is the pressure in the tires and the dependent variable is the fuel consumption in the vehicles. After the determination of the variable comes the logical processing for the research algorithm, where the student has to establish an organization of the application path of his knowledge to know if there is incidence and what type of relationship characterizes it, if it is directly proportional or inversely proportional. This generates the following hypotheses:

Hypothesis of the problem (Hi): the air pressure in the tires affects the fuel consumption of the vehicles

Null hypothesis (Ho): the air pressure in the tires does not affect the fuel consumption of the vehicles

Having determined the hypotheses, we proceed to the selection of the mathematical model to be applied. To select the right model the student should investigate the results of previous research on the subject, which requires a study of the tires and their characteristics to see the correlation between the variables. In the case study the teacher provides a table to the student with measurements of the tire pressure and the fuel consumption percentage of approximately 4 months (17 weeks), where the student from the table should give solution to the problem:

Time (weeks)	Tire Pressure (Psi)	Fuel Consumption (%)
1	37,7	100
3	30,45	101,8
8,5	26,1	**Y**
12	21,75	
17	20,3	

Fig.2: Incomplete measurement chart

Looking at the table we have the data but the fuel consumption data corresponding to the eight and a half weeks onwards is missing, which can be obtained by linear interpolation from the trend in the values recorded in the table.

Based on this criterion the mathematical formula or model to be developed is established:

$$Y = Y1 + \frac{(x - x1)(Y2 - Y1)}{(x2 - x1)}$$

Developing this equation that defines the model to give continuity to the data table proceeds to complete the table obtaining the following data:

Time (weeks)	Tire Pressure (Psi)	Fuel consumption (%)
1	37,7	100
3	30,45	101,8
8,5	26,1	102,88
12	21,75	103,96
17	20,3	104,32

Fig. 3 Complete measurement table

With the data obtained by the model, we proceed to graph for a better interpretation of the same.

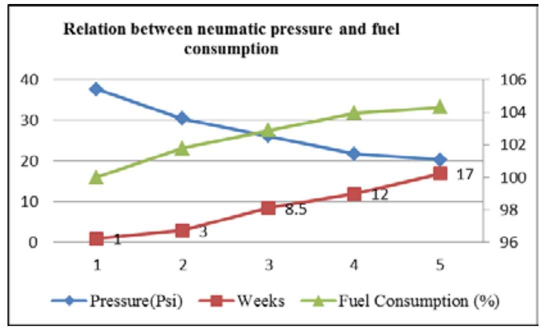

Fig. 4 Graph of ratio tire pressure and fuel consumption

With the revision of the graph, an assessment of the mathematical model is made to establish the solution of the study problem. As it is perceptible in the graph is fulfilled the hypothesis of investigation that raises that the air pressure in the tires influences in the fuel consumption of the vehicles.

For the verification of the solution of the problem we proceed to compare with similar studies made by experts and the comparison is made. In the present case, the information provided by Bridgestone Expert was used as an expert criterion. The website shows a graph that establishes the relationship between the pressure of heavy-duty tires and fuel consumption.

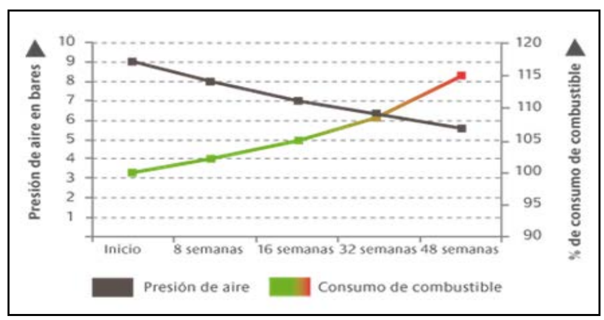

Fig. 5 Graph of tire pressure ratio and the fuel consumption according to Brigdestone.

Comparing both graphs, it is perceived that the trends are similar with lines of linear trends of inversely proportional relationship between variables, thus verifying the validity of the model.

Example of problem for students of Electromechanical Engineering: Relationship of power factor and apparent power as a measure of energy loss in the circuit.

The present problem shows the development of the conceptual model in an application form for students of electromechanical engineering. It is part similar to the previous one of a question as a problem situation: Does the power factor in a power grid affect energy losses? On the basis of this question, the student describes the problem, determines the specific characteristics of the problem, identifies the variables, in this case the independent variable is the power factor and the dependent variable is the apparent power as a measure of energy loss. With the variables identified, an analysis of the most appropriate model is performed to relate the variables and to see the incidence level, reviewing the results obtained in a critical and evaluative way from the hypothesis of the study.

Hypothesis of the problem (Hi): the power factor affects the loss of energy in the electrical network

Null hypothesis (Ho): the power factor does not affect the loss of energy in the electric network

Considering the hypotheses, one proceeds to the choice of the mathematical model to be applied. The implementation of the model requires the review of previous research on the subject of study, and similar to previous cases the teacher provides the table with the measurements.

Tension(V)	Power Receiver (Watt)	Power Factor	Apparent Power (Watt)	Intensity(A)
110	40	0.96	41.7	0.38
110	40	0.87	46.0	0.42
110	40	0.62	64.5	0.59
110	40	0.51	78.4	0.71
110	40	0.44	90.9	0.83
110	40	0.35	114.3	1.04
110	40	0.25	160.0	1.45

Fig. 6 Table of data of electrical variables

From the data of the table we use the power factor and apparent power variables to apply a linear regression model for the study variables, using statgraphics centurion software version 16, the following results are obtained:

Dependent variable (S): Apparent power
Independent variable (Pf): Power factor
Linear: Y = a + b * X

	Least Square	Standard	Stadistical	
Parameter	Estimated	Error	T	Value-P
Intercept	168.146	17.2171	9.76622	0.0002
Pending	-145.305	27.7139	-5.24303	0.0033

Fig. 7 Coefficients

Source	Sum of square	Gl	Medium square	Ratio-F	Value-P
Model	8780.78	1	8780.78	27.49	0.0033
Residue	1597.13	5	319.425		
Total (Corr.)	10377.9	6			

Fig. 8 Variance Analysis

Correlation coefficient = -0.919839
R-square = 84.6103%
R-square (adjusted for g.l.) = 81.5324%
Standard error est. = 17.8725
Mean absolute error = 12.9992

The results of fitting a linear model to describe the relationship between Apparent Power (S) and power factor (Pf) are shown. The equation of the adjusted model is:

$$\mathbf{S} = 168.146 - 145.305 * \mathbf{Pf}$$

Since the P-value in the ANOVA table is less than 0.05, there is a statistically significant relationship between Apparent Power (S) and power factor (Pf) with a confidence level of 95.0%.

The R^2 statistic indicates that the adjusted model explains 84.6103% of the variability in Apparent Power. The correlation coefficient is equal to -0.919839, indicating a relatively strong relationship between the variables.

Since the P-value is less than 0.05, there is indication of a possible serial correlation with a confidence level of 95.0%.

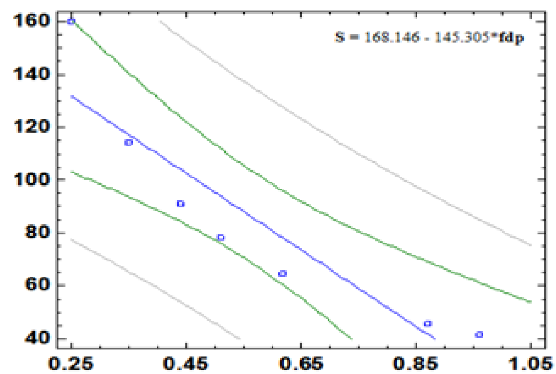

Fig. 9 Graph of the adjusted model

The model shows the level of relation between the variables, for a better understanding we proceed with the graphical representation of the variables from the data of the table of fig. 6:

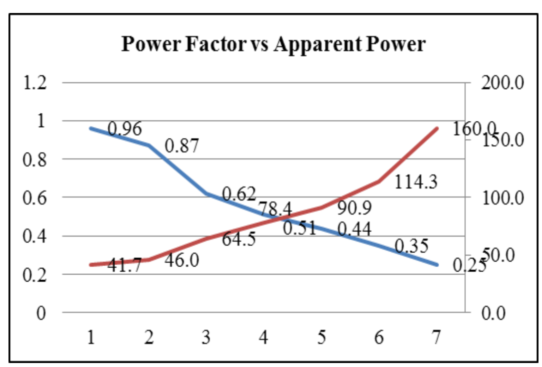

Fig. 10 Graph Ratio Power factor and Apparent Power

With the results it is observed that there is an inversely proportional relation, as the power factor decreases the apparent power increases, which means an increase of the energy loss.

V. EVALUATION OF THE PROPOSED MODEL

For the evaluation of the model, a questionnaire was initially applied to 40 students from the initial levels of the automotive engineering and electromechanical engineering career at the Equinoctial Technological University (UTE) Headquarters Santo Domingo.

The instrument was composed of several questions prevailing for the study the two shown in fig.11, which shows the percentage results at the beginning when they did not apply the proposed model and the later when they are already trained with the proposed model.

You master the steps for resolution of mathematical problems				
	Before	%	After	%
Full Agree	5	12.5	13	32.5
Agree	14	35	15	37.5
Indifferent	19	47.5	9	22.5
Disagree	2	5	3	7.5
Full Disagree	0	0	0	0
	40	100	40	100
The application of concepts, definitions and formulas in problems results in:				
	Before	%	After	%
Very easy to apply	5	12.5	11	27.5
Easy to apply	8	20	16	40
Neither easy, nor difficult	22	55	10	25
Difficult Application	5	12.5	3	7.5
Very difficult to apply	0	0	0	0
	40	100	40	100

Fig. 11 Evaluation of the model

From the comparative analysis shows how there was an improvement in the criteria, when weighting the data obtained according to the attitudinal criteria of the students, an improvement between the before and after is perceived.

VI. CONCLUSION

The incorporation of mathematical models as a learning strategy in students of engineering sciences careers through research projects contributes to the development of mathematical knowledge in a practical way.

The application of mathematical models through problem-based learning facilitates the development of students' mathematical competences and allows establishing evaluative criteria in their professional training.

The teachers at the initial levels should investigate and deepen the alternatives of problem situations that should work with the students according to the professional profile and follow the suggested steps for applying mathematical models.

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