# Location-Pricing Problem in the Closed-loop Supply Chain Network Design under Uncertainty

# F. Jolai, A. Zabihian, R. Tavakkoli-Moghaddam, P. Memari

**Abstract**— This paper presents a bi-objective credibility-based fuzzy mathematical programming model to design a location-pricing decision problem in a closed-loop supply chain with a single product under uncertainty. This problem aims to maximize the total supply chain profit by determining the optimal number of the facility location, collection and distribution centres (CDCs), the assignment of the customer zones to the CDCs and the CDCs to the plants, the price of the new product and the incentive value of the returned product that should be offered for used product. In order to cope with the uncertainties, an appropriate fuzzy method based on the credibility measure is developed. To validate the presented model, a numerical example is taken into account.

*Keywords*— Location-pricing decision, Closed-loop supply chain, Fuzzy mathematical programming, Credibility theory.

#### I. INTRODUCTION

A Closed-loop supply chain has been more attention in recent years for increasing the supply chain profit. It includes all reverse logistic activities as collecting, remanufacturing and refurbishing in addition to all forward activities [1]. Beside the environmental concern, a lot of countries force their companies to undertake take-back responsibilities to reduce waste. A closed-loop supply chain is driven by high profitably and growing attention on an environment. In the other words, by collecting the used product from a customer, companies can gain some value that this value increase companies profit and reduce raw materials consumption. So, recycling the used product and taking it back to customers may actually reduce waste, which will decrease a harmful effect of these waste in the environment. The objective of a closed-loop network design that consist both forward and reverse networks is to determine the number of locations, capacity of facilities, inventories policy and amount

of flow between the facilities [2]. In this paper, we represent a comprehensive and practical mixed-integer nonlinear multiobjective possibilistic model for the integrated closed-loop supply chain network design to decide on both the optimal location for distribution, collection and recovery center, optimal amounts of inventories that should be carried between these facility location centers, optimal price of the new product and refund value as an incentive value that should be offered for the used product, in order to maximize the supply chain profit. Facility location decision is one of the most important decision in closed-loop supply chain management. The most studies in this area consider this issue because opening or closing facility location is very expensive and it takes a long time [1]. This problem evaluates, in which the potential facility location should be selected for P facilities. First, for modelling the reverse supply chain network, a simple single-product uncapacitated model was used. By taking the time, it becomes more complicated with a multi-product capacitated and multiobjective problem [3]. All of the aforementioned problem are in an NP-hard class. So, many heuristic and meta-heuristic algorithms have been developed for solving these problems. The genetic algorithm [4], simulate annealing [5], Tabu search [6] and scatter search [7] are used for solving a mixed-integer linear programming model. A bi-objective mixed-integer linear programming model was developed for the integrated logistics network to avoid sub-optimality. The two main objectives of this paper are to minimize the total cost and maximize the responsiveness of a logistic network. For solving the problem, they represent an efficient memetic algorithm that uses a new dynamic search strategy by considering three different local search [8]. However, there are two drawbacks for using stochastic methods. In the first one, there are not enough historical data for an uncertain parameter to obtain the exact random distribution of an uncertain parameter and the second one is in most previous studies in this area. The uncertainty was modelled under a scenario-based stochastic programming which lead computationally challenging problem [3]. For satisfying aforementioned drawbacks, the fuzzy set theory [9] with the ability to handle different kinds of uncertainties is a good alternative. A bi-objective possibilistic mixed-integer programming model was proposed to cope with uncertainty in the closed-loop supply chain network design

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problem. In this paper, the network design decision was integrated in both forward and reverse supply chain networks. Also, it includes strategic network design decisions with a tactical material flow to avoid sub-optimality. An integrated fuzzy solution method was developed for solving this possibilistic optimization model [3]. A pricing decision model for a fuzzy closed loop with retail competition was proposed in the marketplace which the fuzziness was related to consumer demand, collecting cost and remanufacturing cost. The goal of this paper is how plant can reach optimal decision with respect to fuzzy environment. [10] proposed an uncertain reverse logistic network with different product recovery, whose uncertainty was related to demand and return quantities was proposed. They focused on inventory control and production planning optimization. Also, by using fuzzy trapezoidal numbers, they modelled the problem. For obtaining a solution to the inventory control and production planning, they considered a two-phase fuzzy mixed-integer optimization algorithm.

#### **II. PROBLEM DEFINITION**

A mixed-integer non-linear multi-objective possibilistic programming model for both forward and reverse supply chain is proposed. A new product that produces in plant is transfer to distribution center and then to customer zone through forward flow. In the reverse flow the used product first, collect at collection center and then these collected products shipped to recovery center for remanufacturing and then transfer to distribution center. We consider that each customer in customer zone go to distribution center themselves to buy the product which the demand of this customer depend on some parameter such as price and distance between customer zone and distribution center where by decreasing the price or a distribution center is opened at closer location to the customer in a certain zone k, more of the customer in that zone k will buy the product. The demand of a customer in zone k that go to distribution j will be  $D_{kj} = k_k e^{-kp} x_{kj} \alpha_{kj}$  where k is the multiplier price sensitivity of demand,  $x_{kj}$  is a binary variable which takes the value 1, if customer in zone k go to distribution j and 0 otherwise,  $\alpha_{kj}$  is a parameter between 0 and 1, depending on the distance between zone k and j. In this study we assume that customer returns the end life of the product and these end life of the product collect at collection center. It is obvious that amount of return product from customer depend on distance between customer zone and collection center. Similar to new product demand, number of returned product account by  $k_k(1 - e^{-br})x_{kl}\beta_{kl}$  where b is incentive sensitivity of the collected amount,  $x_{kl}$  is a binary variable which takes the value 1 if customer from zone k go to collection center l, 0 otherwise,  $\beta_{kl}$  is a binary variable depending on the distance between customer zone k and collection center l. Since, data in real world situation are inaccessible, especially in long horizon, most of the parameter fixed in closed loop supply chain network design have an ambiguous nature. In order to satisfying this drawback, we use an appropriate possibility distribution [11]. Also, we proposed a decision horizon consist multiple periods in proposed model and we determine flow quantities between facility due to demand and return and other periodic base parameter at each period. The main assumption that we consider in this paper are as follows:

- There isn't any scrap in our considered closed loop network design
- All demand of customer must be satisfying and all the return product must be collect
- In the forward flow product follows a push mechanism
- In the reverse flow product follows a pull mechanism
- Location of plants and customer zones are determined in advance

III. MODEL FORMULATION

Indice	S:					
i	index for fixed location of plant <i>iel</i>					
j	index for candidate location for distribution center $j \in J$					
k	index for fixed location for customer zones <b>kek</b>					
ı	index for candidate location for collection centre					
m	index for candidate location for recovery center $m \in M$					
t	index for time period tel					
Param	neters:					
$k_k$	number of people in zone <b>k</b>					
Dkjt	demand of customer in zone $k \in K$ that go to distribution center $j \in J$					
	at period t					
Bklt	amount of return products from zone $k \in K$ to collection center $l \in L$					
	at period <b>t</b>					
$\beta_{kl}$	parameter between 0 and 1 depending on the distance between ${m k}$					
	and l					
s	the expected value from return product					
C	cost of a producing a new product					
k	the price sensitivity of demand					
Ь	the incentive sensitivity of return amounts					
Ĵ,	fixed cost for opening distribution center <b>j</b>					
$\tilde{g}_{l}$	fixed cost for opening collection center <i>l</i>					
<i>b</i>	fixed cost for opening recovery center m					
cõij	transportation cost from plant <i>i</i> to distribution center <i>j</i>					
$c\widetilde{u}_{jk}$	transportation cost from distribution $i$ to customer zone $k$					
cqu	transportation cost from customer zone $\boldsymbol{k}$ to collection center $\boldsymbol{l}$					
$c\widetilde{p}_{lm}$	transportation cost from collection center $l$ to recover center $m$					
$\widetilde{\boldsymbol{\varphi}}_{j}$	processing cost at distribution center <b>j</b>					
β <sub>1</sub>	processing cost at collection center <i>l</i>					
$p\tilde{x}_j$	maximum capacity of distribution center <i>j</i>					
$p\tilde{z}_m$	maximum capacity of recovery center m					
tdjk	delivery time from distribution center $\mathbf{j}$ to customer zone $\mathbf{k}$					
py	maximum capacity of collection center <i>l</i>					
$p\widetilde{p}_i$	maximum capacity of plant <i>i</i> at each period					
tẽ <sub>kt</sub>	expected delivery time of customer $\mathbf{k}$ in period $\mathbf{t}$					
D <sub>t</sub>	$[j td_{ik} \ge t\tilde{e}_{ik}]$					
Vari	ablas:					
vuru						
p	optimum price value offered for unit product					

r	optimum incentive value offered for unit used product

*o*<sub>*ijt*</sub> amount of products transferred from plant *i* to distribution center *j* 

at period **t** 

- $u_{jkt}$  amount of products transferred from distribution center j to customer zone k at period t
- $q_{klt}$  amount of products transferred from customer zone k to collection center l at period t
- $p_{lmt}$  amount of products transfer from collection center l to recovery center m at period t
- $h_{mjt}$  amount of products transfer from recovery center *m* to distribution center *j* at period *t*
- $x_j$  1, if a distribution center is opened at location j; 0, otherwise
- $y_l$  1, if a collection center is opened at location l; 0, otherwise
- Z<sub>m</sub> 1, if a recovery center is opened at location m; 0, otherwise
- $x_{kj}$  1, if customer at zone k is served by a distribution center j; 0, otherwise
- $ch_{mi}$  transportation cost from recover center *m* to distribution centre *j*

## IV. MATHEMATICAL MODEL

$$\max w_{1} = \sum_{k \in K} \sum_{j \in J} (p - c) k_{k} e^{-kp} x_{kj} \alpha_{kj} + \sum_{k \in K} \sum_{j \in J} (s - r) k_{k} (1 - e^{-br}) x_{kl} \beta l$$

$$\min w_{2} = \sum_{j} \tilde{f}_{j} x_{j} + \sum_{l} \tilde{g}_{l} y_{l} + \sum_{m} \tilde{b}_{m} z_{m} + \sum_{l} \sum_{j} \sum_{k} (c \tilde{u}_{jk} + \tilde{\varphi}_{j}) u_{jkl} + \sum_{l} \sum_{i} \sum_{j} (c \tilde{o}_{ij} + c) o_{ijl}$$

$$+ \sum_{l} \sum_{k} \sum_{l} c \tilde{q}_{kl} q_{kll} + \sum_{l} \sum_{m} \sum_{m} (c \tilde{p}_{lm} + \tilde{\beta}_{l}) p_{lml} + \sum_{l} \sum_{m} \sum_{j} (c \tilde{h}_{mj} + \tilde{\tau}_{m}) h_{mjl}$$

$$s t :$$

$$\sum u_{jkl} \ge \sum D_{jkl} \quad \forall k, k$$
(1)
(2)
(3)

$$\sum_{i} q_{klt} \ge \sum_{i} B_{klt} \qquad \forall \mathbf{k}, \mathbf{t}$$
(4)

$$\sum_{i} o_{ijt} + \sum_{m} h_{mjt} = \sum_{k} u_{jkt} \quad \forall j, t$$
(5)

$$\sum_{j} u_{jkt} = \sum_{l} q_{klt} \qquad \forall \mathbf{k}, \mathbf{t}$$
(6)

$$\sum_{k} q_{klt} = \sum_{m} p_{lmt} \qquad \forall \mathbf{k}, \mathbf{t}$$
(7)

$$\sum_{l} p_{lmt} = \sum_{j} h_{mjt} \qquad \forall \mathbf{j}, \mathbf{t}$$
(8)

$$\sum_{j} o_{ijt} \le p \tilde{p}_{i} \qquad \forall i , t$$
<sup>(9)</sup>

$$\sum_{i} o_{ijt} + \sum_{m} h_{mjt} \le x_{j} p \tilde{x}_{j} \qquad \forall j, t$$
(10)

$$\sum_{k} q_{klt} \le y_{l} p \tilde{y}_{l} \qquad \forall l, t$$
(11)

$$\sum_{l} p_{lmt} \leq z_{m} p \tilde{z}_{m} \qquad \forall m, t$$
(12)

$$\mathbf{x}_{j}, \mathbf{y}_{l}, \mathbf{z}_{m}, \mathbf{x}_{kj}, \mathbf{x}_{kl} \in \{0, 1\} \quad \forall j, l, m, k$$
 (13)

$$\mathbf{O}_{ijt}, \boldsymbol{u}_{jkt}, \boldsymbol{h}_{mjt}, \boldsymbol{q}_{klt}, \boldsymbol{p}_{lmt} \ge 0 \qquad \forall i, j, k, l, m, t$$
(14)

# V. CREDIBILITY-BASED FUZZY CHANCE CONSTRINED PROGRAMMING MODEL

In most of real life situations, the input parameters of a logistics network design problem are tainted by high degree of epistemic uncertainty. To cope with this challenging issue, a new hybrid credibility-based chance constrained programming model is proposed in this research. Let  $\hat{\boldsymbol{\varepsilon}}$  be a fuzzy variable with membership function  $\boldsymbol{\mu}(\boldsymbol{x})$ , and let  $\boldsymbol{r}$  be a real number. Based on [12] the credibility measure is defined as follows:

$$\operatorname{cr} \{ \tilde{\varepsilon} \le r \} = \frac{1}{2} (\sup \mu(x) + 1 - \sup \mu(x))$$
  
Noteworthy, since  
$$\operatorname{pos} \{ \tilde{\varepsilon} \le r \} = \sup \mu(x) \text{ and } \operatorname{Nec} \{ \tilde{\varepsilon} \le r \} = 1 - \sup \mu(x),$$

the credibility measure can also be defined as follows:

$$\operatorname{cr}\left\{\tilde{\varepsilon} \leq r\right\} = \frac{1}{2} \left( pos\left\{\tilde{\varepsilon} \leq r\right\} + Nec\left\{\tilde{\varepsilon} \leq r\right\} \right)$$

Accordingly, the credibility measure could be defined as an average of the possibility and necessity measures. Also, the expected value of  $\hat{\varepsilon}$  can be determined based on the credibility measure as follows [12]:

$$E\left[\tilde{\varepsilon}\right] = \int \operatorname{cr}\left\{\tilde{\varepsilon} \ge r\right\} dr - \int \operatorname{cr}\left\{\tilde{\varepsilon} \le r\right\} dr$$

Now, assume that  $\hat{\boldsymbol{\varepsilon}}$  is a trapezoidal fuzzy number denoted by four prominent points  $\tilde{\boldsymbol{\varepsilon}} = \left(\varepsilon_{(1)}, \varepsilon_{(2)}, \varepsilon_{(3)}, \varepsilon_{(4)}\right)$ .

It can be proven that if  $\hat{\epsilon}$  is a trapezoidal fuzzy number and  $\alpha > 0.5$  then:

$$\operatorname{cr}\left\{\tilde{\varepsilon} \leq r\right\} \geq \alpha \leftrightarrow r \geq (2 - 2\alpha)\varepsilon_{(3)} + (2\alpha - 1)\varepsilon_{(4)}$$
$$\operatorname{cr}\left\{\tilde{\varepsilon} \geq r\right\} \geq \alpha \leftrightarrow r \leq (2\alpha - 1)\varepsilon_{(1)} + (2 - 2\alpha)\varepsilon_{(2)}$$

According to above mentioned descriptions and justifications, the proposed credibility based fuzzy mathematical programming can be formulated as follows:

$$\max_{1} = \sum_{k \in K} \sum_{j \in J} (p - c) k_{k} e^{-kp} x_{kj} \alpha_{kj} +$$

$$\sum_{k \in K} \sum_{j \in J} (s - r) k_{k} (1 - e^{-br}) x_{kl} \beta l$$

$$\min E(w_{2}) = \sum_{j} E(\tilde{f}_{j}) x_{j} + \sum_{j} E(\tilde{g}_{j}) y_{j} + \sum_{m} E(\tilde{b}_{m}) z_{m} +$$

$$\sum_{i} \sum_{j} \sum_{i} (E(c\tilde{o}_{ij}) + c) o_{ij} + \sum_{i} \sum_{j} \sum_{k} (E(c\tilde{u}_{jk}) + E(\tilde{\phi}_{j})) u_{jki} +$$

$$\sum_{i} \sum_{k} \sum_{l} E(c\tilde{q}_{kl}) q_{kli} + \sum_{i} \sum_{l} \sum_{m} (E(c\tilde{p}_{lm}) + E(\tilde{\beta}_{l})) p_{lmi} +$$

$$\sum_{i} \sum_{m} \sum_{j} (E(c\tilde{h}_{mj}) + E(\tilde{\tau}_{m})) h_{mji}$$
s. t. constraint (3)-(8)
$$(17)$$

$$\operatorname{cr}\left\{\sum_{j} o_{ijt} \leq p\tilde{p}_{i}\right\} \geq \delta_{k} \qquad \forall i,t,k$$
(18)

$$\operatorname{cr}\left\{\sum_{j} o_{ijt} + \sum_{m} h_{mjt} \leq x_{j} p \tilde{x}_{j}\right\} \geq \mathcal{G}_{k} \qquad \forall j, \mathbf{t}, \mathbf{i}$$
(19)

$$\operatorname{cr}\left\{\sum_{j} q_{klt} \leq y_{l} p \tilde{y}_{l}\right\} \geq \partial_{k} \qquad \forall l, \mathbf{t}, \mathbf{j}$$
(20)

$$\operatorname{cr}\left\{\sum_{j} p_{lmt} \leq z_{m} p \tilde{z}_{m}\right\} \geq \pi_{l} \qquad \forall l, m, t$$
(21)

$$\mathbf{x}_{j}, \mathbf{y}_{l}, \mathbf{z}_{m}, \mathbf{x}_{kj}, \mathbf{x}_{kl} \in \{0, 1\} \quad \forall j, l, m, k$$
 (22)

By considering the expected value of trapezoidal fuzzy numbers, the above-mentioned credibility-based chance constraint programming model can be converted to the following crisp equivalent MILP model:

$$\max_{1} = \sum_{k \in K} \sum_{j \in J} (p - c) k_{k} e^{-kp} x_{kj} \alpha_{kj} +$$

$$\sum_{k \in K} \sum_{j \in J} (s - r) k_{k} (1 - e^{-br}) x_{kl} \beta l$$

$$\min_{1} E(w_{2}) = \sum_{j} \left( \frac{f_{i(1)} + f_{i(2)} + f_{i(3)} + f_{i(4)}}{4} \right) x_{j} + \sum_{l} \left( \frac{g_{l(1)} + g_{l(2)} + g_{l(3)} + g_{l(4)}}{4} \right) y_{l}$$

$$+ \sum_{i} \sum_{k} \sum_{k} \left( \left( \frac{co_{ij(1)} + co_{ij(2)} + co_{ij(4)} + co_{ij(4)}}{4} + c \right) \right) \rho_{ijl} +$$

$$\sum_{i} \sum_{k} \sum_{k} \left( \left( \frac{cq_{kl(1)} + cq_{kl(2)} + cq_{kl(3)} + cq_{kl(4)}}{4} \right) \right) q_{kll}$$
(23)

$$+\sum_{t}\sum_{j}\sum_{k}\left(\left(\frac{cu_{jk(1)}+cu_{jk(2)}+cu_{jk(3)}cu_{jk(4)}+\varphi_{j(1)}+\varphi_{j(2)}+\varphi_{j(3)}+\varphi_{j(4)}}{4}\right)\right)\mu_{jkt} + \\ +\sum_{t}\sum_{l}\sum_{m}\left(\left(\frac{cp_{im(1)}+cp_{im(2)}+cp_{im(2)}+cp_{im(4)}+\beta_{l(1)}+\beta_{l(2)}+\beta_{l(3)}+\beta_{l(4)}}{4}\right)\right)p_{imt} + \\ \sum_{t}\sum_{l}\sum_{m}\left(\left(\frac{ch_{mj(1)}+ch_{mj(2)}+ch_{mj(3)}+ch_{mj(4)}+\tau_{m(1)}+\tau_{m(2)}+\tau_{m(3)}+\tau_{m(4)}}{4}\right)\right)h_{mjt}$$

s.t:

$$\sum_{j} o_{ijt} \leq \left[ \left( 2\delta_k - 1 \right) p p_{i(1)} + \left( 2 - 2\delta_k \right) p p_{i(2)} \right] \qquad \forall \mathbf{i}, \mathbf{t}, \mathbf{k}$$
<sup>(25)</sup>

$$\sum_{i} o_{ijt} + \sum_{m} h_{mjt} \leq x_{j} \left[ \left( 2\mathcal{G}_{i} - 1 \right) p x_{j(1)} + \left( 2 - 2\mathcal{G}_{i} \right) p x_{j(2)} \right] \quad \forall j, t, i$$

$$(26)$$

$$\sum_{k} q_{klt} \leq y_{l} \left[ \left( 2\partial_{j} - 1 \right) p y_{l(1)} + \left( 2 - 2\partial_{j} \right) p y_{l(2)} \right] \qquad \forall \mathbf{l}, \mathbf{t}, \mathbf{j}$$

$$(27)$$

$$\sum_{l} p_{lmt} \le z_{m} \left[ \left( 2\pi_{l} - 1 \right) p z_{m(1)} + \left( 2 - 2\pi_{l} \right) p z_{m} \right] \qquad \forall l,m,t$$
(28)

$$\mathbf{x}_{j}, \mathbf{y}_{l}, \mathbf{z}_{m}, \mathbf{x}_{kj}, \mathbf{x}_{kl} \in \{0, 1\} \quad \forall j, \mathbf{l}, \mathbf{m}, \mathbf{k}$$
 (29)

$$\mathbf{o}_{ijt}, \boldsymbol{u}_{jkt}, \boldsymbol{h}_{mjt}, \boldsymbol{q}_{klt}, \boldsymbol{p}_{lmt} \ge 0 \qquad \forall i, j, k, l, m, t$$
(30)

It should be noted that in the above-mentioned formulation, we have assumed that the chance constraints should be satisfied with confidence level greater than 0.5  $(i \, e_{\cdot}, \delta_k, \vartheta_i, \partial_j, \pi_l \succ 0.5)$ 

## VI. THE PROPOSED FUZZY SOLUTION APPROACH

To solve the MOPMILP model, an interactive method is applied in this research. To cope with multiple objective problems, various methods have been proposed. Among these methods, fuzzy interactive methods are one of the most attractive approaches in this area because of their ability in measuring and adjusting the satisfaction level of each objective function based on the decision maker preferences in an interactive and progressive way. Noteworthy, the proposed interactive method applied in this paper uses the TH aggregation function [13] to convert the original bi-objective model to an equivalent single objective one. The steps of the proposed fuzzy interactive method can be summarized as follows.

1- Use the expected value of imprecise parameters to convert the fuzzy objective functions into their crisp ones.

2- Determine the minimum acceptable confidence level for each chance constraint,  $(i \, e_{.}, \delta_k, \vartheta_i, \partial_j, \pi_l)$  to convert the chance constraints into their equivalent crisp ones.

3- Specify the a-positive ideal solution  $(\alpha - PIS)$  and  $\alpha$ -negative ideal solution  $(\alpha - NIS)$  for each objective function. To obtain the  $\alpha$ -positive ideal solutions and the corresponding objective functions values, i.e.,  $w_1^{\alpha-PIS}, x_1^{\alpha-PIS}$  and  $w_2^{\alpha-PIS}, x_2^{\alpha-PIS}$ , the equivalent crisp model should be solved for each objective function separately, and thereafter the  $\alpha$ -negative ideal solutions can be estimated as follows.

 $w_1^{\alpha-NIS} = w_1 x_2^{\alpha-PIS}, \ w_2^{\alpha-NIS} = w_2 x_1^{\alpha-PIS}$ 

4- Determine a linear membership function for each objective function as follows:

$$\mu_{1}(x) = \begin{cases} 1 & w_{1} \prec w_{1}^{\alpha - PIS} \\ \frac{w_{1}^{\alpha - NIS} - w_{1}}{w_{1}^{\alpha - NIS} - w_{1}^{\alpha - PIS}} & w_{1}^{\alpha - PIS} \leq w_{1} \leq w_{1}^{\alpha - NIS} \\ 0 & w_{1} \succ w_{1}^{\alpha - NIS} \end{cases}$$

$$\mu_{2}(x) = \begin{cases} 1 & w_{2} \prec w_{2}^{\alpha - PIS} \\ \frac{w_{2}^{\alpha - NIS} - w_{2}}{w_{2}^{\alpha - NIS} - w_{2}^{\alpha - PIS}} & w_{2}^{\alpha - PIS} \leq w_{2} \leq w_{2}^{\alpha - NIS} \\ 0 & w_{2} \succ w_{2}^{\alpha - NIS} \end{cases}$$

which  $\mu_h(x)$  denotes the satisfaction degree of *hth* objective function.

5- Convert the bi-objective equivalent crisp model into a single-objective one using the TH aggregation function that results to the following model.

$$\max w(x) = \gamma w_0 + (1 - \gamma) \sum_{h=1,2} v_h \mu_h(x)$$
  
s,t:  
$$w_0 \le \mu_h(x) \qquad h=1,2$$

$$x \in F(x)$$
  
 
$$\gamma, w_0 \in [0,1]$$

where F(x) indicates the feasible region involving the constraints of equivalent crisp model and  $w_0$  denotes the minimum satisfaction degree of objective functions (i.e.,  $w_0 = min_h\{\mu_h(x)\}$ ). Additionally,  $v_0$  and  $\gamma$  indicate the importance of the *hth* objective function and the coefficient of compensation, respectively. Indeed, the TH aggregation function actually looks for a compromise value between the min operator and the weighted sum operator based on the value of  $\gamma$ .

6- Determine the importance of the fuzzy goals  $(v_h)$  and the value of compensation coefficient  $(\gamma)$  based upon the decision maker preferences and solve the resulting single-objective crisp model. If the decision maker is satisfied with the obtained efficient solution, then stop and select the current solution as the final decision; otherwise go to the step 2 for seeking a new efficient solution by altering the required parameters such as  $\delta_k, \vartheta_i, \partial_j, \pi_l$  and  $\gamma$  according to the revised and updated preferences of the decision maker.

### VII. COMPUTAYIONAL EXPERIMENT

To demonstrate the validity and usefulness of proposed solution approach, numerical example is considered and the results of this numerical example showed in this section. The size of our test problem is shown in table 1 which it includes the number of distribution center, customer zone, collection center and recovery center. Triangular fuzzy parameters are generated based on [14], the three prominent point for imprecise parameter are estimated. The most likely  $(c^m)$  value of each parameter by using uniform distribution is generated at first and it is assumed that the related crisp value are equal to the most likely when the proposed crisp model is considered. For computing the most pessimistic  $(c^p)$  and most optimistic ( $c^{\circ}$ ) value of a fuzzy parameter  $\tilde{c}$ , two random number  $(r_1, r_2)$  are generated between 0.2 and 0.8 based on uniform distribution. So, based on these two random number,  $(c^{p})$  and (c°) are generated as follows [3].

$$c^{0} = (1+r_{1})c^{m}$$
 and  $c^{p} = (1+r_{2})c^{m}$ 

Random generation for the most likely values based on – uniform distribution are presented in table 2.

Table 1. The number of facilities in our test problem

Facilities	Number	Number	Number of	Number	Number
	of plant	of	potential	of	of
		customer	distribution	potential	potential
		zone	center	collection	recovery
				center	center
Number	1	5	4	4	3

 Table 2. The sources of random generation of the most likely values

parameters	Corresponding random
	distribution
fj	~uniform (180000,260000)
<b>g</b> 1	~uniform (180000,260000)
<i>b</i> <sub>m</sub>	~uniform (300000,400000)
co <sub>ij</sub>	~uniform (4,10)
cu <sub>jk</sub>	~uniform (4,10)
cqki	~uniform (4,10)
cp <sub>lm</sub>	$\sim$ uniform (4,10)
ch <sub>mj</sub>	$\sim uniform(4,10)$
$\varphi_j$	~uniform (1.5, 3)
βı	~uniform (1.5, 3)
τ <sub>m</sub>	$\sim$ uniform (2,4)
pp;	~uniform (500,750)
$px_j$	~uniform (180,300)
$py_1$	~uniform (220,350)
pz_	~uniform (250,350)
$td_{jk}$	~uniform (5,8)
te <sub>kt</sub>	~uniform (4,6)

For solving the possibilistic and crisp model mathematical model are coded with the solver BARON 9.3.1 with GAMS 23.6. Under different feasibility and importance weight  $v_h$  of objective function. It is noted that the value of  $\gamma$  is set 0.4 for this numerical example. The result of this numerical example is shown in table 3. As it can be seen in table 3 the Opt% Gap of the result increases by increasing the  $\alpha$  – *level* of the problem. It can be concluded that the TH method is appropriate and qualified method for solving the auxiliary MOLP problem, since it can obtain efficient solutions. TH method is more appropriate when decision makers have a tendency toward obtaining balanced efficient solutions and it pays more attention to minimum satisfaction level of objectives.

Table 3. The result of test problem with different  $\alpha$ -level

$\alpha$ – level	$\mu(w_1)$	$\mu(w_2)$	<i>w</i> <sub>1</sub>	W <sub>2</sub>	Opt% Gap	CPU time (s)
0.5	0.96	0.96	112375.30	475.23	4.5	1273
0.6	0.96	0.90	118836.05	469.10	6.7	930
0.7	0.91	0.90	123837.39	471.84	5.3	1375
0.8	0.83	0.85	137839.21	463.24	7.11	1581
0.9	0.79	0.92	163928.09	461.37	17.9	1126
1	0.85	0.95	164640.11	457.02	22.01	1384

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