

# Research on Volatility of Return of Chinese Stock-Market Based on Generalized Hyperbolic Distribution Family

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**Abstract** *Financial time series often present a nonlinear characteristics, and the distribution of financial data often show fat tail and asymmetry, but this don't match with the standpoint that time series obey normal distribution of return on assets, etc, which is considered by linear parametric modeling in the traditional linear framework. This paper has a systematic introduction of the definitions of GH distribution family and related statistical characteristics, which is based on reviewing the basic properties of the ARCH/GARCH model family and a common distribution of its disturbance. And select the Shanghai Composite Index and the Shanghai and Shenzhen (CSI) 300 index daily return rate index to estimate volatility model. GH distribution is used for further fitting to disturbance. This is done after take full account of the effective extraction of the model for the disturbance distribution information. The results show that the GH distribution can effectively fitting residuals distribution of the volatility models about series on return rate.*

**Keywords:** *Chinese stock-markets; generalized hyperbolic (GH) distribution; ARCH/GARCH model; series on return rate; volatility model*

## I. INTRODUCTION

The linear gaussian time series modeling has got a great deal of development, since Box and Jenkins (1970) provide general theoretical summary and modeling framework of autoregressive moving average (ARMA) model. The traditional economic time series modeling follow this linear relationship for a long time, however, many phenomena are not all linear in real economic life. In the early 1950s, Moran (1953) implied the limitations of the linear model in the Canada bobcats data modeling process. He noted the "strange" phenomenon in the data, that is the residuals of the sample points greater than the mean were significantly less than those residuals of the sample points less than the mean. This phenomenon can be explained in the follow-up literature as there are so-called "control effect" at different stages of population fluctuation (Tong,1990).The research for control effect and the set-up process including some non-standard features of the model breakthrough the scope of the traditional gaussian time series model. The description for the non-standard features is extremely rich, including of non-normality, Non-symmetry cycle, double form, the nonlinear relationship of delay variables, the prediction changes of state space, the sensitivity of the initial value, etc.

Fan Jianqin (2005) [1]summarized the modeling of the

nonlinear characteristics above. It can be ascribed to two kinds of methods, one is to continue to use the linear framework of the general autoregressive moving average (ARMA),and appropriately select white noise distribution so that the model constructed can fit the corresponding non-linear characteristics as far as possible, for example, allowing the conditional expectation of lagged variable with a nonlinear form. But it is obviously biased that the practical modeling of the traditional time series usually identify and judge the "right" white noise distribution function according to the specific observation data. Thus, the other is set the model into nonlinear form directly, such as the random variable can be expressed as nonlinear function of its corresponding lagged variable, which show due empirical value.

About theoretical exploration of nonlinear time series, its early development focused on exploring a variety of non-linear parameter model, including research ARCH/GARCH model of financial data volatility and its generalized form (Engle,1980;Bollerslev,1986) ecology and threshold model of economic data (Tiao and Tsay1994),etc. On the other hand, the latest progress of the non-parametric regression methods provide another effective selection method for nonlinear time series modeling (Hardle, Lutkepohi and Chen,1997;Fan and Yao,2003).

The distribution of return of financial assets has an important significance to financial assets investment, risk management and so on. Early linear model play a crucial role in analyzing finance time series. At the same time, empirical evidence and random sampling from financial market indicate the distribution of financial data in reality often show fat tail and asymmetry. Xiao Qingxian (2003) point out that the time series don't necessarily have stability even though the edge distribution of financial data sequence or transformed financial data sequence follow normal distribution in financial econometrics model. Therefore, the normal Gaussian distribution of traditional linear model fit the actual financial data distribution with great limitations.

In the process of looking for a more reasonable distribution hypothesis, stationary distribution and truncated stationary distribution one after another describe the actual return distributions instead of normal distribution. Since function analytical formulas of stationary distribution are not unique, difficult to estimate parameter, there is inconsistent judgment on the

choice of the cut-off point about truncated normal distribution, these shortages limit their research in the field of application (Cao Zhiguang, etc, 2005)[11]. In 2003, Feng Jianqiang and Wang Zhixin[13] studied the form of distribution function of various of synthesis index returns in Shanghai and Shenzhen stock markets and estimated parameters of distribution function, with stable Paretian distribution and t distribution as alternative, on the basis of describing all kinds of distribution function of the stock returns. Barndorff and Nielsen put forwarded Generalized Hyperbolic Distribution (the following short for GH distribution) in 2007. Eberlein and Keller (1985) [4] applied it to the financial sector, the tail of GH distribution is more suitable for describing finance time series with features as peak, heavy tail, biased and so on, it can be in good agreement with stylized facts of return of financial assets and also can be suitable to describe correlation properties of the returns fluctuation, so GH distribution has been developing rapidly in the finance field.

Stock returns change is a single variable time series, how to capture and explore the information of distribution characteristics which contains is the primary problem to study and discuss the characteristics of stock returns. Considering the tail of GH distribution is more suitable for describing finance time series with features as peak, heavy tail, biased and so on, it can be well consistent with "real distribution" of return of financial assets, and also be suitable to describe related characteristics of return volatility, these advantages can be effectively compensate for fitting and estimation error produced by the assume that the series on return rate obey normal distribution in Linear modeling. Therefore, this paper will use GH distribution to analyze specifically specific distribution of characteristics of return volatility in China's stock market. The remaining parts are as follow, the second part has a brief review of several disturbance distribution, and gives the definitions of GH distribution and relevant statistical characteristics. The third part chooses the Shanghai Composite Index and the Shanghai and Shenzhen (CSI) 300 index daily return rate as empirical object, first of all introduce volatility model to estimate the parameters, and then select representative disturbance sequences to do further fitting analysis with the help of GH distribution. The fourth part ends this article.

## II. ARCH/GARCH MODEL AND GH DISTRIBUTION FAMILY

### A. disturbance distribution of ARCH/GARCH Model

In financial time series, especially income sequence of financial assets, often with time-varying characteristics, we can often find the phenomenon of aggregation of volatility. That is sequence volatility is very large in some period of time, but pretty small in another period of time. Conditional variance of the residual is not only a function of time, it will be affected by last volatility. In order to describe the feature of financial sequence, Engle put forward ARCH model at first in 1982, which captured conditional heteroscedasticity of the sequence by moving averages of residual square that is not expected in the

past. But, ARCH model often require a high model order to obtain better fitting results in the practical applications, which increases the instability of the model and the difficulty of model estimates.

Usually assumed that the residual distribution is normal distribution when carrying out maximum likelihood estimation on the model. But, the peak and heavy tail phenomenon of financial sequence even conditional distribution, will make the assumption of the normal distribution cause errors to model set. Hence, it's necessary to introduce some heavy tail distribution beyond normal distribution to describe the characteristic the financial data, two kinds of common heavy tail distribution is t distribution and Generalized error distribution (GED). The probability density function of t distribution have a heavier tail than normal distribution, it can describe the heavy tail characteristic of returns, but t distribution lack of good statistical properties compared with normal distribution. In GED with  $\nu$  as its degree of freedom,  $\nu$  controls the form of distribution, when  $\nu$  equals to 2, GED is normal distribution; when  $\nu$  greater than 2, the tail is more thinner than normal distribution; when  $\nu$  less than 2, the tail is heavier than normal distribution, which is a form of residual distribution often used.

### B. GH distribution family

Jensen and Lunde (2001) supposed that the random error term of GARCH model obey son distribution of GH distribution that is Normal Inverse Gaussian distribution (NIG) in the volatility research of U.S. stock returns. The results show that fitting effect of NIG distribution is better than the classical t distribution. Eberlein and Prause (2002) point outed in allusion to the estimation of modeling the skewness of the return on assets that if allowing the existence of conditional bias, the fitting of GH distribution to conditional variance and the excess earnings can obtain more accurate prediction effect.

#### (1) GH distribution

McNeil, Freya and Embrechts (2005) give the general definition of the GH distribution. Supposing  $W (W \geq 0)$  is one-dimensional random variable and obey Generalized Inverse Gaussian distribution, denoted as  $GIG(\lambda, \chi, \psi)$ , we call

$$X = \mu + W\gamma + \sqrt{W}AZ \quad (1)$$

obey d-dimension GH distribution, denoted as  $X \sim GH(\lambda, \chi, \psi, \mu, \Sigma, \gamma)$ , where,

- (i)  $Z \sim N_k(0, I_k)$  and  $W$  is independent of  $Z$ ,
- (ii) structure matrix  $A \in R^{d \times k}$  and  $\Sigma = AA^T$  is positive definite,
- (iii) location parameter  $\mu \in R^d$  and drifting parameter  $\gamma \in R^d$ .

The density function of  $GIG(\lambda, \chi, \psi)$  is

$$f_{GIG}(w) = \left(\frac{\psi}{\chi}\right)^{\lambda/2} \frac{w^{\lambda-1}}{2K_{\lambda}(\sqrt{\chi\psi})} \exp\left(-\frac{1}{2}\left(\frac{\chi}{w} + \psi w\right)\right)$$

where,  $K_{\lambda}(x) = \frac{1}{2} \int_0^{\infty} w^{\lambda-1} \exp\left[-\frac{1}{2}x\left(w + \frac{1}{w}\right)\right] dw$

is Bessel function of the third kind and parameters meet the following relationship:

when  $\lambda > 0, \chi \geq 0, \psi > 0$ ;

when  $\lambda = 0, \chi > 0, \psi > 0$ ;

when  $\lambda < 0, \chi > 0, \psi \geq 0$ .

From type (1), we can note that  $X|W = w$  obey normal distribution

$$X|W = w \sim N_d(\mu + w\gamma, w\Sigma) \quad (2)$$

That is, given  $W$ , the conditional distribution of  $X$  obey Gauss normal distribution with  $\mu + w\gamma$  as mean and  $w\Sigma$  as variance.

It could be confirmed that the density function of  $d$ -dimensional GH distribution is

$$f_{GH}(x; \lambda, \chi, \psi, \mu, \Sigma, \gamma) = C \frac{K_{\lambda-d/2}(\sqrt{(\chi + Q(x))(\psi + Q(\mu + \gamma))}) e^{(x-\mu)'\Sigma^{-1}\gamma}}{(\sqrt{(\chi + Q(x))(\psi + Q(\mu + \gamma))})^{d/2-\lambda}} \quad (3)$$

where,  $C = \frac{(\sqrt{\chi\psi})^{-\lambda} \psi^{\lambda} (\psi + Q(\mu + \gamma))}{(2\pi)^{d/2} |\Sigma|^{1/2} K_{\lambda}(\sqrt{\chi\psi})}$ ,

$Q(x) = (x - \mu)'\Sigma^{-1}(x - \mu)$  is the function of  $x$ .

The expectation vector and covariance matrix of  $x$  are

$$E(X) = \mu + \gamma \left(\frac{\chi}{\psi}\right)^{1/2} \frac{K_{\lambda+1}(\sqrt{\chi\psi})}{K_{\lambda}(\sqrt{\chi\psi})}, \quad (4)$$

$$COV(X) = \left(\frac{\chi}{\psi}\right)^{1/2} \frac{K_{\lambda+1}(\sqrt{\chi\psi})}{K_{\lambda}(\sqrt{\chi\psi})} \Sigma + \gamma\gamma' \left(\frac{\chi}{\psi}\right) \times \left[ \frac{K_{\lambda+2}(\sqrt{\chi\psi})}{K_{\lambda}(\sqrt{\chi\psi})} - \left(\frac{K_{\lambda+1}(\sqrt{\chi\psi})}{K_{\lambda}(\sqrt{\chi\psi})}\right)^2 \right] \quad (5)$$

GH distribution maintain closure for linear transformation, that is if

$$X \sim GH(\lambda, \chi, \psi, \mu_0, \Sigma_0, \gamma), Y = BX + b,$$

where  $B \in R^{k \times d}, b \in R^k$ ,

$$Y \sim GH(\lambda, \chi, \psi, B\mu + b, B\Sigma B', B\gamma).$$

In type (1), we define  $E(W) = 1, \bar{\alpha} = \sqrt{\chi\psi}$ , then

$$\psi = \bar{\alpha} \frac{K_{\lambda+1}(\bar{\alpha})}{K_{\lambda}(\bar{\alpha})}, \chi = \bar{\alpha} \frac{K_{\lambda}(\bar{\alpha})}{K_{\lambda+1}(\bar{\alpha})}.$$

Now we can get another parameter representation of GH distribution, denoted as  $X \sim GH(\lambda, \bar{\alpha}, \mu, \Sigma, \gamma)$ . We adopt the parameter represent the following empirical part.

(2) Maximum Likelihood Estimation of GH distribution parameters

This article aims at fitting series on return rate under

the assumption of the univariate GH distribution, consequently mainly investigate unitary GH distribution. Let  $\Sigma = 1, d = 1$  in (3), then we can get density function of the univariate GH distribution for

$$f_{GH}(x; \lambda, \chi, \psi, \mu, \gamma) = \frac{(\sqrt{\chi\psi})^{-\lambda} \psi^{\lambda} (\psi + \gamma^2)^{1/2-\lambda}}{(2\pi)^{1/2} K_{\lambda}(\sqrt{\chi\psi})} \times \frac{K_{\lambda-1/2}(\sqrt{(\chi + (x - \mu)^2)(\psi + \gamma^2)}) e^{(x-\mu)\gamma}}{(\sqrt{(\chi + (x - \mu)^2)(\psi + \gamma^2)})^{1/2-\lambda}}$$

As a result, logarithm likelihood function can be expressed as:

$$\ln L(\theta; x_1, x_2, \dots, x_n) = \sum_{t=1}^n \ln f_{GH}(x_t; \theta) \quad (6)$$

where  $\theta = (\lambda, \chi, \psi, \mu, \gamma)$  is to be estimated parameters,  $x_1, x_2, \dots, x_n$  are observation samples.

Owing to existing the Bessel function in type (6), maximization is not easy. According to literature[1], we can apply EM algorithm to obtain maximum of type (6), and therefore get maximum likelihood estimation of the parameters.

### III. AN EMPIRICAL ANALYSIS

#### A. Descriptive statistics of the series on return rate and Stationary test

In view of our country's adopting the system of price limits in December of 1996, it's considered that the distribution of Shanghai Composite Index returns are different before and after the data. Meanwhile considering the launch time of Shanghai and Shenzhen 300 index is late, the paper select 3443 sample data from the Shanghai Composite Index daily return rate of January 2, 1997 to March 31, 2011, and 2235 sample data from Shanghai and Shenzhen 300 index daily return rate of Jan 7, 2002 to March 31, 2011 as the empirical sample series. These data come from thinking database. Adopting  $r_t = \log(x_t + 1)$  for data to approximately replace logarithmic return, all the data processing and figure generation of this paper are accomplished by R 2.13.0.

We often implement stationary test to the data at first in the actual analysis of univariate time series, DF statistics obtained by ADF unit root test, the ADF test lagged 5 separately are -23.6612 and -19.1812, all the p-value with two decimal places are  $0.01 < 0.05$ , the result indicates that the return data in the range of two samples are all stationary series.

Through table 3.1, it can be seen that distribution characteristic of the Shanghai Composite Index daily return rate in the time period taken is the mean less than the median, which shows left skewed fat tail and peak. The mean of Shanghai and Shenzhen 300 index returns is also less than median, presents left skewed, kurtosis quite approaches to 3. Table 3.2 give normality

test results of Shanghai Composite Index daily return series distribution. P-value of three kinds of statistics are very small, refusing the null hypothesis that series on return rate obey normal distribution.

TABLE I. THE BASIC STATISTICAL CHARACTERIZATION OF RETURN RATE

Sample	Maximum	Minimum	Mean	Median	Standard Deviation	Skewness	Kurtosis
the Shanghai Composite Index	0.094037	-0.093322	0.000337	0.000800	0.017225	-0.230114	4.136601
CSI 300 Index	0.089749	-0.096952	0.000400	0.001000	0.018524	-0.256458	3.095893

TABLE II. NORMALITY TEST RESULTS

	Shapiro-Wilk Test		Jarque-Bera Test		Kolmogorov-Smirnov Test	
	W Statistics Value	p-value	J-B Statistics Value	p-value	K-S Statistics Value	p-value
the Shanghai Composite Index	0.9454	< 2.2e-16	2490.12	< 2.2e-16	0.4733	< 2.2e-16
CSI 300 Index	0.9574	< 2.2e-16	920.2443	< 2.2e-16	0.4729	< 2.2e-16

Make use of nonparametric kernel density estimation to carry out distribution density estimation to series on return rate. First need to select sum parameters and smoothing parameter. In the fitting of nonparametric, the influences of the selection of smoothing parameter on the peak features of return distributions fitting curve are manifested as too big smoothing parameter has led to over-smooth curve, flareout peak; while the smaller smoothing parameter produces not enough smooth curve, kurtosis is on the high side. This paper employs default gaussian kernel

$$K_G(u) = (\sqrt{2\pi})^{-1} \exp(-u^2 / 2), u \in (-\infty, +\infty),$$

the selection of the optimal window-width uses Silverman rule of thumb

$$\hat{h}_{opt} \approx 1.06 \hat{\sigma} n^{-1/5}$$

for reference, where,  $\hat{\sigma}$  is the standard deviation of samples. As shown in figure3.2, when the optimal window-widths are respectively 0.002219 and 0.002699, we can get nonparametric kernel density estimation curve of return distribution within the sample interval between the Shanghai composite index and Shanghai and Shenzhen 300 index with good fitting effect.

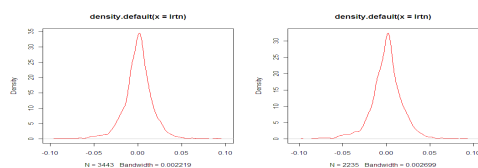


fig3.1 nonparametric kernel density estimation of series on return rate

It's obvious that the descriptive statistics and normality test as well as related kernel density fitting figure of the Shanghai composite index return data and Shanghai and Shenzhen 300 index return data above all indicate the rate of returns distribution is similar to symmetry distribution. But the tail appears the status of multi-peak and left-right asymmetry, among them the Shanghai composite index showed the peak phenomenon, which have a great relationship with the limit frequency of the data range, with uncertain randomness.

B. Standard error term GH distribution fitting

Due to non-normality of series on original return rate, this paper plans to use ARMA, GARCH and ARMA+GARCH conjunctive model to build the model of series on return rate, evaluate corresponding model parameters, test residual items of various fitting model whether meet the assumption of independent and identically distributed follow. Ling Qingquan and Zhang Jianlong(2010) pointed out that according to quasi maximum likelihood estimation, we can use Gaussian hypothesis to evaluate GARCH parameters even if error term disobey normal distribution, and the parameter values have consistency with the true values, the error between them has asymptotic normality. Table 3.3 gives statistical indicator(the lagging order are respectively 2,5,10) of ARCH significance test obtained by ARMA+GARCH model joint estimation in the form of three distribution(normal,t,ged).Statistical indicator of various orders all significantly pass the critical point(p=0.05), illustrating standard error produced by the above model obey the assumption of independent and identically distributed. On the basis of standard residuals are independent and identically distributed, introduce GH distribution and its subclass distribution to fitting on the data.

TABLE III. STANDARD ERROR OF SHANGHAI COMPOSITE INDEX MODEL ARCH EFFECT TESTING STATISTICAL INDICATOR

Model	Distribution Form	Statistical Index	The Lagging Order		
			2	5	10
ARMA(1,1)+GARCH(1,1)	normal	statistic	1.818	2.391	5.130
		p-value	0.4030	0.7929	0.8823
	t	statistic	2.094	2.725	5.267
		p-value	0.3509	0.7423	0.8726
	ged	statistic	1.963	2.615	5.222
		p-value	0.3748	0.7590	0.8759
ARMA(1,1)+EGARCH(1,1)	normal	statistic	2.598	3.575	5.995
		p-value	0.2728	0.6120	0.8157
	t	statistic	2.474	3.582	5.961
		p-value	0.2903	0.6110	0.8185
	ged	statistic	2.698	3.831	6.216
		p-value	0.2595	0.5739	0.7968
ARMA(1,1)+GJR-GARCH(1,1)	normal	statistic	1.566	2.231	4.916
		p-value	0.4570	0.8163	0.8967
	t	statistic	1.342	2.092	4.867
		p-value	0.5112	0.8363	0.8999
	ged	statistic	1.440	2.195	4.926
		p-value	0.4868	0.8215	0.8961

TABLE IV. STANDARD ERROR OF SHANGHAI AND SHENZHEN 300 INDEX MODEL ARCH EFFECT TESTING STATISTICAL INDICATOR

Model	Distribution Form	Statistical Index	the Lagging Number		
			2	5	10
ARMA(1,1)+GARCH(1,1)	normal	statistic	0.2648	1.5346	5.5404
		p-value	0.8760	0.9090	0.8523
	t	statistic	0.4403	1.3783	5.4045
		p-value	0.8024	0.9267	0.8626
	ged	statistic	0.3901	1.3595	5.4220
		p-value	0.8228	0.9287	0.8613
ARMA(1,1)+EGARCH(1,1)	normal	statistic	0.2459	1.6286	5.5998
		p-value	0.8843	0.8978	0.8477
	t	statistic	0.3616	1.4041	5.7101
		p-value	0.8346	0.9239	0.8390
	ged	statistic	0.3792	1.4502	5.6460
		p-value	0.8273	0.9187	0.8441

ARMA(1,1)+ GJR- GARCH(1,1)	normal	statistic	0.1783	1.2164	4.8816
		p-value	0.9147	0.9433	0.8989
	t	statistic	0.1509	1.0020	4.8005
		p-value	0.9273	0.9624	0.9041
	ged	statistic	0.1533	1.0207	4.8242
		p-value	0.9262	0.9609	0.9026

Select the above model through the ARCH effect testing, further analyze the stationary of the residual data. Fig.3.2 and Fig.3.3 provide self-correlation function figure and empirical distribution density figure of standardized residuals squared after the model estimation with Shanghai composite index

ARMA(1,1)+GJR-GARCH(1,1)+t

model as example. It's obvious that the residual term after the model estimation is leveling out, the extraction of model to residual is quite sufficient, standardized residual data after model fitting No longer have the long memory.

Table 3.5 and Table 3.6 provide mean, variance, skewness, kurtosis and empirical statistics of the quantile of Shanghai composite index and Shanghai and Shengzhen 300 index data model through different distribution forms. It can be seen from table 3.5 that the standard error sample of SSE series on return rate after model estimation shows significant peak, negative skewness and other characteristics.

TABLE V. DESCRIPTIVE STATISTICS OF RESIDUAL SERIES AFTER SHANGHAI COMPOSITE INDEX MODEL ESTIMATION

	Minimum	Maximum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Model 1	-0.09419	0.09432	0.00009	0.000573	0.017229	-0.22659	4.146267
Model 2	-0.09416	0.094013	-0.00019	0.00028	0.017227	-0.22639	4.143515
Model 3	-0.09418	0.093859	-0.00026	0.0002	0.017225	-0.22745	4.144652
Model 4	-0.094	0.094404	0.000296	0.000772	0.017226	-0.22818	4.148466
Model 5	-0.09407	0.09408	-0.0001	0.000368	0.017227	-0.22666	4.143855
Model 6	-0.09398	0.093928	-0.00015	0.000307	0.017224	-0.22786	4.144346
Model 7	-0.09409	0.094519	0.00027	0.000767	0.01723	-0.22644	4.146563
Model 8	-0.09416	0.094153	-0.0001	0.000371	0.017229	-0.22583	4.14337
Model 9	-0.09412	0.093982	-0.00017	0.000303	0.017226	-0.22712	4.144451

TABLE VI. DESCRIPTIVE STATISTICS OF RESIDUAL SERIES AFTER SHANGHAI AND SHENGZHEN 300 INDEX MODEL ESTIMATION

	Minimum	Maximum	Mean	Median	Standard deviation	Skewness	Kurtosis
Model 1	-0.097598	0.089911	0.000141	0.000799	0.018513	-0.25944	3.062942
Model 2	-0.103771	0.096456	0.000032	0.0006	0.018545	-0.16456	3.638354
Model 3	-0.102985	0.094883	-0.00018	0.000306	0.018528	-0.1815	3.56305
Model 4	-0.097454	0.089884	0.000257	0.000933	0.018512	-0.25903	3.064743
Model 5	-0.103882	0.097152	0.000251	0.000812	0.018551	-0.15811	3.661962
Model 6	-0.103201	0.095619	0.000009	0.000502	0.018534	-0.17563	3.594445
Model 7	-0.097478	0.090058	0.000268	0.000943	0.018513	-0.25948	3.061945
Model 8	-0.103632	0.096596	0.000154	0.000727	0.018542	-0.16349	3.636151
Model 9	-0.103059	0.095263	-6.6E-05	0.000398	0.01853	-0.17849	3.577606

The skewness and kurtosis of standard error between the models are little fluctuation. It can be seen from table 3.6 that standard error of Shanghai and Shengzhen 300 index returns after model disposal shows negative skewness, the kurtosis value is slightly higher than 3. But skewness and kurtosis of the residual sequence between different models have obvious gap, for example, skewness and kurtosis of model 5 is maximum, skewness and kurtosis of model 7 is minimum.

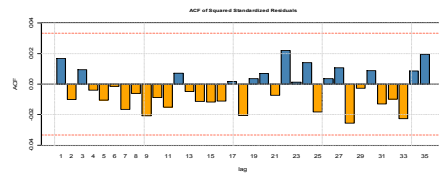


fig3.2 Shanghai composite index ARMA(1,1)+GJR-GARCH(1,1)+t model standardized residuals squared self-correlation function figure

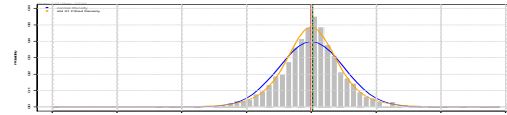


fig3.3 Shanghai composite index ARMA(1,1)+GJR-GARCH(1,1)+t model standardized residuals probability density figure

Note: the model number of table 3.5 and table 3.6 is given in turn according to the order of table 3.3 column 2

Since the skewness of normal distribution is 0 and the kurtosis is 3, as well as the skewness of t distribution is 0, in order to better to do accurate description of the return data, we need to re-examine whether the information extraction of residual sequence is abundant. In consideration of the good fitting effect of GH distribution to the peak, fat tail and biased characteristics of financial time series. For standard residuals, introduce GH distribution and its subclass distribution to fit in the following text. And for Shanghai composite index return data, select model 2 and model 8 whose skewness and kurtosis are quite small in the above nine models to fit, for Shanghai and Shengzhen 300 index returns, adopt two extreme skewness and kurtosis models, model 5 and model 7, to fit. The specific parameter estimations and density function graphs are shown in table 3.5 and figure 3.5-3.7.

Making use of quasi maximum likelihood parameter estimation method referred to in the previous article, in the case of distribution hypothesis is GH distribution, separately fit residual sequences of SSE returns series after estimated by model 2 and model 8, for Shanghai and Shengzhen 300 index return series, the parameters estimated by model 5 and model 7 are shown in table 3.7.

TABLE VII. GH DISTRIBUTION DISTRIBUTION PARAMETERS OF THE RETURN DATA

	Shanghai Composite Index		CSI 300 Index	
GH parameter	Model 2	Model 8	Model 5	Model 7
$\lambda$	-0.179479	-0.14516	-0.0004	1.04209
$\bar{\alpha}$	0.569969	0.55083	0.549707	0.000389
$\mu$	0.0008524	0.001047	0.001069	0.001803
$\Sigma$	0.0171972	0.017262	0.01854	0.018368
$\gamma$	-0.00102	-0.00115	-0.00082	-0.00154
Log-likelihood Ratio	9407.033	9406.579	5933.693	5909.581
AIC	-18804.07	-18803.2	-11857.4	-11809.2
Iterations	502	502	210	494
Convergence	FALSE	FALSE	TRUE	TRUE

In order to observe conveniently and intuitively the fitting of the sample data under different distribution hypotheses, this paper draws up two probability density curves of GH distribution of standard residual sequences estimated by different models. Based on the comparing between sample empirical distribution density curve of nonparametric kernel estimation and sample normal distribution density curve, considering sample numerical span range of the fitting data is quite large, we also provide logarithmic distribution density curves of the three kinds of probability distribution. See fig.3.5-3.8, the red dotted line is sample empirical distribution density curve, the green full line is fitting curve of GH distribution, the blue smoothed curve is normal distribution density curve.

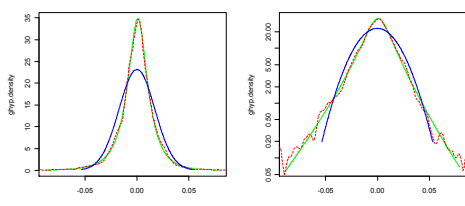


fig3.4 residual sequence distribution density curve (left) and logarithmic distribution density curve (right) of Shanghai composite index based on model 2 estimation

Fig.3.4 and 3.6 are separately three probability density curves, which are fitted by residuals of Shanghai composite index returns based on model 2 and model 8 estimation through GH distribution. Since the gap between skewness and kurtosis of residual sequence estimated by model 2 and model 8 is quite small, in addition to the tails of the two density curves have obvious differences, the empirical distribution density curve and the GH distribution fitting curve are almost overlapped. It indicates that GH distribution has a good fitting effect to residual sequence, especially when kurtosis of residual sequence of Shanghai composite index based on model estimation is quite big, it's better able to reflect the description ability of GH distribution to the fat-tail phenomenon and others of financial time series.

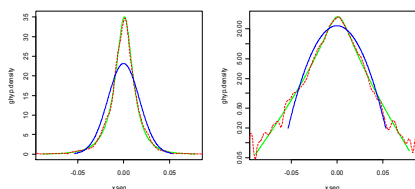


fig3.5 residual sequence distribution density curve (left) and logarithmic distribution density curve (right) of Shanghai composite index based on model 8 estimation

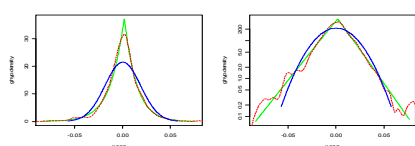


fig3.6 residual sequence distribution density curve (left) and logarithmic distribution density curve

(right) of CSI 300 index based on model 5 estimation

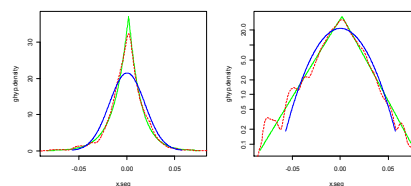


fig3.7 residual sequence distribution density curve (left) and logarithmic distribution density curve (right) of CSI 300 index based on model 7 estimation

Fig.3.6 and 3.7 are separately three probability density curves, which are fitted by residuals of Shanghai and Shengzhen 300 index returns based on model 5 and model 7 estimation through GH distribution. From table 3.6, we can see that skewness and kurtosis of residual sequence estimated by model 5 and model 7 stand for two extremes of nine kinds of models. Its performance on curve for empirical distribution density curve and the GH distribution fitting curve exist obvious gap, GH distribution curve presents the peak characteristic on the top, which is because of kurtosis of residual sequence is not significant. GH distribution loses useful information in the iterative process, it appears partial deviation for the fitting of sample series.

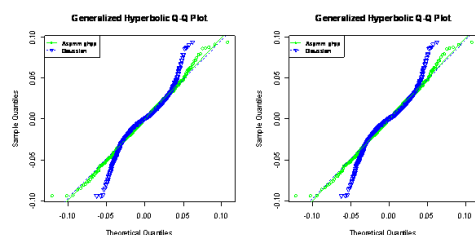


fig3.8 GH distribution quantile figure of residual sequence of Shanghai composite index based on model 2 (left) and model 8 (right)

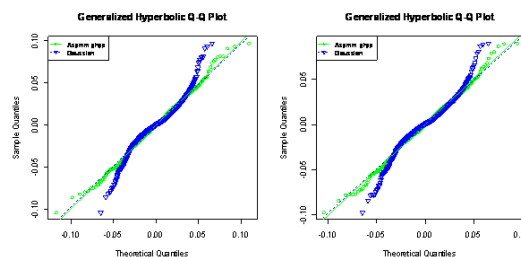


fig3.9 GH distribution quantile figure of residual sequence of CSI 300 index based on model 5 (left) and model 7 (right)

Fig.3.8 and 3.9 are GH distribution quantile figures of residual sequence separately based on four kinds of model estimation of two return series. The curve consists of green circle scattered points is GH distribution quantile line, the curve consists of blue triangle scattered points is normal Gaussian distribution quantile line. It's easy to see from the figure that most of the data fitted by GH distribution are concentrated near the 45° line, obviously better than the fitting of normal Gaussian distribution. Moreover, quantile figures of Shanghai composite index series based on model 2 and model 8 estimation are basically the same, while quantile figures of Shanghai and Shengzhen 300 index based on model 5

and model 7 estimation have greater volatility in both ends, have obvious outlier, the fitting effect of GH distribution is slightly worse than the fitting effect of Shanghai Composite GH distribution.

#### IV. CONCLUSIONS

Because of standardization and convenience of linear time series in modeling theory, the traditional way is generally assumed that financial asset returns obey the normal Gaussian distribution, however, the distribution of reality financial data series typically exhibits stylized fact characteristics of peak, fat tail and biased. The modeling of linear frame usually cause wrong understanding to the distribution of "real return rate". This paper breaks through traditional linear modeling constraints on the basis of literature introduced in the introduction section, and extracts the residual disturbance after the volatility model estimation of series on return rate, introduces the generalized hyperbolic distribution to further fit the distribution features of the disturbance through corresponding statistics describing and reasonable parameter estimation. The result shows that GH distribution can be more effective to fit sample disturbance sequence than the normal distribution in the volatility model.

However, the empirical part of this paper doesn't further discuss the proposition that "the assumption that GH distribution's direct replacement for the original information disturbance distribution of volatility modeling whether exist 'form setting' error. Meanwhile, the article only simulates for univariate time series, lack of corresponding discussion for time-varying volatility, portfolio structure analysis, options and derivatives pricing and other problems of financial markets. In addition, lack of consideration for other parametric representation of GH distribution, the nature of multivariate GH distribution and parameter estimation problem. I believe that the further study of these issues will be resolved effectively accompanied by theoretical research of financial time series and the development of measurement empirical method.

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