Empirical Analysis of the Phillips Curve in the Czech Republic

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Abstract**—**The information about inflation and unemployment is always impatiently expected, analyzed and commented not only by economists but also by inhabitants of the country. This article goes back to the original idea of the Phillips curve as a tool for empirical verification of the relationship between inflation and unemployment. The question that arises is whether the unemployment rate can be used to explain the changes in the inflation rate and on the contrary, in the Czech Republic. The verification of this relationship will be carried out on the basis of econometric models on the basis of the annual time series from 1995 to 2012. In the multivariate modeling of economic time series it is useful to distinguish between short-term and long-term relationships. It is natural that when examining the relationship between economic time series the cointegrated series are interesting.

Keywords**—**Phillips Curve, inflation, unemployment, time series, model ADL, cointegration.

I. Introduction

Inflation, together with unemployment, are considered to be the most important economic indicators of a state. Their current information is sensitively perceived not only by economists and economic analysts, but above all by the inhabitants of the given country. From the point of view of stastiticians, inflation manifests itself in the form of price level inflation [IR], which most often is counted as an outgrowth of the average yearly consumer price index. This means that it is perceived as a percentage change in the average price level for the following 12 months, against the average for the previous 12 months. Unemployment is expressed in the form of the Unemployment Rate [UR]. This means the number of unemployed people as a percentage of the total number of inhabitants of active age.

Analysis of the relationship between the level of wage inflation and the unemployment rate was first carried out in 1958 by A.W. Phillips [21]. The author, on the basis of empirical data pointed out the fact that in the period 1861-1957 in England, there was high unemployment accompanied by a growth in nominal wages. As has been said, the purpose of this work „…is to see whether statistical evidence supports the hypothesis that the rate of change of money wage rates in the UK can be explained by the level of employment and the rate of change of employment…". In other words, this explains whether the changes in the inflation rate could be dependent on changes in the unemployment rate. He assumed, therefore, a one sided, non-linear causal relationship coming from the unemployment rate to the rate of inflation.

After some time, [18], [20], [22] and many other authors joined their efforts to Phillips' work. The aim of their analysis was not only to test the one sided relationship leading from the unemployment rate to inflation [wage inflation, moreover, with the passage of time, was replaced in these models by price inflation], but also the opposite relationship; i.e., from inflation to unemployment. Depending on how the calculation technique was developed, there came into existence estimations of the Phillips Curve in the works of his followers which were more complicated and more demanding from the calculation point of view. The original Phillips Curve was constructed on the basis of the yearly time order, and it was, from the point of view of today's statisticians, a relatively simple statistical regression model [it did not contain a delay in both indicators. Even its quality was not tested].Today, thanks to sophisticated time order analysis methods, which started to be applied at the end of the last century [multi-dimensional time order models, and co-integration analysis]. The Phillips Curve became the new challenge for econometric experts and contributions on this theme constantly appear in prestigious professional journals (for all of them, see, for example [12]).

Because this article deals with the relationship between inflation and unemployment in the Czech Republic, it is necessary to mention many of the Czech authors, who have contributed over the past several years to the research into this question. They are, for instance,: [1], [2], [3], [10], [26].

II. Testing the Relationship Between Inflation and Unemployment in the Czech Republic

Data

Let us return, in this empirical part, to the original idea of the Phillips Curve; i.e., as an instrument for the empirical testing of the relationship between inflation and unemployment, and this in the conditions peculiar to the Czech Republic. In order to test this relationship we will use the

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yearly data from the period 1995-2012. Although the time series might be considered shorter, we tried to avoid unstable period of 90´s distorted by various economic factors. All data were provided by the Czech Statistical Office (CSO) and by the Ministry of Labour and Social Affairs, (MLSA).

The inflation rate in % (IR) was calculated as the growth rate of the Consumer Price Index (CPI)

$$IR_t = \frac{CPI_t}{CPI_{t-1}} \times 100 \qquad (1)$$

which publishes the CSO. The unemployment rate in % (UN) publishes the MLSA. The used data are published in quarterly frequency for the period I/2000–IV/2012. They were transformed into the yearly frequency.

In order to illustrate the nature of the main data, the following Figures 1 and 2 provides overall summarization of development of inflation versus unemployment relationship in the Czech Republic from 1995 to 2012.

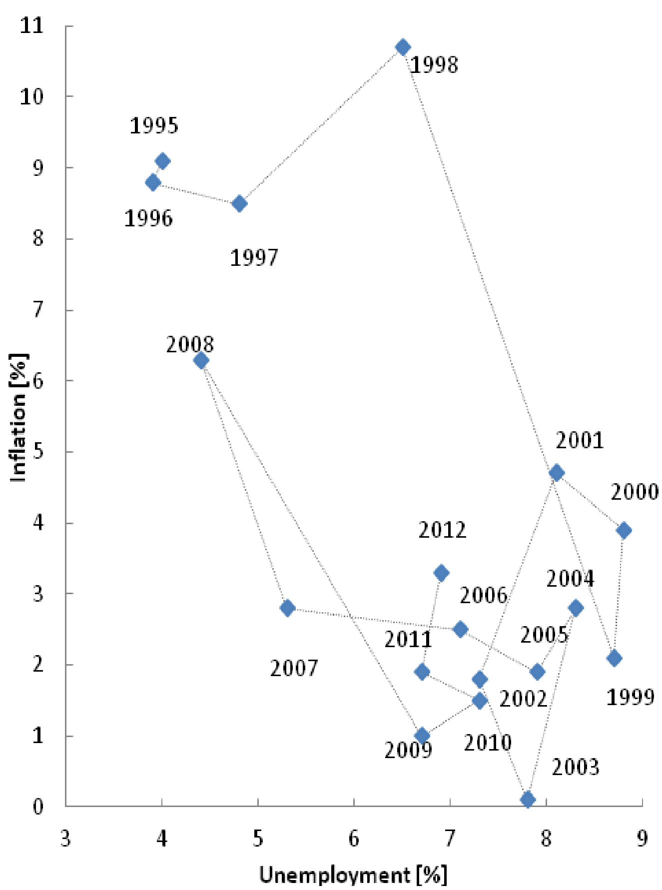

Fig. 1. Phillips Curve in the Czech Republic in the period 1995-2012
Source: data Czech Statistical Office

Fig. 1. shows on the point diagram the relationship between the inflation rate and the unemployment rate during the period 1995-2012. It is apparent that in the short term [e.g., 1996-1999, 2005-2008 and 2008-2009] the relationship between both indicators can be identified fairly easily. However, can the assumed relationship over the entire period under examination be tested?

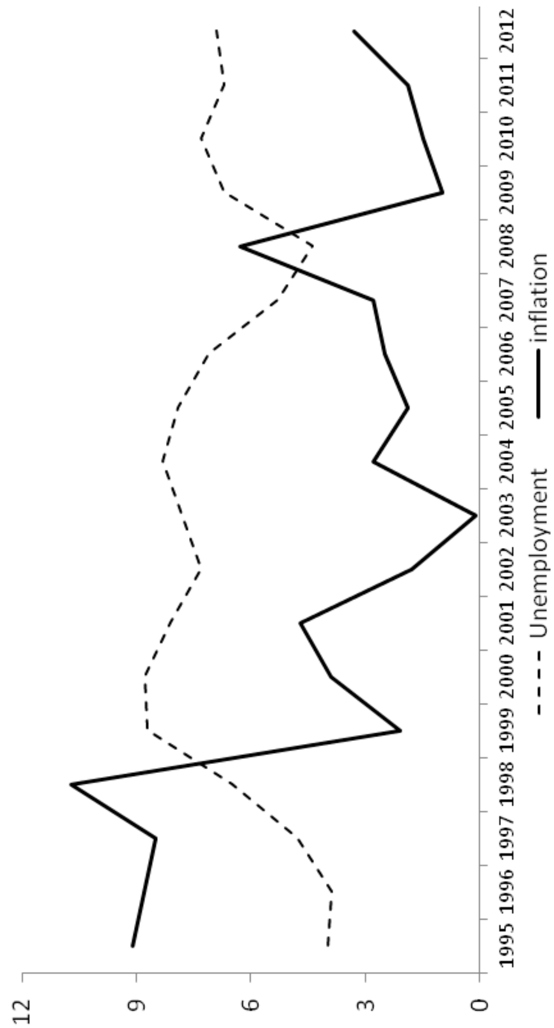

Fig. 2. The inflation rate and the unemployment rate in the Czech Republic in the period 1995-2012
Source: data Czech Statistical Office

Cointegration

In multivariate modeling of economic time series it is useful to distinguish between short-term and long-term relationships. The first type of relationships between the time series exists only in a relatively short period, these relationships over time disappear. The second type of relationships has a long duration, as the time goes it does not disappear. The issue of long-term relationships between time series is closely related to the concept of equilibrium (steady state). In this context it can be understood as a state to which the system is constantly attracted. As the system is exposed to continual shocks, it is never in equilibrium, but it can be in the long-run equilibrium, i. e. in the state, which converges towards equilibrium over time.

In the modeling of economic time series it is logic to follow the hypothesis that the development of time series connected by a theoretically reasoned economic relationship in the long run does not diverge. If the shift in trends of the time series is only short-term, it is lost in time and there is a limit beyond which it cannot go, then we say that the time series are in equilibrium. Statistical expression of this condition is called cointegration. If this limit is not here, then it is not possible to say that they are in equilibrium, from a statistical point of view, these time series are not cointegrated. It is natural that when examining the relationship between economic time series the cointegrated series are interesting. If the time series are not cointegrated, they contain no common element and their exploration as a system is irrelevant as their long-run development is independent.

The idea of cointegration of time series first appeared in the early 80th years in the works of C. W. J. Granger. This idea (as is obvious from its name) is based on the issue of integrated processes, which for the first time was comprehensively dealt by Box and Jenkins (1970).

There are different ways of classifying of economic time series. One of them is a division of the series according the memory: the short and the long memory time series. For series with the short memories, the influence of shock, which is caused by certain factors or groups of factors in one or a few seasons past, gradually disappears. For series with long memory it is not the case, the effect of the shock from history is still present.

Let's look at this issue in detail. Consider the time series generated by stationary process AR(1), i. e.

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \tag{2}$$

where $-1 < \rho < 1$ and $\{\varepsilon_t\}$ is the white noise process, i. e. proces with zero means, constant variances and zero autocovariances. The solution of this equation is

$$Y_t = \sum_{j=0}^{\infty} \rho \varepsilon_{t-j} \ , \tag{3}$$

so, individual coefficients of ε_{t-j} exponentially decline. This means that the impact of shocks that occurred in the past, gradually weakens over time. This property have generally all stationary AR processes and stationary and invertible processes ARMA. These processes are known as processes with short memory.

If $\rho = 1$, than

$$Y_t = Y_{t-1} + \varepsilon_t. \tag{4}$$

This process is called random walk. It can be expressed in the form

$$Y_t = \sum_{j=0}^{\infty} \varepsilon_{t-j} \ , \tag{5}$$

so, all shocks have the same weight. This process has long memory. The first difference of the random walk is the white noise process, which has a short memory.

Generally, suppose $\{X_t\}$ is a process with a short memory. then the process

$$Y_t = Y_{t-1} + X_t \tag{6}$$

has long memory.

Processes with long memory, which after the first difference are transformed into the processes with short memory are called the integrated processes of order one - I(1). In general, the processes that after the *d*th difference are transformed to processes with short memory are called integrated processes of order d - I(d). It follows that the short memory processes are called integrated processes of order zero - I(0).

Stationary processes I(0) and non-stationary processes I(d) differ in unconditional variance and autocorrelation function. While stationary processes have a finite variance, variance non-stationary processes grow indefinitely with $t \to \infty$. The values of the autocorrelation function of stationary processes are independent in time and with increasing time lag exponentially decline. The values of the autocorrelation function of non-stationary processes with $t \to \infty$ converge to one.

Distinguishing the stationary and non-stationary time series is very important in the analysis of their relationships. One of the most popular models of multiple time series is a single-equation regression model. Its construction must be done very carefully, because the use of non-stationary time series may result in a situation which is referred to as a spurious regression.

This situation means that the index determination, *t*-tests and *F*-test indicate the possibility of using the model even in the case of time series, which are unrelated. Because the spurious regression can occur when using a nonstationary time series (series of I(1) type), there is an possibility to remove it by differencing of the analyzed time series (stationarization of time series by using a deterministic function of time variables is not possible, since the generating process does not contain this deterministic element). But it turned out ([4], [13], [16]) that this is a wrong way, because the important information about the long-term characteristics of the relationship between the time series is lost.

The effort to construct a model that would respect both short and long-term relationships has led to the conclusion that for the modeling the undifferentiated time series must be used.

There are a few simple rules about the linear combinations of processes I(0) and I(1):

a) if $\{X_t\} \sim$ I(0), than $\{a + bX_t\} \sim$ I(0),

b) if $\{X_t\} \sim$ I(1), than $\{a + bX_t\} \sim$ I(1),

c) if $\{X_t\} \sim$ I(0) and $\{Y_t\} \sim$ I(0), than $\{aX_t + bY_t\} \sim$ I(0),

d) if $\{X_t\} \sim$ I(1) and $\{Y_t\} \sim$ I(0), than $\{aX_t + bY_t\} \sim$ I(1),

e) *generally*, if $\{X_t\} \sim$ I(1) and $\{Y_t\} \sim$ I(1), than $\{aX_t + bY_t\} \sim$ I(1).

In some cases, the final rule e) does not apply and linear combination of these processes is stationary, i.e., $\{aX_t + bY_t\} \sim I(0)$, such processes (and thus time series) are called cointegrated. The definition generally determines the relationship that may exist between the integrated processes. For two processes, it can be expressed as follows:

Definition 1: The processes $\{X_t\}$ and $\{Y_t\}$ are called cointegrated of order d, b, and are referred to as $\{X_t\}$, $\{Y_t\} \sim$ CI (d, b) if: a) they are both of I(d) type, b) there is a linear combination $\{aX_t + bY_t\} \sim I(d - b)$, where $b > 0$ Vector $[a, b]$ is called cointegration vector.

In the empirical time series econometrics the most interesting is the case when cointegration vector leads to the stationary linear combination, i. e., when $d = b$.In the case of the two processes there is only one cointegration vector, thus only one stationary linear combination exists.

There are at least three reasons why the principle of cointegration can be considered as the central idea of the integrated time series modeling.

I. The stationary linear combination of integrated (non-stationary) time series can be understood as an estimate of equilibrium.

II. Regression containing integrated time series makes sense only if these are cointegrated. The test of cointegration is thus simultaneously a method for distinguishing between the true regression and the spurious regression.

III. The group of cointegrated time series can be described by the error correction model, through which it is possible to distinguish between long-term and short-term relationships between time series.

Error correction model

First, consider stationary time series Y_t and Z_t, a time series of I(0) type. Their relationship can be modeled using a static regression

$$Y_t = c + \beta Z_t + u_t. \tag{7}$$

There can be two situations:

a) u_t is of white noise type,

b) u_t is autocorrelated, its development can be represented by AR(p) model.

In the first case, the parameters can be estimated and tested by standard procedures. More complicated is the second situation. In this situation the least squares method leads to estimates of parameters with underestimated standard errors which means that the hypothesis testing tends to reject the null hypothesis when it has to be taken.

The problem of autocorrelation can be solved using dynamic regression. The dynamisation of static regression means the adding the lagged variables into the model (7). These models are called ADL (p, q, k) (Autoregressive Distributed Lag models).

For example, the model ADL(1,1;1) has form

$$Y_t = c + \alpha_1 Y_{t-1} + \beta_1 Z_t + \beta_2 Z_{t-1} + v_t. \tag{8}$$

When examining the dependence of economic time series we are usually interested in the fundamental problem: how to determine the long-run equilibrium relationship (equilibrium) between endogenous and exogenous time series. In the case of static regression it can be determined easily: the relation (7) can be expressed in the mean values

$$E(Y_t) = c + \beta E(Z_t), \tag{9}$$

so the long-term relationship is given by parameter β, which in this context is called the long-term multiplier. In the case of dynamic regression (8) there are the relations

$$E(Y_t) = E(Y_{t-1}) \text{ a } E(Z_t) = E(Z_{t-1})$$

and therefore

$$(1 - \alpha_1)E(Y_t) = c + (\beta_1 + \beta_2)E(Z_t), \tag{10}$$

so

$$E(Y_t) = c^* + \beta^* E(Z_t), \tag{11}$$

where

$$c^* = \frac{c}{1-\alpha_1} \text{ a } \beta^* = \frac{\beta_1 + \beta_2}{1-\alpha_1}. \tag{12}$$

The long-term multiplier is in this case the parameter β^*. It is interesting that the model (8) can be expressed also in the following form

$$\Delta Y_t = c + \beta_1 \Delta Z_t + \gamma(Y_{t-1} - \beta^* Z_{t-1}) + v_t, \text{ where } \gamma = \alpha_1 - 1. \tag{13}$$

This model is called EC ("error correction"). The long-term relationship is expressed by regressor $(Y_{t-1} - \beta^* Z_{t-1})$, which includes long-term multiplier β^* given by (12). The regressor is referred to as component EC. The rest of the model (13) expresses the short-term relationship between time series. Parameter γ is the amount of differences of short-term relationship and the long-term relationship. It can be interpreted as the speed with which the short-term deviation from equilibrium is lost.

Let us return now to the static regression (7) and consider the time series Y_t and Z_t that are of I (1) type. There can be three situations:

a) u_t has white noise character, i. e. it is of I(0) type,

b) u_t is stationary and autocorrelated, i. e.it is also of I(0) type,

c) u_t is of I(1) type.

In situation a) no problem arises because the time series contained in the model are cointegrated and the regression parameter is a long-term multiplier. Similarly like in the stationary time series it is offered to solve the problems b) and c) by dynamization of the static regression, i. e. to model the relationship between the time series model by ADL model.

Let's look first at the situation c). Consider a model ADL (1,1,1) in the form (8). We know that this model can be converted into the EC model of (13) form. If the time series contained in the model are of I(1) type and a residual component is also of I(1) type, then in the model (8) $\alpha_1 = 1$, since the inclusion of an explanatory variable of I(1) type do not lower the integration rate of dependent variable. Then also $\gamma = 0$ and EC model (13) is transformed into the form

$$\Delta Y_t = c + \beta_1 \Delta Z_t + v_t. \tag{14}$$

This model does not contain long-term multiplier, because in the case of the non-cointegrated time series there is no equilibrium. So, it is not already the EC model. Obviously, the two-dimensional time series where the individual time series are not cointegrated can be stationarized by differencing each time series separately. It should be noted that, if the time series are used in the model (7), it is the spurious regression.

The situation b) indicates that the time series contained in the model are cointegrated. It has been proved (Granger theorem, see [8]), that in this case the model EC exists, because $\gamma \neq 0$. Intuitively, this situation can be explained similarly as the previous case: the inclusion of an explanatory variable of I(1) type, which is cointegrated with the dependent variable reduces the integration order of the dependent variable, so that in the model (8) $\alpha_1 < 1$. Due to the existence of long-term multiplier, equilibrium relationship ($Y_{t-1} - \beta^* Z_{t-1}$) containing the cointegrating vector $[1, -\beta^*]$ exists. The two-dimensional time series containing cointegrated time series is not stationarized by individual differencing of the time series.

When modeling relationships between time series of I(1) type, in the case of cointegrated time series it is not suitable to stationarize individual time series by differencing. If we will still make it, we would lost very important information which is contained in a EC model. It should be emphasized that the importance of EC model lies in the fact that it allows to combine the statistical and econometric approach to modeling of economic time series, it combines the advantages of modeling time series transformed by differencing and untransformed original time series, allowing to simultaneously capture the short-term and long-term relationships.

Unit root testing

There are several statistical tests to determine the order of integration, they are refered as the tests of unit roots. The most frequently used unit root test is the Dickey Fuller test. This test is used to distinguish whether the time series are of I(0) or I (1) type.

Consider the process

$$Y_t = \rho Y_{t-1} + u_t; \quad u_t \sim \mathrm{IID}(0, \sigma_u^2);\ Y_0 = 0. \tag{15}$$

When testing the hypothesis H_0: $\rho = \rho_0$, for $|\rho_0| < 1|$, the test criterion $t = (\hat{\rho} - \rho_0)/S_\rho$, where S_ρ is the estimate of the standard error of the parameter ρ estimate, has asymptotic standard normal distribution. In small samples this statistics has approximately distribution t. In the case $\rho_0 = 1$ it is not true.

In [6] the critical values for statistics t and $T(\hat{\rho} - 1)$ for the following models when the null hypothesis $\rho_i = 1$ for $i = a, b, c$ holds ware published

$$Y_t = \rho_a Y_{t-1} + u_t, \tag{16}$$

$$Yt = \mu_b + \rho_b Y_{t-1} + u_t, \tag{17}$$

$$Y_t = \mu_c + \gamma_c t + \rho_b Y_{t-1} + u_t. \tag{18}$$

In practice, the autocorrelation structure of the residual components of the generating process (15) can be richer. In this case the unit root testing is based on the model

$$Y_t = \rho Y_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta Y_{t-i} + u_t. \tag{19}$$

Statistics $T(\hat{\rho} - 1)$ and $(\hat{\rho} - 1)/S_\rho$ have limit distribution tabulated in the above mentioned Dickey tables for $T \to \infty$. Like in the case of AR (1) process, the model can be extended for the case where the generating process contains a constant and deterministic trend. These tests are called Augmented Dickey-Fuller tests ("ADF tests").

Cointegration testing

Testing of cointegration in a single-equation models can be based on an assessment of whether residuals of the static model (no lagged variables) in the form

$$\alpha \boldsymbol{X}_t = u_t \tag{20}$$

have a character of I(1) or I(0). In the first case, the series are not cointegrated and in the second case they are cointegrated.

In order to obtain residuals. the cointegration vector has to be estimated. Assume that all series of vector $\boldsymbol{X}_t$ are of I(1) type and are cointegrated by only one cointegration vector $\boldsymbol{\alpha}$. This vector can be estimated using the least squares method, this is based on the relation (20), in which one series is taken as dependent variable and the others as explanatory variables. If the residuals are of I(0) type, the regression is called as cointegration regression, if the residuals are of I(1) type, it is called as a spurious regression.

The testing is based on the residuals of the estimated regression model. It is tested the hypotheses that the residuals contain unit root, i. e. there is not cointegration. When the residuals are stationary time series, there is the cointegration. For testing, ADF test is use.

Another possibility for cointegration testing is based on the EC model in the form.

$$\Delta Y_t = c + \beta_1 \Delta Z_t + \gamma(Y_{t-1} - \beta^* Z_{t-1}) + v_t. \quad (19)$$

When parameter $\gamma = 0$, than there is no cointegration. For testing this hypothesis t-test can be used.

Weak exogeneity

The concept of weak, strong and super exogeneity was suggested by [27].

Each type of exogeneity has different purpose. For the parameter estimates the weak exogeneity is important, strong exogeneity is important for the construction of forecasts and analysis of policy definition is based on concept of super exogeneity. All three definitions specify the parameters of the model to which relate the exogeneity.

Joint probability density l-dimensional stochastic process $\{\boldsymbol{X}_t, t = 0, \pm1, \pm2, ...\}$ in time t has form

$$D_S(\boldsymbol{X}_t, \boldsymbol{X}_{t-1}, \boldsymbol{X}_{t-2}, ... | \boldsymbol{\omega}) = D_S(\boldsymbol{x}_t | \boldsymbol{\omega}), \quad (20)$$

where $\boldsymbol{x}_t = \{\boldsymbol{X}_{t-s}, s \geq 0\}$ and $\boldsymbol{\omega}$ is parameter vector. This joint distribiution is for $t = 0, \pm1, \pm2, ...$ called „data generatin proces". It holds

$$D_S(\boldsymbol{x}_t | \boldsymbol{\omega}) = \prod_{i \leq t} D_S(\boldsymbol{X}_i | \boldsymbol{x}_{i-1}; \boldsymbol{\omega}), \quad (21)$$

where $D_S(\boldsymbol{X}_i | \boldsymbol{x}_{i-1}; \boldsymbol{\omega})$ is conditional probability density under the condition $\boldsymbol{x}_{i-1}$.

If $X_t' = (Y_t', Z_t')$, where Y_t has dimension (m x 1) and Z_t has dimension (n x 1), than

$$D_S(\boldsymbol{X}_t | \boldsymbol{x}_{t-1}; \boldsymbol{\omega}) = D_P(\boldsymbol{Y}_t | \boldsymbol{y}_{t-1}, \boldsymbol{z}_t; \boldsymbol{\lambda}_1) D_M(\boldsymbol{Z}_t | \boldsymbol{y}_{t-1}, \boldsymbol{z}_{t-1}; \boldsymbol{\lambda}_2). \quad (22)$$

For $t = 0, \pm1, \pm2, ...$ $D_P(.|.)$ is conditional process and $D_M(.|.)$ marginal process. $\boldsymbol{\lambda}_1$ are parameters of conditional process and $\boldsymbol{\lambda}_2$ are parameters of marginal process (in more detail [16], [27], [28]).

Econometric model should correspond with conditional or marginal process depending on which of them is consistent with economic theory. It is clear, however, that econometrically interesting are usually models corresponding with conditional processes.

The concept of weak exogeneity is connected with the problem of parameter estimation of econometric model. Process $\{\boldsymbol{Z}_t\}$ is considered as weakly exogenous with respect to the group of parameters $\boldsymbol{\lambda}_1$, if marginal process $\{\boldsymbol{Z}_t\}$ does not contain any useful information for estimating of parameters $\boldsymbol{\lambda}_1$, i.e. if it is possible to obtain efficient parameter estimates $\boldsymbol{\lambda}_1$ only on the basis of conditional process.

Weak exogeneity will be specified more precisely. Process $\{\boldsymbol{Z}_t\}$ is weakly exogenous for parameters $\boldsymbol{\lambda}_1$, if it is possible to express the process of generating data by conditional and marginal process, i.e.

$$D_S(\boldsymbol{X}_t | \boldsymbol{x}_{t-1}; \boldsymbol{\omega}) = D_P(\boldsymbol{Y}_t | \boldsymbol{y}_{t-1}, \boldsymbol{z}_t; \boldsymbol{\lambda}_1) D_M(\boldsymbol{Z}_t | \boldsymbol{y}_{t-1}, \boldsymbol{z}_{t-1}; \boldsymbol{\lambda}_2), \quad (23)$$

and

(a) $\boldsymbol{\eta}$ is function only of parameters of the conditional process $\boldsymbol{\lambda}_1$, i. e. $\boldsymbol{\eta} = \boldsymbol{g}(\boldsymbol{\lambda}_1)$,

(b) parameters $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are variation free.

The condition (a) indicates that the parameters $\boldsymbol{\lambda}_1$ provide sufficient information needed to obtain parameters $\boldsymbol{\eta}$. The condition (b) assumes that the parameters ($\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$) belongs to the parametric space $\boldsymbol{\Lambda}_1$ x $\boldsymbol{\Lambda}_2$. Parameters $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are variation free if the parameter space $\boldsymbol{\Lambda}_1$ of parameters $\boldsymbol{\lambda}_1$ is not a function of parameters $\boldsymbol{\lambda}_2$ and parametric space $\boldsymbol{\Lambda}_2$ of parameters $\boldsymbol{\lambda}_2$ is not a function of parameters $\boldsymbol{\lambda}_1$. In econometric terminology this means that there is no mutual restriction linking parameters $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$, so that the knowledge of the parameters $\boldsymbol{\lambda}_2$ provides no information about the parameters $\boldsymbol{\lambda}_1$.

Weak exogeneity testing

Test of weak exogeneity can be practically done by testing of the presence of residuals from the marginal model in the conditional model as another explanatory variable. If these new explanatory variables does not belong into the conditional model, the real variables on the right side of the conditional model are supposed to be weak exogenous.

For testing of this hypothesis the likelihood ratio test, Wald test or Lagrange multiplier test are used. In special case when $m = n = 1$, i. e. one-equation conditional and marginal models, it is possible to test the weak exogeneity using standard t-test.

This type of test of weak exogeneity was suggested by [7]. In papers [7] and [14], the issue is analyzed in more detail and other more complex testing procedure is designed. Also [17] proposed a test procedure for weak exogeneity testing.

Empirical cointegration analysis

For testing this hypothesis, we use the standard methods which are used for the analysis of the time order, namely the cointegration analysis. The basic assumption of a possible, mutual, relationship between the time orders is, that they result from similar developments. Therefore, it is necessary to test whether the time orders under examination [Figure 2] are stationary [I(0)], or non-stationary [I(1)], because analysis of the relationships between the time orders make sense only if these time orders are integrated into the same order. From the ADF test [6], individual roots [Table I] are the result of both time orders being non-stationary, type I(1).

TABLE I. UNIT ROOT TESTS

1995-2012	y_t		Δy_t	
	t_{ADF}	**Prob.**	**t_{ADF}**	**Prob.**
IR	-1.85192	0.0623	-6.71773	0.0000
UR	-2.61244	0.1088	-3.68034	0.0011

Source: Own calculations

The Engle-Granger test of co-integration [8], which arises from analysis of the residue of the static regressive models of both time orders, from which we have eliminated the possibility of spurious regression [Table II]. The time orders are, therefore, co-integrated, and we can identify a long term relationship between them.

TABLE II. UNIT ROOT TEST OF a_t

$\hat{a}_t$	**t_{ADF}**	**Prob.**
	-2.632112	0.0119

Source: Own calculations

A. From unemployment to inflation

If we assume that there is a one sided direction causing the time order of the unemployment rate on the time order of the inflation rate, and if we know that $\hat{a}_t$ from Table II is auto-correlated, it is sufficient to estimate the relationship between both time orders in the ADL (*Autoregressive Distributed Lag model*, [8]) [Table III] in the form:

$$Y_t = c + \alpha_1 Y_{t-1} + \beta_1 X_t + a_t. \qquad (24)$$

TABLE III. ESTIMATE OF THE ADL MODEL

Dependent variable: IR				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	8.785245	2.956477	2.971525	0.0095
IR(-1)	0.388793	0.170633	2.278531	0.0378
UR	-0.960111	0.374551	-2.563364	0.0216
R-squared	0.573678	Durbin-Watson stat		2.3678
F-statistic	10.09233	Prob(F-statistic)		0.0016
Diagnostics tests			**Statistics**	**Prob.**
Breusch-Godfrey Serial Correl. LM Test:			0.558695	0.5851
Jarque-Bera Test			0.529154	0.7675
ARCH Test			0.009699	0.9229

Source: Own calculations

The following ADL model of the dependence of inflation on unemployment in the Czech Republic can be written like this:

$$\hat{IR}_t = 8.785245 + 0.388793 IR_{t-1} - 0.960111 UR_t, \qquad (25)$$

from which it arises that the inflation rate in time t directly depends on its value in time $t - 1$ and indirectly points to the unemployment rate in time t. Diagnostic control of the model indicates, the unsystematic make up of the model has the same properties of the processes of white noise [Table III]. The re-written equation of the ADL model into the form of the ECM (*Error Correction Model*; [8]) model, like so:

$$\Delta Y_t = c + \beta_1 \Delta X_t + (\alpha_1 - 1)[Y_{t-1} - \frac{\beta_1}{\alpha_1 - 1} X_{t-1}] + a_t, \qquad (26)$$

we get:

$$\Delta \hat{IR}_t = 8.7852 - 0.9601 \Delta UR_t - \\ - 0.6112(IR_{t-1} + 1.5708 UR_{t-1}), \qquad (27)$$

where we gain through the parameters $(\alpha_1 - 1) = -0.61121$ information about the speed with which the system reacts to deviations from equilibrium. The value of the long term multiplier $\beta_1/(1 - \alpha_1) = -1.57084$ indicates that in the period 1995-2012 in the Czech Republic was confirmed the long term indirect orientation of the dependence of the inflation rate on the unemployment rate, because the increase of the unemployment rate by one percentage point caused a drop in the inflation rate by an average of 1.57 percentage points.

B. From inflation to unemployment

If we consider the opposite directional flow, i.e., the dependence of unemployment on inflation, we get the ADL model from Table IV.

TABLE IV. ESTIMATE OF THE ADL MODEL

Dependent variable: UR				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	4.116266	1.421115	2.896505	0.0117
UR(-1)	0.522758	0.171439	3.049242	0.0087
IR	-0.239689	0.101610	-2.358907	0.0334
D1	2.439173	1.090637	2.236467	0.0421
R-squared	0.732137	Durbin-Watson stat		1.267881
F-statistic	12.75518	Prob(F-statistic)		0.000272
Diagnostics tests			**Statistics**	**Prob.**
Breusch-Godfrey Serial Correl. LM Test:			2.025284	0.1746
Jarque-Bera Test			0.099651	0.9514
ARCH Test			0.027736	0.8700

Source: Own calculations

This model is in the form:

$$\hat{UR}_t = 4.1163 + 0.5228 UR_{t-1} - 0.2397 IR_t + 2.4392 D_t \qquad (28)$$

from which it arises that the unemployment rate in time t directly and proportionately depends on its value in time $t - 1$ and indirectly proportionately on the inflation rate in time t. The model contains the artificially changed D_t (in which 1998 = 1, other periods = 0), for correcting deviations in the time order of the inflation rate, which emerged as a consequence of the financial crisis in the Czech Republic in the final years of the 20th century. Diagnostic control of the model indicates that the unsystematic make up of the model has the properties of the process of white noise [Table IV]. Upon rewriting the ADL model equation in the ECM form, we get:

$$\Delta \hat{UR}_t = 4.1163 - 0.2397\Delta IR_t - \\ - 0.4772(UR_{t-1} + 0.5022IR_{t-1}). \quad (29)$$

The reaction speed of the system to deviations from equilibrium is given by the parameters $(\alpha_1 - 1) = -0.47724$ and the value of the long term multiplier $\beta_1/(1 - \alpha_1) = -0.50224$ shows that increasing the inflation rate by one percentage point caused a drop in the unemployment rate by an average of 0.5 a percentage point. The long term indirect proportionate dependence of the unemployment rate on the inflation rate was confirmed, and overturned, in the Czech Republic in the period 1995-2012.

Empirical testing of weak testing

From the point of view of the above mentioned results, when in the Czech Republic during the period under examination, it was proven not only the dependence of the inflation rate on the unemployment rate, but also that of the unemployment rate on the inflation rate; it would be useful to test the exogenity of both time orders, in order to confirm or overturn the above discovered results.

In the classic regressive model it is assumed that the explanatory change is not correlated with the unsystematic make up of the model. In the case of the relationship between two stochastic time orders, this does not apply, however. If, for instance, the time order of the unemployment rate is not correlated with the unsystematic make up of the model, then the development of the inflation rate is contingent on the development of the time order of the unemployment rate and not the other way around.

The unemployment rate is then changed exogenuously and the inflation rate is changed endogenuously. If this condition does not apply, a one directional relationship between the time orders is not guaranteed. Also, it is necessary, for modelling the relationship, to use the double equation model; VAR, in which the alternative relationship is also dealt with. Therefore, we will carry out the exogenity test [7] for both time orders.

We shall, first of all, reveal the marginal model for both time orders [Table V], and the residue of this model will be used for testing the exogenity in the conditional models [from Table III and Table IV].

TABLE V. MARGINAL MODEL (STANDARD ERRORS IN ( ) & T-STATISTICS IN [ ])

	UR	IR
UR(-1)	0.933693	0.138178
	(0.04741)	(0.12550)
	[19.6947]	[1.10103]
IR(-1)	0.127511	0.679280
	(0.05716)	(0.15133)
	[2.23059]	[4.48885]

Correl. LM Tests			Heteroskedasticity Tests		Jarque-Bera Tests		
Lags	LM-Stat	Prob	Chi-sq	Prob	Ser.	JB	Prob
1	4.9737	0.2900	15.0258	0.2400	UR	0.8440	0.6557
2	4.7733	0.3114			IR	0.0302	0.9850

Source: Own calculations

TABLE VI. TEST OF THE EXOGENITY OF THE UNEMPLOYMENT RATE IN THE CONDITIONAL MODEL

Dependent variable: IR				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	8.286318	3.839299	2.158289	0.0488
IR(-1)	0.400931	0.185182	2.165064	0.0481
UR	-0.893465	0.496260	-1.800396	0.0934
RESID_UR	-0.157018	0.731719	-0.214588	0.8332
R-squared	0.575075	Durbin-Watson stat	2.402854	
F-statistic	6.315674	Prob(F-statistic)	0.006225	

Diagnostics tests	Statistics	Prob.
Breusch-Godfrey Serial Correl. LM Test:	1.755196	0.2145
Jarque-Bera Test	0.787919	0.6744
ARCH Test	0.003953	0.9537

[a.] Source: Own calculations

From the values of the test criteria t for the parameters of the residual explanatory changes $RESID\_UR_t$ [Table VI], it appears that the unemployment rate is exogenuous to the other parameters of the conditional model [parameters do not change], and the model from Table III can be used to model estimates of the inflation rate.

In the case of the second model, from the values of the test criteria t $RESID\_IR_t$ (Table VII), it emerges ($\alpha = 0,1$), that the inflation rate (IR) is not exogenuous from the point of view of the other parameters of the conditional model [the parameters changed] and the model from Table IV cannot be used for the model for estimating the inflation rate, because the inflation rate is endogenuously changed.

TABLE VII. TEST OF THE EXOGENITY OF THE INFLATION RATE IN THE CONDITIONAL MODEL

Dependent variable: UR				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	2.498754	1.560836	1.600908	0.1334
UR(-1)	0.659190	0.173257	3.804706	0.0022
IR	-0.053672	0.135589	-0.395844	0.6986
D1	2.422238	1.002242	2.416819	0.0311
RESID_IR	-0.237121	0.125327	-1.892019	0.0810
R-squared	0.789971	Durbin-Watson stat		1.651335
F-statistic	12.22408	Prob(F-statistic)		0.000241
Diagnostics tests			**Statistics**	**Prob.**
Breusch-Godfrey Serial Correl. LM Test:			1.490106	0.2675
Jarque-Bera Test			0.562210	0.7549
ARCH Test			1.462591	0.2542

Source: Own calculations

III. CONCLUSION

The aim of this article was to model the relationship between inflation and unemployment in the Czech Republic in the period 1995-2012 in the context of the Phillips Curve. On the basis of theoretical assumptions we, first of all, constructed a model of the dependence of the inflation rate on the unemployment rate, and in the second phase we turned this relation around and analysed the model of the dependence of the unemployment rate on the inflation rate.

From the results it appears that during the period under analysis, the long term one directional indirect proportionate dependence of the inflation rate on the unemployment rate is as was assumed by [21] in his original work. However, if we wanted to analyse the dependence of the unemployment rate on the inflation rate, it is not possible to construct this model in a one directional way [i.e., in the form of the ADL model].So, from the point of view of the endogenity of the inflation rate, only the two equation VAR model can be used. This model contains the two sided relationship between the time orders.

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