

Proposal of Realistic Mathematics and Mobile Computing

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Abstract—The purpose of this paper is to show how mobile computing impacts on learning using Realistic Mathematics. We developed a proposal made by a mobile application, activity-based Learning Problem Solving approach. A quantitative and qualitative methodology was used to assess the impact of this proposal on the Teaching and Learning Processes. Also it was used as a case study the concept of function and worked with 66 students in the first semester at College. Finally we concluded that combining Mathematics Realistic and Mobile Computing motivates students, allows them to focus better and solve in less time and correctly the problem posed in the activity. These results give a different way to exploit the mobile computing in Education

Keywords—Mobile Computing, technology, constructivism, function.

I. INTRODUCTION

MOBILE devices have made great progress in recent years and are influencing all activities. For this reason groups like New Media Consortium, EDUCAUSE and Learning Initiative point out through their projects Horizon Report [1]-[3], the use of mobile devices in education as one of the developing research lines in recent years.

Additionally, there is "educational" software on the market, especially built for mobile devices and developed for different purposes: fun, acquiring proficiency in a subject, present information about a course, solving puzzles, etc. These materials are disorganized and there is not a study. About their influence in the Teaching-Learning Process (TLP). Disciplines like Mathematics Education allow researchers to investigate how to get inside a classroom with these devices linking technology and theoretical work on learning.

One goal of this work is to take advantage of both disciplines, Mobile Computing and Mathematics Education, in order to teach calculus, focusing on the concept of function.

In different researches regarding the difficulties students have in learning mathematics, authors note that one issue is due to the way how the content is structured in the classroom, preferentially oriented to the purely mathematical aspect,

based on exercise and algorithms management without a link to real-world and science problems. This leads to a lack of interest of students in the study of mathematics because they don't conceive its use in their studying process as mentioned by Aravena [4], [5], and Biembengut [6].

Artigue [7] states:

"... According to the difficulties found, the traditional teaching and in particular, university education, even if they have other ambitions, tends to focus on algorithmic and algebraic practice of calculus and evaluate the acquired skills in that domain. This phenomenon becomes a vicious cycle. To have acceptable levels of success, it is assessed what students can do better, and this is, in turn, is considered as essential by students because that is what is assessed ...".

A. Problem Statement

In the absence of contextualized mobile computing applications that support teaching and learning processes, we take advantage and measure usability provided by M-Learning to develop a teaching proposal for the topic of function, as a case of study.

It is also considered that the Mobile Computing could support and enhance the teaching-learning process, as current students have a mobile culture Kurlovsky [8], which implies that they have prior knowledge that should be exploited as mentioned Ausubel [9].

"If I had to reduce all of educational psychology to just one principle I would say that: the most important factor influencing learning is what the learner already knows. Ascertain this and teach accordingly"

B. Theoretical Aspects

In this section we point out the concepts involved in the construction of the tools used in this research.

At first Mobile Computing is described and its relation towards education, which gives rise to M-Learning. Later education methodologies employed and finally some learning problems that were addressed in the research.

C. Definition of Mobile Computing

There are several definitions for Mobile Computing systems, in this paper we take the approach outlined by B'Far [10]:

"Mobile Computing Systems are Informatics Systems that can be moved easily in a physical way and whose computing capabilities can be used while moving" (p. 3).

According to this definition mobile informatics systems

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distinguishes themselves from other computer systems, taking into account the differences between the tasks they are designed to perform, how they are constructed, and how they are used. This identifies four characteristics of mobile devices: mobile user, mobile device, mobile application and mobile network.

A mobile user is defined under the following conditions of mobility:

- Being on the move, at least occasionally, alternating between known and unknown locations.
- The user's attention is not focused primarily on the computational task performed. When in motion requires attending other activities.
- Require short times of response from a system with a high degree of interactivity.
- Change frequently or abruptly between tasks.
- Require digital access to information anytime, anywhere.

On the other hand we have the characteristics of a Mobile Computing System, which is known as mobility dimension by B'Far [10]:

- Location awareness,
- Network connectivity quality of service (QOS),
- Limited device capabilities (particularly storage and CPU),
- Limited power supply,
- Support for a wide variety of user interfaces,
- Platform proliferation, and
- Active transactions.

The location can be seen on two levels: to obtain information about the location of the device and how this information is used in the system functionality.

On the other hand the information technologies based on Internet, has been involved in all human activities including education, giving rise to what is known as E-Learning. This is defined as the use of Internet technologies to provide a wide range of solutions that improve the process of obtaining knowledge by Chuanto [11]. The development of E-Learning techniques has given rise to other related activities such as T-Learning, U-Learning, M-Learning, (television, ubiquitous and mobile, respectively). The latter is described as the intersection of distance learning using the Internet (E-Learning) and Mobile Computing.

M-Learning is divided into two categories: first, the actor is independent of his context and location, only makes use of the mobile device for learning, such as listening to music in English to learn the language; in the second, the actor is placed in a context that is necessary to carry out the learning process. This second category is known as context-aware mobile learning Chuanto [11].

The context required for the M-Learning is, in the information used to characterize the learning situation in order to be relevant to the interaction between student and mobile application.

Also, three levels of context are identified: computer, user and physical.

It is worth noting that the mobile application as a research tool proposed in this document complies with the dimensions of mobile computing and of context-aware M-Learning.

PROBLEM BASED TEACHING

In this article "Problem-Based Learning" (PBL) is used because some of the aims of the research is to address calculus problems and meet the guidelines of the Instituto Politécnico Nacional (IPN).

According to that described by García [12] the PBL complies with the following aspects.

It has particular impact on the student, being a working active methodology.

Allows the development of thinking skills, from the standpoint of critical and analytical, which are consolidated and endure over time and are open to other disciplines.

Looks for a comprehensive and plural development in students.

The protagonist of the Teaching-Learning Process (TLP) is the student.

Students work in discussion and reflection groups

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A. Mathematics Education

In Mathematics Education the fact that work in the classroom is contextualized has gained importance. This means that the concepts should be studied in a real situation instead of an abstract one by Solokowski [13].

On the other hand we also have Realistic Mathematics (REM Realistic Mathematics Education) which is based on the interpretation of the Mathematics of Freudenthal as "a human activity". From this perspective, students must learn Mathematics in order to mathematize an issue from a realistic situation and his own mathematization (i.e the context of the problems must be mathematically true for the student) by Freudenthal [14].

It's about learning Mathematics through rediscover the concepts involved within real problems, similar to how these were discovered for the first time in history. Reverse the process of teaching first formal and methodological aspects of mathematics and later its implementation. Seeking to address these issues in parallel, described in [15]-[17]

B. Objective

Evaluate, with a technological and educative methodology, the use of Mobile Computing in the TLP of Calculus using an innovative application that take advantage of the Mobile Computing potential.

II. METHODOLOGY

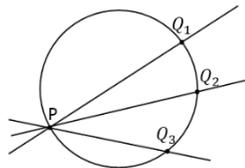
In order to accomplish the objective these methodological tools were used:

- A. *Diagnostic Questionnaire.*
- B. *Application Design.*
- C. *Activities Design..*

A. Diagnostic Questionnaire

The diagnostic questionnaire consisted of nine questions and the tenth is on student opinion. This, was applied to 69 freshmen engineering and it was rated between 0-5, zero was assigned to the problem that was solved incorrectly, while the five for the correct answer. If no data it means that the student did not answer the question. The questions focused on review both the conceptual and the algorithmic part of the students.

The diagram shows a circle and a fixed point P on the circle. Lines PQ are draw from P to points Q on the circle and are extended in both directions. Such lines across a circle are called secants, and some examples are show in the diagram



- (a) How many different secants could be draw in addition to the ones already in the diagram?
- (b) AS Q gets closer and closer to P what happens to the secant?

Fig. 1 First question of the questionnaire

The questions are divided between the algorithmic and those requiring the concept, we observed that most of the students show deficiencies related to the concept.

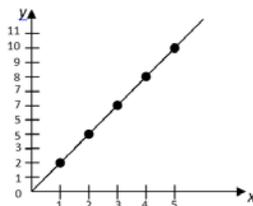
The problems are numbered from one to nine, in several of them are asked questions marked with the letters a, b, c, d, e and f.

The first problem, by having two subsections is divided into 1a and 1b. Fig 1.

The second problem has not subsections, so in this case is just the number 2. Fig 2.

Water is flowing into a tank at constant rate, such that for each unit increase in the time the depth of water increase by 2 units. The table and graph illustrate this situation

Time(x)	0	1	2	3	4	5
Depth(y)	0	2	4	6	8	10
Difference		2	2	2	2	2

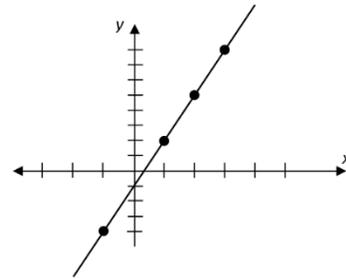


What is the rate of increase in the depth when $x=2 \frac{1}{2}$?

Fig. 2 Second question of the questionnaire

The third problem has five paragraphs, hence it is split into 3a, 3b, 3c, 3d, and 3e. Fig 3

The graph below represents $y=3x-1$



- (a) What is the value of y when $x=a$?
- (b) What is the value of y when $x=a+h$?
- (c) What is the change in y as x increases from a to $a+h$?
- (d) What is the average rate of change in y in the x-interval a to $a+h$?
- (e) Can you use the result (iv) to obtain the rate of change of y at $x = 2 \frac{1}{2}$? At $x = X$? If so, who?

Fig. 3 Third question of the questionnaire

A total of 12 responses were reviewed, with each subsection. It is important to note that these 12 questions that were asked in the diagnostic questionnaire are algorithmic questions.

The following problems with their subsections are: 4, 5a, 5b, 5c, 7a, 7b, 7c, 7d, 7e, 7f, 8a, 8b, 8c, 9a, 9b, 9c. These problems are of algorithmic type or using the memory. For example the question 5 in Fig. 4

What is the formula for the rate of change for the equation $y=x^n$?

- (a) $y = 3x^3$
- (b) $y = 4$
- (c) $y = \frac{2}{x^2}$

Fig. 4 Question five of the questionnaire

The questions 1a, 1b, 2, 3a, 3b, 3c, 3d, 3e, 6a, 6b, 6c y 7g are the concept and 4, 5a, 5b, 5c, 7a, 7b, 7c, 7d, 7e, 7f, 8a, 8b, 8c, 9a, 9b y 9c are algorithmic questions.

III. RESULTS

In this section we present the results of the diagnostic questionnaire applied to the group of 69 freshmen engineering of first semester of Calculus.

The responses were organized into categories according to the strategy they used to solve the problems of the diagnostic questionnaire, these categories are described below.

Category called Substitute: This is one of the most common mistakes, the student substitute a value in an expression whether or not that was asked in an exercise.

Category called "Confuses concepts": When a student answers a question with a different concept, that may be correct but the learner is confusing two concepts.

Category called "Reduce its size": This error only occurs in Question 1 of the diagnostic questionnaire, since the students state that the secant line reduces its size.

Category called "Same": This error only occurs in the question 7g. The student states the average change and the derivative are the same.

The results obtained are shown in the following graph in Fig. 5.

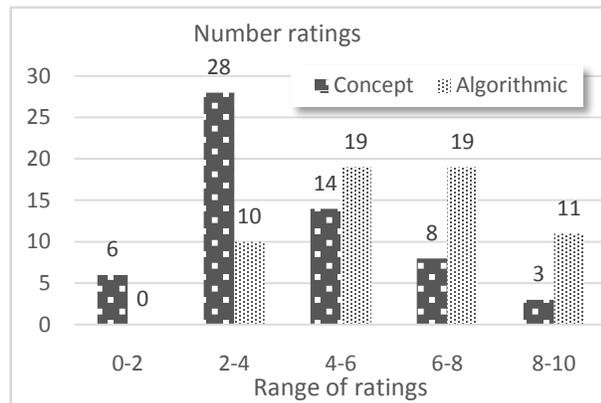


Fig. 5 Diagnostic Questionnaire Answer Graph. y the number of students that had obtained the rating in the range in the x axis. This graph is for t two type of question: concept and algorithmic

With these data it is possible to estimate using a Student's t test, with a confidence of 99%, the average population for algorithmic questions mentioned as subscript to and for the indicated subscript c concept to obtain that:

$$\bar{x}_a = 6.1907 \pm 0.5952 \text{ with } \alpha=0.01$$

$$\bar{x}_c = 4.2062 \pm 0.6303 \text{ with } \alpha=0.01$$

These results show that students can better respond to the questions algorithmic that the concept.

By taking the average of the ratings, only the ones responded well (rating between 1 and 5) we have the following. The algorithmic questions have an average rating of 4.39 and the concept questions 3.96. This means that the students who answer correctly found easier the algorithmic questions.

The graph in Figure 2 shows the percentage response rate for each question (right or wrong). Where the average of the questions answered incorrectly is greater for the group of concept questions than the group of algorithmic questions. (46.98% and 30.48%, respectively)

This difference is increased with the analysis of the difficulties experienced by the students to respond. The error detected more frequently is that students substitute values in the expressions provided to them no matter what the exercise they were asked to respond. This happened both in Question 2 as in 3e.

The second most common mistake is to confuse the concepts they were asked

Porcentaje de respuesta

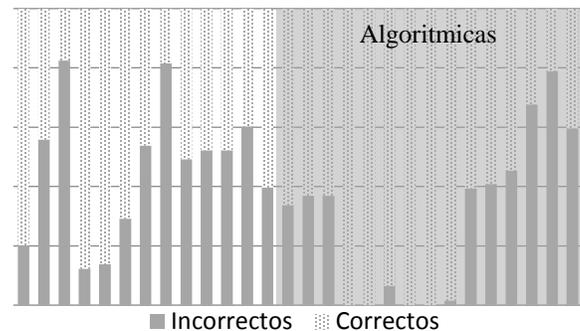


Fig. 6 Show how algorithmic questions are correctly answered more frequently

IV. APPLICATION DESIGN

The application has the ability to acquire data under controlled conditions by the student and stores that data considering the time they were captured for granting independence in the form of use to the student and thus encourage innovation and independent learning, which allows a significant learning.

In the case of the camera, the most frequently feature found in mobile devices of students in Escuela Superior de Cómputo (ESCOM, IPN) in a previous study, it is possible to "paint" on it, similar to augmented reality, different base curves on the image to approximate the behavior of the object in the picture (Figure 7).

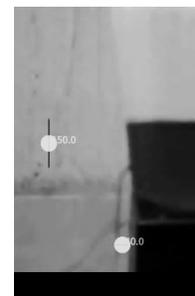


Fig. 7 Interface that allows "measure" over a taken picture

The student should consider that the data provided by the application are in pixels and must convert these, if required, to a standard metric unit.

In addition to allowing the student to obtain the algebraic expression of the different curves drawn on the image, it is possible to perform measurements in the units specified, setting the conversion between pixels and units established.

The application allows the student to take measurements with the different features of the device, compass, microphone, accelerometer, etc.

For example, the level of the device can be used to measure angle changes according to time and draw the respective graph. See Figure 8.

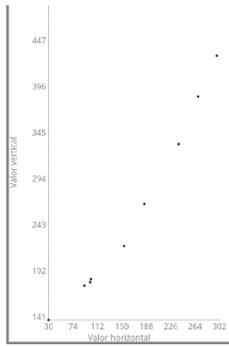


Fig. 8 The application allows the graphical view of the taken measurements.

The application also stores the data on the device for later analysis, share data with peers and teachers using standard formats and communicating with Moodle.

After training students in the use of the application, which is intuitive, a problem to be solved in groups is proposed. These proposed problems are referred to as activities. Each activity has instructions for the student and the teacher, questions to be solved, these activities can be obtained from the web using a Moodle server or also to be printed on paper in order to have them on hand while capturing data.

An activity example is described in next.

ACTIVITY: WATER LEAK

Motivation:

After class two students stop by and watch a pipeline on the garden with a water leak, they say to each other:

- Look, a water leak

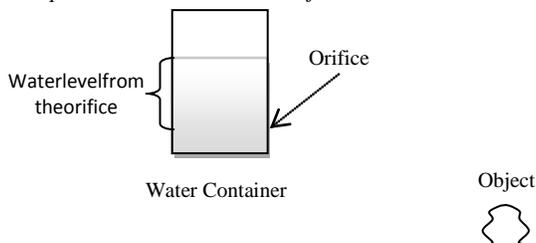
- Yeah, and they tell us to save

- Imagine that we could use that water as a weapon and could destroy a starship

They laugh and go report the water leak.

Problem:

A water container having an orifice which generates a leak is in the school. How full should be the container for the water to reach dipping an object that is located at a certain distance as shown in the figure. Find the rule of correspondence between the height of the water level and the place that should be the object to be wet.



Questions:

Before starting with the activity answer the following questions according to your own experience.

- If the relative position between the container and the object is given. What should be the water level from the orifice in order to wet the object?
- Which path will follow the trickle?
- How the container shape changes the path of the trickle?
- The path of the trickle is affected by the water volume contained?

What kind of container will you use?

Changing the container width modifies the distance of the object to be wet?

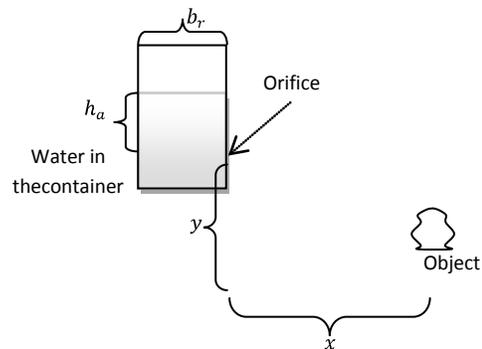
How does the orifice position affects the reach of the trickle?

Use your mobile device to discover the rule of correspondence between the water level and the place that should be the object to be wet.

1. Propose
2. Shoot a video.
 - a. Shoot a video of the described situation with your mobile device taking into account that the shoot let you perform measurements.
 - b. Once you shooted it determine the shape of the trickle and how it changes trough time.
 - c. Repeat the shooting with different containers in order to establish how its shape influences the path of the trickle.
 - d. Perform the necessary tests to answer the stated questions.

Analysis

Once you have experimented with the videos we propose you to label some distances involved in the problem in order to answer some qualitative questions.



- b_r : Base of the container
- h_a : Water height from the orifice
- x : Horizontal distance of the object to the base of the container
- y : Vertical distance of the object to the orifice in the container

The distance h_a is a data that we can't control because it changes according to the water leak, it changes through time. Same happens with distance x . These are dependent variables. On the other hand b_r and y are variables that we can fix

Analytic Expressions:

Supposing that the path of the trickle is a parable determine the constants (a, b) of the following expression for a given moment

$$y = ax^2 + b$$

How can we rewrite this expression involving time?

What is the relation between constants a and b with h_a or b_r ?

Results

Write a final expression that could solve the original problem, i.e., if I have a container (height and width, if needed both or one of them) and an amount of water, in which position should be the object to be wet as soon as the container starts leaking

Conclusions

To conclude argue with your classmates about how this problem reflects the concept of function: domain, image or range, correspondence rule, dependent and independent variables and functions composition.

V. ANALYSIS

The observations made to students while they were solving activity shown in Fig 8.

Now we present the conclusions reached by the group of students about the activity applied. To solve it the group was divided into 7 teams.

Team 6 concluded that the amount of water is an important factor influencing the pressure on the orifice and this is reflected in the distance of the water.

Team 6 also noted that they weren't able to express the function algebraically they just got conclusions regarding the practice and knowledge obtained. But this allowed them to understand what was really happening instead of the sole algebraic expression that did not tell them anything. This team commented that using different applications within their phone to solve a math problem is something totally new. Exemplified the fact that they are used to take video or take photos to things outside of what they work at school, noting that most of them have used the camera to take a picture to the board, instead of taking notes.

Team 2 indicated that the function is decreasing as the pressure at which water leaks depends on the height of the orifice and the shape of the container.

For this team to use their phones was something different and interesting because they solve a math problem using applications included in their mobile devices and the application installed by the teacher helped them a lot, they could draw a graph and measure the distance between the object and the vessel containing the liquid. This makes mathematics realistic, not abstract.

Team 5 said that the scope of the trickle to the object depends on the pressure with which it leaves the container and what is obtained is a decreasing function as this is due to the height of the liquid that wets the object and the loss of pressure as the liquid is ejected.

Team 7: concluded that the function being sought depends upon the amount of water and the container that serves as an experiment, it was also noted that it is decreasing because water within it, is depleted.

Mobile usage required that students had to recall less formulas, and better focused on solving the problem. In both cases, when using mobile as they do not, students

utilized their previous experiences to solve problems. The use of mobile computing motivates students to work, expressed motivates use tools that are "flourish".

VI. CONCLUSION

In this first phase of the research that has been developed and shown in this article, we note the importance of the use of Mobile Computing in Education, specifically in Mathematics Education, currently young students have access to Mobile culture, so that the application designed and the activities formulated were directed to use what the student knows. On the other hand the design of the application and the proposed activities were aimed to work the deficiencies found in undergraduate Computer Science students of first semester, as a result of applied a diagnostic questionnaire, where when contrasted with the literature found great similarity, and students prefer to solve algorithmic issues to situations where the concept of function is involved more realistically.

The impact of working with mobile devices allowed the student to solve the activity using the application installed on their phones in different moments of resolution, such as drawing the coordinate plane and graph a parabole corresponding to the water leak, but this was achieved by the students because they shot a video when the water container was emptied through the orifice made in it.

The usability of their phones was a fundamental key, just like the portability of these.

The impact of mobile computing that was evaluated was the Usability part of M-Learning and found the following: Having worked with the activities and the designed application we performed an evaluation questionnaire and one on the usability and found that the students were able to develop different skills such as reflection, communication and discovery.

In relation to learning obtained, it was more significant, it was not mechanical.

It is considered that the contribution of Mobile Computing consisted of a comprehensive development of the student in order to achieve learning that is not based on memory.

These results extends the use of mobile computing in the M-learning to not only exploit the features it has in common with a personal computer: networking, processing, watching videos, etc.. It takes advantage of the device mobility itself and the various sensors with which it is equipped.

The mobile device is not only a teaching tool that replaces the calculator. Also allows students to use the data they can get with his mobile and interpret it, to draw conclusions that may subsequently generalize.

It is possible to use a hybrid methodology that has qualitative and quantitative components, to analyze the usability of a mobile system.

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