Project-based Learning in Basic Course of Technical Physics: Computer-controlled Experiments and Agros2D Modeling

Michal Růžička, Michal Rejthárek, Štěpán Major

Abstract— This paper is deals with exploit of Arduino microcomputers and finite element software Agros2D in basic course of hydromechanics. The Arduino is software and many hardware components and microcomputers developed for teaching robotics. The Arduino microcomputers such as UNO can be used to control experiments and also for subsequent data processing. In the case of hydromechanics, the microcomputer can used to control valves and flow-sensors in experimental devices intended to study fluid dynamics. This device was developed by students and it's possibilities are discusses in the article. Because real liquids are compressible as well as material of pipeline is deformable. The system allows study effect flow rate of compressible liquid and other problems. Students can use software based on Finite Element Method for analysis of behavior of these effects. The Agros2D was chosen as suitable for students. This software was primarily developed for the educational purpose. This tool is utilized in learning of Finite Element Method. This software is able to solve complex problems described by partial differential equations. Both of these tools are appropriate for project-based learning, which form the basis for technical education. Experience shows that the students are interested in learning. Results of student tests shows, that the participants on this form of project-based learning achieve much better results in both, theoretical and practical tests (in mechanics, physics, informatics).

.

***Keywords*—**Educational software, Computer-controlled experiment, Flow mechanics, Thermomechanics

This project was prepared by finantial support "Metoda konečných prvků a automatizace experimentu ve výuce mechaniky a technických laboratoří na katedře technických předmětů" 18/I zč. SV 2119 - work place 1440.

Michal Růžička is specialist in the CAD and CAM education, he tought courses devout to the Autodesk Inventor, Solid Works and Auto CAD. Michal Růžička is also interested in programming microcontrolers such Raspberry Pi or Arduino Robots and automatization. He teaches programming C++ and C#. This author also deals with database issues in school practiceMichal Růžička is at the University of Hradec Králové, faculty of Education, Department of Technical Subject, Rokitanského 62, Hradec Králové III, 500 03, Czech republic (e-mail: Michal.Ruzicka@ uhk.cz).

Michal Rejthárek is student at the University of Hradec Králové, faculty of Education, Department of Technical Subject, Rokitanského 62, Hradec Králové III, 500 03, Czech republic (e-mail: Michal.Rejtarek@ uhk.cz).

Stepan Major is working at the University of Hradec Králové, faculty of Education, Department of Technical Subject, Rokitanského 62, Hradec Králové III, 500 03, Czech republic (e-mail: Stepan.Major@ uhk.cz).

I. The Current State of the Problem

A. *Finite Element Method at School*

IN the case of natural sciences, mechanics and engineering the partial differential equations are utilized for description of complex problems. These complicated systems of equations are solved using finite element method (in the following text, we will use the abbreviation FEM). Currently number of simulation programs based on FEM method is used in practice. Many are distinguished by friendly interface. These suitable interfaces do not require advanced knowledge of mathematics, at least in the case relatively simple tasks. Such user-friendly graphical interface allows creating two or three dimensional models purely on the basis of intuitive steps. Such process of solutions of physical model includes these stages: creating of model, meshing, and solving. The final step is then analysis of results. Students learn to use these programs in not only at university, but also on high schools focused on technology or natural sciences. In the learning this problems is often used e-learning (as in all subjects oriented on computer utilization) and project-based learning [1] - [5].

Students usually meet FEM for the first time in technical graphics lessons. Today, probably all professional CAD programs used for mechanical components modeling some FEM solver. This solver allows basic strength analysis. An example of such software is SolidWorks. This software is widely used at technically oriented high schools. The second most commonly used software is Autodesk Inventor. FEM solvers that are part of these two programs do not allow more complicated work with the FEM model, as a transformation of networks or sub-modeling. The students usually get acquainted with software devices designed primarily for FEM simulation first at university. Due to high prices of professional computational software such as ANSYS, the educational institutions prefer freeware programs such as FEMINA. An example of these programs developed for educational purposes is Agros2D software. The system is based on Hermes2D library. This device is freeware available for use at no monetary cost, making them ideal for use at school and in universities. This program can also be used for creating simulations used in teaching. In doing so, these simulations can be used effectively even, if the students are not at all acquainted with FEM method. In this article is special interest dedicated to the use for analysis of experiments realized within teaching of hydromechanics at our university.

Especially subjects related to the teaching of finite elements method are mechanics and applied mathematics. The knowledge acquired by students in these two courses, is further utilized in the design exercises. The students usually get acquainted with the FEM software interface in the first lesson [5] - [8]. After the students get acquainted with software interface, they will start preparing models (this is an activity close to their skills obtained by technical graphics lesions). and they are looking for a solution of problems. Student's knowledge from the first lesson can be tested easily by online questionnaire assessment with multiple choice test [1], [5], [8]. However, this part of the course is quite minor. The FEM solvers are working with very large files when running, therefore software user must be sure that your local drive has space for it and he must be able to work with this files. Therefore, first lecture is also devout to problematic such as creating.

The most important part of the course consists of tasks in which students creates model that describes typical engineering problems.

One of the first tasks, that is solved by student's analysis of two dimensional truss-structure such as bridge truss. This is example of structural analysis. Following lectures are dedicated to the problem of „Plane Stress Bracket", also this task is example of 2-Dimensional object. Subsequently students pass to the problems which require 3-dimensional modeling. In order to practice and master this part of course student musts solve about ten examples of this type. After finishing this basic lecture, students begin with complicated problems in this succession: "Effect of self weight", "Distributed loading", "vibration", "buckling", "Non-linear Behavior of materials" and "Non-linear analysis". The reason why it was chosen just this sequence of tasks is logical sequence of steps involving models to be start from easiest problem to the tasks describing more complicated reality. Whereby, the student can use one original model, which is connected to other parts.

B. Using Computer-controlled Experiments in Teaching

During the last few decades, as manufacturing costs have decreased electronic devices, relatively complex electronic measuring devices are also used in the basic course of physics. These measuring devices are able to communicate with PC or those devices are directly controlled by computer. Unfortunately, most of these school laboratories are devote purely to electricity and magnetism or their technical applications. An example of such a laboratory is described in paper [9]. This illustration of computer-aided educational laboratory is devout to student's familiarization with problematic of photovoltaic cells.

Perhaps most common example of computer aided school laboratory that is devout to others problems, than electricity is laboratory equipped software IP COACH. This software was developed by CMA (Centre for Microcomputer Applications) purely for educational purposes. Also, CMA itself was founded by science educators of the University of Amsterdam with goal to improve science education with innovative and attractive technology. This software has been made possible to implement prefixed format for every laboratory exercises used in the curriculum at high school or at base physic course at university. One great advantage of this solution is that students do not have to pay attention to system scaling and calibration. Detailed textbooks were published about this software. These textbooks contain many tasks for students with exact instructions and explanation of physical phenomena [10].

C. Robotics and Arduino at School

Nowadays, emphasis is placed on the robotics teaching. Recently a number of inexpensive learning platforms have emerged in the world, such as Raspberry Pi. One of these resources is particularly widespread and it's advantage is modular character. This tool is Arduino. This name is used for both, open source software and hardware developed by Arduino company, respective Smart Projects company [11] - [13]. There is a whole range of Arduino microcontrollers and microcontroller kits.

These microcontrollers differ in performance and other parameters, as well as in use. These single-board microcontrollers for building digital devices and interactive objects that can control small robots or different type of contrivance mechanism based on stepper motors and the sensors for measurement of physical quantities. These tools are most often known due to their use on various walking robots. However, all these sensors can be effectively utilized in the school laboratory to teach physics.

The Arduino was especially developed for education, therefore is optional for utilization in basic course of programming and robotics. The software Arduino is freeware and Arduino hardware components are mostly cheap. So cheap, that it is possible to be used by students at home.

The Arduino software can work on considerable number of different boards both official boards produces by Smart Projects company or compatible such as Freeduino or Freeduino and futher more (It is not possible enumerate all). Most of these units are based on the processor ATmega328. The basic and most often used official boars are (the processor is listed in bracket, if the it is used other processor than ATmega328): Arduino Micro (ATmega32U4), Arduino Nano, Arduino Nano, Arduino Mini, Arduino Mega (ATmega1280), Arduino Uno, Arduino Leonardo (Atmega32U4), Arduino Robot, Arduino Esplora and other.

The Arduino used number different of measurement devices and detectors not only to determine basic electrical quantities such electric-current or voltage, but also more sofisticated measurement instruments such as ultrasonic ranging detector for non-contact measurement and obstacles detection. There are also, for example different types motion sensors or two and triple axis accelerometers [11], [12]. Also, sensors for measurement humidity and temperature of environment are available. In this article Arduino was used for Bernoulli's principle teaching, so flew sensors were used. Sensors based on Ventury tube principle are also available for Arduino and these sensors can be used for low rate measurement in combination with other sensors for distance measurement.

II. Mechanics and Theory of Elasticity

A. Material Mechanics at High School

Students at elementary school are already familiar with the most basic form of the Hooke's law. Mostly it is in 8 or 9 class (students about 14 year old). At this age, they are only familiar with the basic form of this law for the tensional loading. At the high school curriculum differs strongly according to the school orientation, technically oriented schools, these high schools especially schools focused on mechanical engineering they place considerable emphasis at the teaching mechanics. At these schools mechanics is taught as a separate subject. The issue of flexibility and strength is lectured for one whole year. The students will learn about process of deformation in the elastic region, meaning of concepts such as yield stress, ultimate stress and Young's modulus. The students are learned to solve simple tasks such as beam deflection in these different cases: cantilever beam, beam simply supported at ends, this beams are under load of both concentrated load and uniformly distributed load or load distributed by some function (most often linear). Course of mechanics is in all this cases reasonably to supplement with teaching in the laboratory. These demonstration experiments have great impact on the students. Many school labs have a tension testing machine available. These devices are ideal to demonstrate Hooke's law, i.e linear growth of deformation to the yield value σ_Y and the ultimate strength σ_U. Other tests are possible realized with much less professional equipment. Despite this fact, they are often neglected in teaching. Such tests are described in this paper, in the following paragraph.

B. Material Mechanics and Student Project

The most frequently implemented elasticity test is at schools tension test. The experimental sample (flat bar or beam) is loaded by tensional forces to the final rupture. The acting forces are relaitivelly high and testing device requires strong reliable frame a relatively powerful power plant. The test is mostly performed to final rupture of specimen. The eperiment is based on simple formula:

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{SE} \quad , \qquad (1)$$

where ε is relative elongation defined as ratio between increments of length and original length of sample $\Delta l / l_0$, S is crossection of sample and F is loading force, see Fig. 1.

If we focus on the bending test, experiments can be realized with a less advanced and also cheaper device, due to the fact that there is no need for a massive and durable frame.

Here we will show a student project aimed at determining the modulus of elasticity from the bending test. This experiment was designed by students of the Department of Technical Education, for basic course of technical measurements. First, let's get acquainted with the experiment from a physical point of view, and in the next phase we look at the possibility of it's automatisation. The Fig. 1 shows beam simply supported at ends. If a force is applied to the beam, for example in the center of the beam, the beam deflected downwards, see Fig. 2. The deflection can be calculated by equiton:

$$\delta_{\max} = \frac{Fl^3}{48EI} , \qquad (2)$$

where $\delta_{\max}$ is deflection, l is length of beam (respectively distance between supports), I is the area moment of inertia of the cross-section. In general, it is most appropriate use rectangular profile of beam.

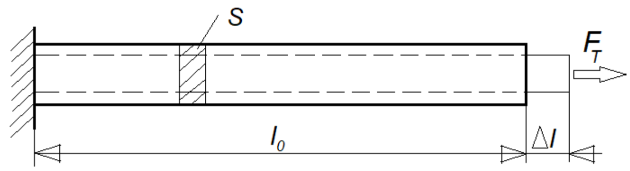


Fig. 1 The tension test and elongated beam: l_0 – initial length of beam, Δl – elongation of the beam, S –cross-section of the beam, F_T–tension force. Elongated beam is drawn in a thin line.

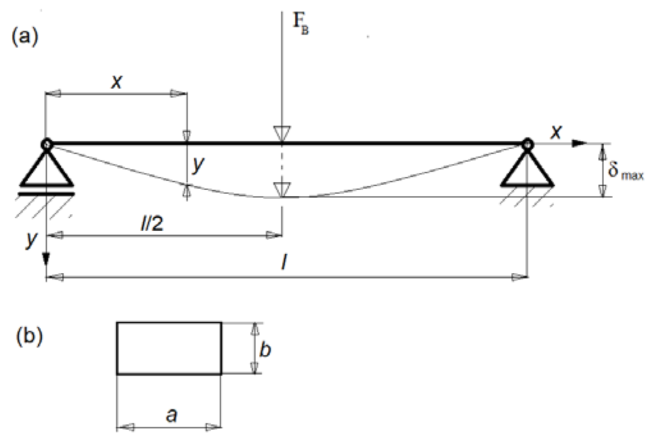


Fig. 2 (a) Beam simply supported at ends – concentrated load F_B at the center. (b) The cross-section of the beam.

Now, when we know the deflection value, the beam dimensions and the load force, we can calculate the E value. The calculation of this equation is somewhat more complicated than the tensile test (1), but if we compare these two equations (1) and (2) we can easily understand the advantage of determining E from the bending test. Let's assume that the deformation is in both examples measured by same device. The cheaper and less accurate sensors (for the student kit) measures with less accuracy and therefore it is better to measure it with rather large easily detectable deformations. The deformation that we can still measure with this gauge is v_a. This value is put into (1) and (2), i.e $\Delta l = v_a$ and $\delta_{\max} = v_a$. We also assume that these two tested beams have same dimensions and its cross-section is $a \times b$. Then, the area moment of inertia is :

$$I = \frac{ab}{12}, \quad (3)$$

So with v_a and (1), (2) and (3) we got these two formulas:

$$\frac{v_a}{l} = \frac{F_T}{abE}, \quad (4)$$

where F_T is tensional force and v_a related length increment. This form for bending:

$$v_a = \frac{F_B l^3}{48 \frac{ab^3}{12}}. \quad (5)$$

The force F_T required to reaching the same measure v_a in the case of tension is many times higher in the case of bending (concentrated load - bending force F_B):

$$F_T = \frac{F_B l^2}{4b^2}. \quad (6)$$

When using a configuration experiment shown on the Fig. 2, we can also measure deflection in other points than just in the center of the beam, see Fig. 2 deflection in the distance x from the support:

$$y(x) = \frac{F_B x}{12EI}\left(\frac{3l^2}{4} - x^2\right). \quad (7)$$

Generally, the deflection in at any point of the beam can be determinated using diferential equaton:

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{E(x)I(x)}. \quad (8)$$

The significance of variables in this equiton is clearly visible in Fig. 3. The solution of this equaton can be written as:

$$y = \frac{F_B bx}{6lEI}(l^2 - x^2 - d_2{}^2), \quad (9)$$

for the case $0 < x < d_1$. In the case $d_1 < x < l$ solution

$$y = \frac{F_B bx}{6lEI}\left[\frac{l}{d_2}(x-d)^3 - (l^2 - d_2{}^2)x - x^3\right]. \quad (10)$$

Maximal value of deflecion is in this cas:

$$\delta_{max} = \frac{F_B d_2 (l^2 - d_2^2)^{3/2}}{9\sqrt{3}lEI}. \quad (11)$$

We are now able to determine the Young modulus of elasticity. The disadvantage of bending tests is that they are only suitable for elastic area of deformations. This test can not be used for determination of yield stress σ_Y or ultimate stress σ_U. Now students can devote further measurement improvements. Thus, for example, it's automation.

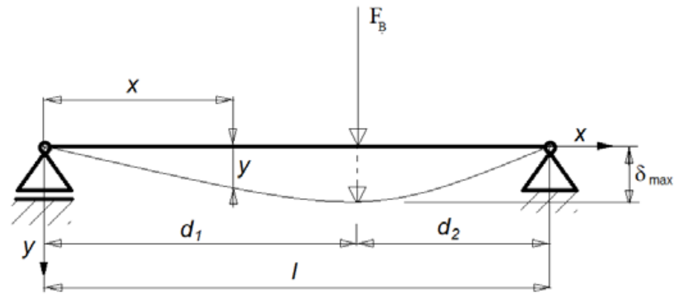


Fig. 3 Beam simply supported at ends – concentrated load F_B at a general point on the beam.

C. Automatization of Experiments

We now have experimental device devout to the measurement of elastic module. This device consists of a frame on which the support of the nose is placed and a gauge is placed. The loading and hence the deflection rate are controlled manually. The easiest way to achieve a certain load on the beam is to hang the weight on this beam. Thus, the loading force is $F_B = m.g$, where m is weight and g is earth gravity. The deviation value is then also deducted manually. However, the whole process can be successfully automated. First, the deflection can be measured by a position sensor that can communicate with the computer.

If we measure relatively large deflections, an ultrasonic probe can be used for measurement. This sensor is used with Arduino robots. This sensor will be placed on the frame above the bent beam and will transmit to the PC information about the drop of the beam underneath, see Fig. 4.

In the previous paragraph, however, we have described the possibility that the elastic modulus can be calculated from the deflection even if the deflection was not measured in the center of the beam, see (9) and (10). It would be advisable, if the the device will be able to measure the deflection for more than one point on the entire length of beam. The simplest solution is to place multiple sensors on the frame.

The second posible solution is place the sensor on the movable platform and scan the surface with it. The movement can be secured by ball screw or trapeziodal screw and stepper motor. These elements can be obtained as accessories for Arduino robotic kit. The measurement assembly with movable sensor is shown in Fig. 5. Similarly, it would be advisable to automate the loading process. Also in this case it is conveniently use the trapeziodal screw and stepper motor. The bending force developed can then be readily measured by a electronic forcemeter.

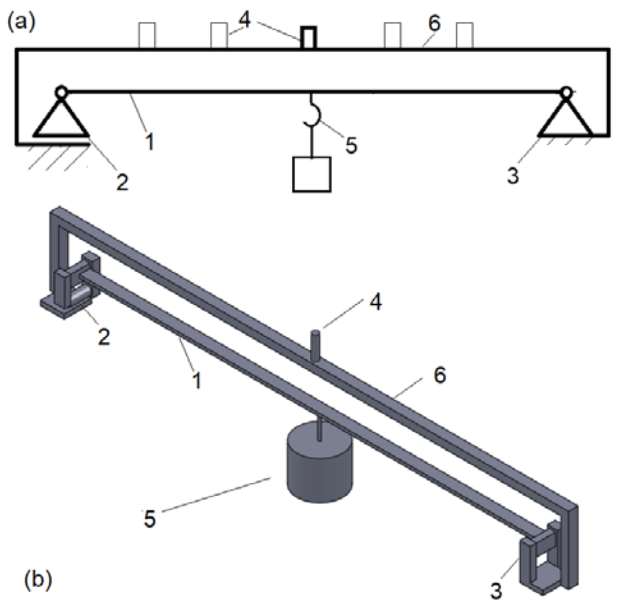


Fig. 4 (a) Schematic representation of testing device with multiple pieces of sensor (the other sensors in the line are drawn in a thin line), (b) 3D model of testing device with one sensor. 1 – sample beam, 2 – movable support, 3 – fixed support, 4 – sensor, 5 – weight, 6 – frame.

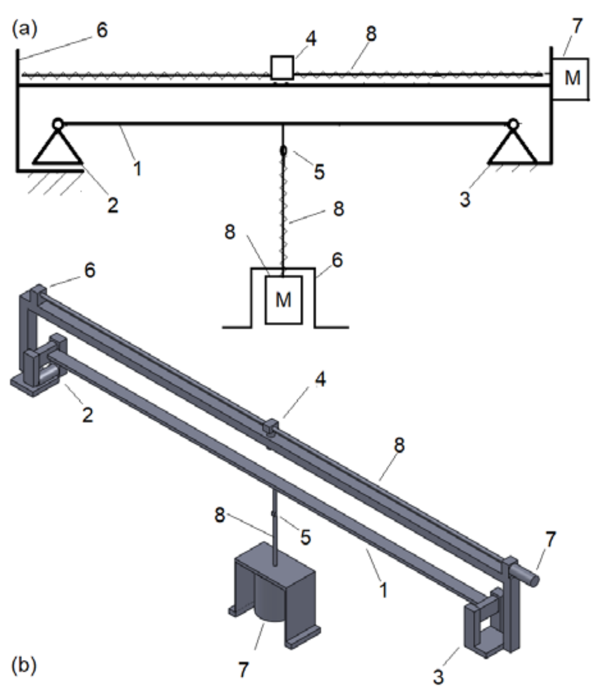


Fig. 5 (a) Schematic representation of testing device with movable sensor and loading mechanism, (b) 3D model of testing device with movable sensor. 1 – sample beam, 2 – movable support, 3 – fixed support, 4 – sensor, 5 – loading mechanism, 6 – frame, 7 – stepper motor, 8 – ball screw.

D. Finite Element Simulation of Beam Loading and Agros2D Descrition

As has already been shown in the previous paragraph, the students are able to determine deflection at any point of the beam. This is possible using (9) and (10), respectively the when deflection is known and load force can be derived Young's elastic modulus *E*. However, students can use numerical simulations besides the analytical solution, which was shown in the previous paragraph. The Agros software allows create simple model of beam under bending, see Fig.6

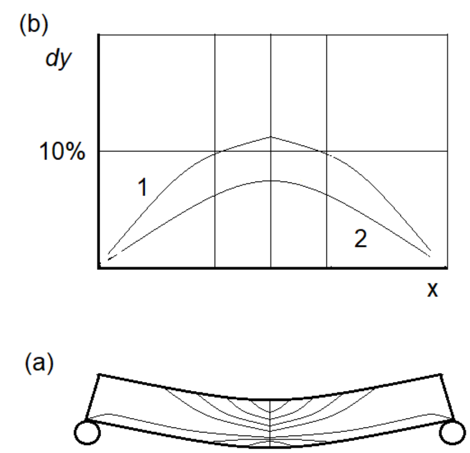


Fig. 6 (a) The FEM model of the beam. (b) Calculated deflections: 1- difference between analytically obtained deflection, 2- difference between FEM obtained deflection (on the upper surface of sample).

The Agros2D software is a versatile multiplatform application designed for solution of physical fields. This platform was written in C ++ and uses the Hermes2D library [21] – [25]. Hermes2d herself is based on hp-FEM (adaptive finite element method of higher order precision) to solve partial differential equations. The application is developed at the Department of Theoretical Electrical Engineering of FEL West Bohemian University in Pilsen and is distributed under the GPL license. Its principal part is a user interface serving for complete preprocessing and postprocessing of the tasks (it contains sophisticated tools for building geometrical models and input of data, generators of meshes, tables of weak forms for the partial differential equations and tools for evaluating results and drawing graphs and maps). The Agros2D is working in the operating system Windows.

The basic features of this software are coupled physical fields, that means that the user can blend two or more physical fields in one problem. specifications, our Publishing House may not be able to include your paper in the Proceedings. The Agros2D allows both Simulation and analysis of linear and nonlinear problems [21]. The Agros2D software implements Newton's and Pickard's methods for solution of nonlinear problems. Another advantage of this software is Automatic space and time adaptivity. This feature has been taken over from Hermes2d [16]. Another important feature of the Agros2D is adaptive time stepping for transient phenomena analysis. Another, also useful functions or features of Agros2D are: utilization of curvilinear elements, quadrilateral meshing and particle tracing.

In general, however, the starting user does not need to understand higher mathematics. The software is very user friendly. The great advantage for use in education is that the software used only two dimensions for model construction. The program can be used not only by students at university, but also by students in the secondary education (high schools). In this article the Agros2D software was used for study of and simulation of flew of incompressible and compressible liquid through Pipeline. The pipeline in the model is both firm non-deformable and flexible (deformable). This flexible pipeline is expanded by the pressure of the liquid. All these problems can be relatively easy to simulate in the program as well as high school students. Although these problems would be hardly solved by methods based on analytical methods, now it can be solved by students without understanding to komplex differential equations.

E. Question for Students: Neutral Curve in the Beam and the Measured Deflection

After creating this FEM model shown in Fig. 6, students can compare both results, i.e., the results obtained analytically and the results obtained using the FEM model. Very interesting is when students conduct a study of the accuracy of deflection determination in relation to the thickness of the test beam. To this test, it is advisable to use a very flexible material such as plastic, because, at school we will usually not be able to exert enough load to be able study this problem on metals.

Students will practically always detect deviations between the two results, but most of these deviations, they will explain only by some gauge inaccuracy or or explains the error as a result of the number rounding during calculations. However, if the proper beam was used, there is a systematic error here, which students should find. Because, the deflection is measured on the beam surface and the calculated deflection is related to the neutral curve (netral thread), the measured value of deflection differ slightly from value obtained analyticaly. However, for example, in the case of metal, the deviation is so small that we are not able to measure it in school conditions. However, in the case of plastic, this difference can be recorded.

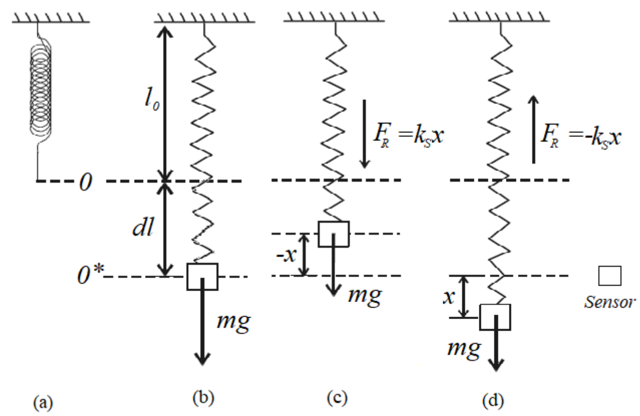
The students will prepare a table in which they will compare, measured deflection over the entire length of beam, with both theretical results, i.e. obtained by the calculations and simulations must be compared with the results of the measurement. The graph of deviation between calculated end measured deflection is shown in Fig. 6. The graph shows the deviation between the measured value and the calculated value The advantage of the FEM model is the fact, that the students can monitor the movement of any point on the surface of the model. Verify truthfulness of the explanation of the deformation distribution in the body. If students perform this analysis, they will better realize which half is stressed and that pressure.

The values measured by students are in 95% in compliance with values obtained by simulation in Agros2D software.

III. Spring Oscillator and Pendelum

A. Spring Oscillator and it's Theory

One of the tasks that students can solve at school is the harmonic oscillator formed by the ball on the spring. This problem includes theory of elasticity and dynamics. The harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force F_R, proportional to the displacement x. The harmonic oscillator is shown in Fig. 7.



The spring oscillator: (a) The spring without weight, it's length is l_0; (b) the spring with weight, it's length $l = l_0 + dl$. The oscillator is in balance in position 0*.; (c) The oscillator deflection x and the restoring force of spring is F_R.; (d) The deflection is - x and the restoring force has opposite direction. The sensor records presence of the weight in the position 0*.

The restoring force F_R which caused oscillation of weights at the end of the spring occurs in response to deformation of the spring. Size of this force is given by equality:

$$F_R = -k_S x\,, \tag{12}$$

Where k_S is spring stiffness. A simple harmonic oscillator is an oscillator that is neither driven nor braked by external forces such as frictional forces. It consists of a weight with mass m, which experiences a single force F_R. This force pulls the weight in the direction of the point $x = 0$. The value of this force depends only on the mass's position x and a stiffness constant k_S. The Newton's motion law says that the driving force of body that moves with acceleration a and is:

$$F = ma = m\frac{d^2x}{dt^2}\,, \tag{13}$$

The acceleration is defined as second time derivation of position vector. The oscillator in state of balance can be described by equation, which is we get, when (12) and (13) are merged:

$$m\frac{d^2x}{dt^2} = -k_S x\,. \tag{14}$$

The (14) represents the second order differential equation. It's solution is

$$x(t) = A\cos(\omega t + \varphi), \tag{15}$$

where ω is angular frequency. The parameter *A* is amplitude, i.e maximal deviation from weight position in the state of equilibrium. This variable is defined as:

$$\omega = \sqrt{\frac{k_S}{m}} = \frac{2\pi}{T}. \tag{16}$$

The parameter *T* is oscilation period defined as $T = \frac{1}{f}$ and *f* is oscillator frequency. Similarly, a pendulum can be described in a similar way.

B. Mathematical Pendulum and Gravity

The mathematical pendulum is idealization of real pendulum and it can be described as weight (mass) fixed on the massless rod or cord. The weight swings on this rod. This rod also always remains taut, see Fig. 8.

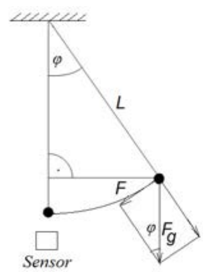


Fig. 8 pendulum with weight (mass of weight is m), length of rod is *L*. The sensor is detecting presence of the pendulum in the equilibrium position.

The balance equation of pendulum is

$$\frac{d^2\varphi}{dt^2} = -\frac{g}{L}\varphi. \tag{17}$$

The parameter is *g* is gravity. The length *L* is length of te rod, respectively distance between rotation axis and oscillating mass. The importance of other parameters is clear from Fig. 8.

The solution of (17) can be expressed as:

$$\varphi(t) = \varphi_0 \cos\left(\sqrt{\frac{g}{L}}t\right). \tag{18}$$

Where φ is deflection angel and φ_0 it's amplitude. The oscilation periode and angular frequency are:

$$T_{Pen} = 2\pi\sqrt{\frac{L}{g}}. \tag{19}$$

A very good school assignment is to determine gravity *g* using a pendulum, to which students use this relationship derived from (19):

$$g = \frac{4\pi^2 L}{T_{Pen}}. \tag{20}$$

C. Automated Determination of Quantity of Oscillation

In both experiments the students must to measure period *T*, respectively T_{Pen}. Because the oscillation of the weights is relatively fast, the students are always counting the time for 10 transits of weights through equilibrium position. Both experiments can be simply automated. It is only nedeed to count number of transitions through equilibrium position. The sensor used in Arduino to detect the obstacle in front of the can be used to do this. This system is working in this way: (1) Each time a barrier is detected, the logical one is recorded and (2) an instant time is assigned to this record. The control program for this detector is so simple, that students are able to write it during the lesson. Due to the fast data processing with the program, students can, for example, change the length of the rod *L* and compare the results for the variable pendulum length.

IV. Hydro and Thermo-Mechanics in Basic Course of Physics

A. Hydromechanics Course at School and it's Principal Experiments

An important part of the basic physics course is also hydromechanics or fluid mechanics [14], [15]. When hydromechanics is taught, the students are first acquainted with difference between incompressible and compressible fluid. The hydromechanics can be divided in the hydrostatics and hydrodynamics. In the case of hydrodynamics for compressible flew only qualitative description is taught. This is because quantitative description of these problems is possible only by using higher mathematics. On the other hand, in the case incompressible flew the mathematical description is significantly simpler and more comprehensible to students. In the case of hydrostatics are physical laws taught also for gases.

Probably most important laws or principles taught in basic course of hydromechanics or hydrodynamics is Bernoulli's principle [14], [15], [16]. Most simple form of Bernoulli's principle for incompressible flows is taught in the seventh class at elementary School (in Czech Republic). The teaching of this issue can be appropriately complemented by a number of illustrative experiments. We can create a model set of different tubes in which we will measure the flow. These assemblies may include different flow sensors. These sensors are used for building blocks for teaching programming and robotics at secondary schools. These schools are oriented towards mechanical and electrical engineering. It is possible to integrate robotics and mechanics and physics in a suitable way in certain student projects. In this article, we will become acquainted with the integrated teaching of Bernoulli principle, as part of the student project in robotics and technical mechanics. At the same time, we will show the use of Agros2D in education.

B. Thermodynamics and Mechanics of Gases at Basic Course of Physics

Gases mechanics is a branch of fluid mechanics, which is devout to ideally compressible. In reality, also the gases are not compressible in zero volume. When compressing gases, phenomena that study thermodynamics begin to appear [17]. Thermodynamics is a branch of physics concerned with heat and temperature and their relation to energy and work. Students are familiar with the three laws of thermodynamics at technical high schools. This course builds on the basic knowledge obtained in the previous study and is based on the assumptions, that students are familiar with terms such as temperature. It is then possible to interpret such phenomena as thermodynamic processes: Adiabatic process, Isothermal process, Isobaric process and others. These processes can be implemented in a relatively simple laboratory.

V. Bernoulli's principle and Student Experiments

A. Introdaction

Bernoulli's principle is a seemingly counterintuitive statement about how the speed of a fluid relates to the pressure of the fluid [14] - [16]. The Bernoulli's principle is closely locked with the law of conservation of energy. In reality the basic Bernoulli's equation is law of conservation for sum of potential and kinetic energy of volume unit of fluid. Bernoulli's principle can be applied to various types of fluid flow, resulting in various forms of Bernoulli's equation. In the physics different forms of Bernoulli's equation are known for different types of flow. The simple form of Bernoulli's equation is valid for incompressible flows. The Bernoulli's law in this form says that, in a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline.

B. Incompressible Flows

When Bernoulli's principle is applied on the incompressible flow, e.g liquids and gases whose density can be considered to be constant in studied region [14] - [16]. This flow is also characterised by low Mach number [14], [16]. Where the Mach number can bee characterised as a dimension less quantity defined as the ratio of flow velocity past a boundary to the local speed of sound. Bernoulli originally performed his experiments only on liquids, so his equation in its original form is valid only for incompressible flow. A common form of Bernoulli's equation, valid at any arbitrary point along a streamline, is [14] - [16]:

$$\frac{v_1^2}{2} + gh_1 + \frac{p_1}{\rho} = \frac{v_2^2}{2} + gh_2 + \frac{p_2}{\rho} \ . \tag{21}$$

Where: v is the fluid flow speed at a point 1 or 2 (see. indexes in (12)) on a streamline, g is the acceleration due to gravity, h is the elevation of the point 1 or 2 above a reference plane, with the positive h-direction pointing upward – so in the direction opposite to the gravitational acceleration, p is the pressure at the chosen point, and ρ is the density of the fluid at all points in the fluid.

An important condition is there is no friction between fluid and pipe wall. The fluid flow is steady without turbulence. The meaning of indexes is shown at Fig. 9.

C. Compressible Flows

The basic and the most general form of Bernoulli's equation for compressible flows can be expressed as [14] - [16]:

$$\frac{v_1^2}{2} + \int_{p_1}^{p} \frac{dp}{\rho(p)} + \psi(\vec{r}_1) = konst. \tag{22}$$

where p is the pressure and ρ is the density, which is function of pressure p. The flow speed is v and Ψ is the potential associated with the conservative force field, often the gravitational potential. The potential Ψ is function of positron, i.e Positioning vector $\vec{r}$ or coordinates $x, y. z.$ If this equation is applied on the incompressible flow, the potential Ψ has same meaning as product gh in the (21).

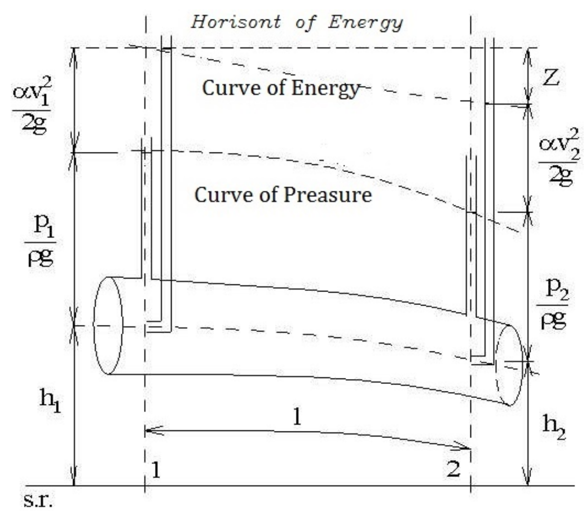


Fig. 9 Fluid flow through the pipe, from the point at elevation h_1 to the point at elevation h_2. The pipe is characterised by cross-sections S_1 and S_2

D. Experimental Device

The students are preparing functional model for study of Bernoulli's principle. This model is prepared from to types of pipes: Firmly un-deformable made from hardened transparent plastics and second prepared from soft plastics with a known modulus of elasticity. The elasticity modulus was E = 0.5 GPa. Students have base desk Arduino Micro with two flew sensors based on principles of Venture tube, and one sensor for deformation measurements. Device Assembly is clearly visible from Fig. 7.

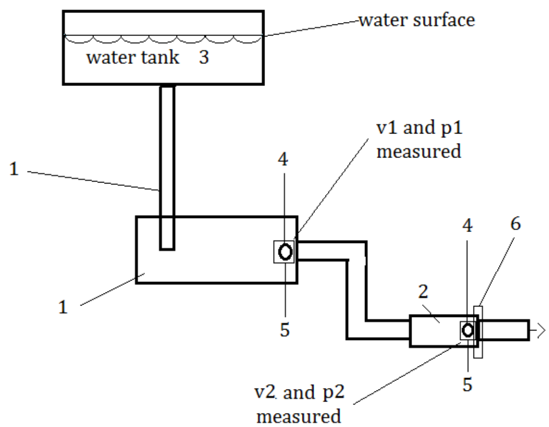


Fig. 10 Pipeline system with sensors: 1 – Tube from hard plastics, 2 – Exchangeable tube - hard plastics/ soft plastics, 3 – water tank, 4 - speed sensors, 5 – pressure sensors, 6 – deformation measurement – tube diameter and its expansion.

This is illustration of project-based learning of programming, mechanics and physics [18] - [20].

These flew sensors controlled by Arduino measured and record the flow speed at point 1 and 2. The deformation of diameter of elastic tube from the soft plastics is measured by deformation sensor. Also the pressure in the tubes at the point 1 and two is measured and recorded by sensors controlled by Arduino Micro.

The pressure, respectively flow speed is controlled by height of water tank over the basic plane. Also the height of the tank level is measured by the sensor controlled by Arduino Micro. The influence of water compression is very small and hardly measurable with available Equipment. Therefore, it has been decided to measure the deformation of the soft tube. And thus modify the model. The advantage is that the deformation of the tube is easily connected to the pressure changes in the pipeline by students. The growth of tube diameter is measured and the simulation is working with diameter of tube. For tube diameter increment Δd can be used same sensor as in the paragraph C in previous chapter II.

E. Simulation of Flow in Pipes

The students prepared 2D model of Pipeline system in Agros2D software: This model used as input data geometrical parameters of system especially the height of the individual tubes over the basic plane and diameter of tubes. Very important input value is the height of water surface in the water reservoir, this value was measured during the experiments. The task of FEM simulation is determine, the speed at the point 1 and 2 (These values are a function of the height of water surface in the water reservoir.), also the pressure at this two points was determined. These values are compared with measured. The influence of water compression is very small and hardly measurable with available Equipment. Experiments conducted by students show compliance 95% with FEM model.

VI. School Experiments and Gas Thermomechanics

A. Adiabatic Process and it's Theory

The adiabatic process can be described as compression or expansion of gases without transfer of heat or matter between a thermodynamic system and environment. So we will assume such a process: an ideal gas is compressed adiabatically (no heat was added, and also no matter was added in the vessel), work is done on it and the temperature og gas in the vessel increases. Otherwise in case of an adiabatic expansion, the gas does work and its temperature drops. These processes actually have a whole range of technical applications. For example, Adiabatic compression actually occurs in the cylinders of a car, where the compressions of the gas-air mixture take place so quickly that there is no time for the mixture to exchange heat with its environment. Further, this principle is utilized in for example in some refrigeration plants.

This process can be described in the case ideal gas by simple equation:

$$PV^n = konst. \tag{23}$$

where P is pressure, V is volume, and for this case $n = \gamma$. The process described by this form is a reversible (i.e., no entropy generation) process and process can be represented by the polytropic process equation. The parameter n can be defined as:

$$\gamma = \frac{C_P}{C_V} = \frac{f+2}{f} = \frac{k_T}{k_S}, \tag{24}$$

where C_P being the specific heat for constant pressure, C_V being the specific heat for constant volume, γ is the adiabatic index, and f is the number of degrees of freedom (f = 3 for monatomic gas, 5 for diatomic gas and collinear molecules e.g. carbon dioxide). The parameters k_T and k_S are isothermal compressibility and isoentropic compressibility respectively.

B. Adiabatic Process and Experiments

In this paragraph, the simple and accurate experiment to determine the adiabatic coefficient γ of air is presented. This experiment with its simplicity meets the requirements on the basic school experiment. The experiment has a moderate cost since only a large glass syringe, a glass bottle with two holes and two rubber stoppers, and a common pressure gauge with corresponding output are required. The method allows one to measure experimentally the isothermal compressibility, k_T, and the adiabatic or isoentropic compressibility k_S of air. The experimental setup is shown in Fig. 10. The volume changes are measured using glass syringe. The shift of syringe piston is measured by sensor and its movement is controlled by servomotor. The pressure changes were measured with pressure device on which is able import data from measurement for further analysis. The control of the experiment was realised utilizing Arduino components.

The volume changes are measured determined from piston displacement in syringe. In this way, students are able to determined pressure increase ΔP_T and ΔP_S, the measurement under constant volume decries ΔV. Finally the parameter γ will be determined equation:

$$\gamma = \frac{\Delta P_S}{\Delta P_T} \tag{25}$$

VII. Pedagogical Research

In this section we will compare other results of students of (bachelor study program) who attended lessons in such organized laboratories with others students. These students whose skills are evaluated are divided into three groups:

1. The students who have been involved in the preparation of such modified school laboratories.
2. The students who attended laboratory lessons and solved the problems described in the text, or other similar tasks. Thus, tasks that combine knowledge from programming, physics and mechanics.
3. The group of students that not attended these lessons.

A. Methodology

The evaluation of the students' results was based on their classification in other subjects, which are closely related to these problems. The results achieved in these lessons have been used for comparison: (1) two-semester course of mechanics, (two assessments); (2) three-semester course of PC work, which includes the basics of programming; (3) the one-semester course of automation and robotics; (4) two-semester course of the basics of technical physics; (5) a three-semester course of technology. Each evaluation includes a classification from the theoretical and practical part of the subject. The results of the comparison are shown in Table 1.

The table gives two school grades for each lesson, i.e. a classification from the practical part and from the theoretical part. Next to the average classification grade, we also use the median value in the given group. Both average and median classification are shown in Table 1. In the table are used these abbreviations: AC - average classification and MC - median classification; Pr - practical part of course and Th – theoretical part of course. Comparative groups are numbered in the table in the same order as in the previous paragraph.

B. Results and Discussion

The results shown in the Table 1 clearly show fact, that the students, who participated in the preparation of new school laboratory are best. The students who take parts on new lessons have very good results and third group have worse results than previous groups. Therefore, it is desirable to have as many students as possible on these courses. Generally speaking, such a lesson has a very good impact on students.

Table 1. The results of students in different subjects:

subject		Group 1		Group 2		Group 3	
		AC	MC	AC	MC	AC	MC
Physics I	Th	1.6	1.4	2.2	1.7	2.9	2.9
	Pr	1.2	1	2.6	2.2	2.5	2.5
Physics II	Th	2.1	1.5	2.7	2.3	3.1	3,0
	Pr	1.7	1.2	1.6	1.5	2.1	2.0
Mechanics I	Th	2.0	1.7	2.8	2.8	3.7	3.6
	Pr	1.9	1.5	2.5	2.5	3.0	3.0
Mechanics II	Th	1.5	1.5	2.8	2.7	2.8	2.7
	Pr	1.7	1.6	2.7	2.5	2.7	2.5
PC course I	Th	1.1	1.1	2.0	1.9	2.1	1.7
	Pr	1.0	1.0	2.3	2.3	2.7	2.2
PC course II	Th	1.5	1.5	2.9	2.7	3.1	3.1
	Pr	1.7	1.5	2.9	2.9	3.4	3.2
PC course III	Th	2.3	1.9	2.7	2.4	2.9	2.9
	Pr	1.7	1.7	2.4	2.4	3.1	3.0
Automatization and Robotics	Th	1.8	1.1	2.3	2.0	2.7	2.4
	Pr	1.2	1.2	1.9	1.9	2.9	2.7
Technology I	Th	1.9	1.9	1.6	1.4	2.0	1.7
	Pr	1.5	1.5	2.2	2.2	1.9	1.7
Technology II	Th	2.0	2.0	2.1	1.7	2.5	2.5
	Pr	2.3	2.0	2.7	2.2	3.7	3.6
Technology III	Th	1.7	1.4	2.5	2.5	2.4	1.3
	Pr	1.3	1.3	1.6	1.4	2.5	1.7

VIII. Conclusion

This is illustration of project-based learning of programming, mechanics and physics [18] - [20]. Our school will try to develop further this approach. These students that take part in this course and work on task described in text achieve much better results in both, theoretical and practical tests.

Acknowledgment

This project was prepared by finantial support "Metoda konečných prvků a automatizace experimentu ve výuce mechaniky a technických laboratoří na katedře technických předmětů" 18/I zč SV 2119 - pracoviště 1440.

References

[1] M. Aparacio, F, Bacao, T. Oliveira, "Cultural Impacts on E-learning System Success", *The internet and Higher Education* 31 (2016), Elsevier, pp. 58-70.

[2] C. L. McDermott, E. F. Redish, "Resource Letter: PER-1: Physics Education Research". *American Journal of Physics*. **67** (9) 1999, pp. 755–767,

[3] R. Duit, H. Niedderer, H. Schecker (2006). "Teaching Physics". *Handbook of Research on Science Education*, pp. 606

[4] C. L. McDermott,Guest Comment: "How we Teach and How Students Learn---A Mismatch?", *American Journal of Physics*. 61 (4) 1993, pp. 295–298.

[5] J. Chaskalovic. *Finite Elements Methods for Engineering Sciences*, Springer Verlag, 2008

[6] O.C. Zienkiewicz, R. L. Taylor, J. Z. Zhu, *The Finite Element Method: Its Basis and Fundamentals*, Butterworth-Heinemann, 2005

[7] I. Babuska, U. Banerjee, J.E. Osborn, "Generalized Finite Element Methods: Main Ideas, Results, and Perspective", *International Journal of Computational Methods*. 1 (1) 2004, pp. 67–103

[8] D.L. Logan, *A first course in the finite element method.* Cengage Learning, 2011

[9] K. Zachariadou, K. Yiasemides , N. Trougkakos, "A low-cost computer-controlled Arduino-based educational laboratory system for teaching the Fundamentals of Photovoltaic cells", Eur. J. Phys. 33, 2012, pp.1599–1610.

[10] R.F.Tinker, *Microcomputer- based Labs: Educational Research and Standards*, Springer, 1992

[11] M. Banzi, M. Shiloh, *Getting Started with Arduino*; 2014

[12] T. Karvinen, K. Karvinen, V. Valtokari*: Make: Sensors* Haftad, Engelska, 2014.

[13] J. Purdum, *Beginning C for Arduino: Learn C Programming for the Arduino and Compatible Microcontroller*, Apress, 2014.

[14] H. Lamb, *Hydrodynamics* (6th ed.). Cambridge University Press, 1993

[15] L.D. Landau, E.M. Lifshitz, *Fluid Mechanics*. *Course of Theoretical Physics* (2nd ed.). Pergamon Press, 1987.

[16] H. Chanson, *Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows,* CRC Press, Taylor & Francis Group 2009. T. Markham, Project Based Learning. *Teacher Librarian*, 39(2) 2011, 38-42

[17] M. Bailyn, *A Survey of Thermodynamics*. New York, NY: American Institute of Physics, 1994.

[18] J.G. Greeno, "Learning in activity", R. K. Sawyer (Ed.), *The Cambridge handbook of the learning science*s. New York: Cambridge University Press, 2006

[19] R.K. Sawyer, *The Cambridge Handbook of the Learning Sciences.* New York: Cambridge University Press, 2006.

[20] P. Solin, K. Segeth, I. Dolezel: *Higher-Order Finite Element Methods*, Chapman & Hall/CRC Press, 2003.

[21] P. Karban, F. Mach, P. Kůs, D. Pánek, I. Doležel, "Numerical Solution of Coupled Problems Using code Agros2D", *Computing*, 2013, Volume 95, Issue 1 Supplement, pp. 381-408.

[22] P. Solin, L. Korous, "Adaptive Higher-order Finite Element Methods for Transient PDE Problems Based on Embedded Higher-order Implicit Runge–Kutta Methods", *Journal of Computational Physics*, 2012, Volume 231, Issue 4, pp. 1635–1649.

[23] L. Korous, P. Solin, "An adaptive hp-DG method with Dynamically-Changing Meshes for Non-Stationary Compressible Euler Equations", *Computing*, 2013, Volume 95, Issue 1 Supplement, pp. 425-444

[24] P. Solin, J. Cerveny, I, Dolezal, *Arbitrary-Level Hanging Nodes and Automatic Adaptivity in the hp-FEM*, Math. Comput. Simul., 2008, Volume 77, pp. 117 – 132.