Applying Problem-Based Learning Model and Creative Design to Conic-Sections Teaching

Wen-Haw Chen

***Abstract*—**To help students integrate and apply learned mathematical concepts is definitely an important topic for a mathematics teacher. It is known that problem-based learning (PBL) model is an effective approach to help students self-learning from the problems. In particular, it can help them to integrate the knowledge and concepts previously learned in the classroom. This paper presents an idea in teaching and learning geometry to integrate the concept of creative design into a problem-oriented learning model. Our method is based on the learning of conic sections contents included in mathematics curriculum. In the learning process, students learn how to recognize the problem given by the teacher and applying classroom knowledge into practice. Further, through this learning activity's emphasis on problem-based learning, students acquire creative thinking skills and professional skills as they can use them to solve interdisciplinary and real-situation problems. Our goal is to set a model of teaching to integrate the conic sections concepts in high school mathematics. In addition to master the geometric concepts, students will create their own creation with geometric knowledge and team cooperation under this teaching model. It will enhance the effectiveness of teaching and learning related geometric concepts, and hence enable students to integrate and apply the material.

***Keywords*—**problem-based learning model, creative design, conic- sections teaching.

I. Introduction

An important topic in teaching and learning mathematics is how to help students integrate and apply learned mathematical concepts. Therefore, to provide learning experiences that allow students to be actively engaged in the material as they work cooperatively with others to apply the concepts in real-life contexts is always a challenge facing a mathematics teacher. Some students invested a lot of time and effort in every chapter of their mathematics curriculum. They may cope with a small range of assessment testing. However, they often meet the lack of the ability to integrate learned concepts, which is often "see the tree forest" problem. This problem leads in addition bad performance to a wide range of assessment tests, and the study of related college course such as calculus. While continuing to practice solving problems may improve the performance of examinations, but only the real mastery of relevance and flexibility of learned mathematics concepts will enhance positively students' mathematics literacy.

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Mathematics teachers should avoid entering a field that has predominantly been dominated with traditional teaching practices as the focus is oftentimes driven by lecture and performing simple computations. They have to on the contrary create authentic learning experiences that allow students to take ownership when students face difficult concepts. One of these difficult concepts in mathematics is conic sections and their properties (for example, the reflection properties of conic sections). Students find difficulty in working with the equations, the complexity of the numerous new terms, and realizing the graphs of equations and the reflection properties of standard conic sections. Then they may fail to integrate the concepts and apply these concepts in a real-life situation. Therefore, how to provide students with a memorable, engaging, and interactive learning experience is then crucial to an educator because it will allow students to enjoy their learning while gaining great understanding of the material and to be able to apply the learned notions.

In this paper, we will present an approach to enhance the effectiveness of geometry teaching through the problem-based learning (PBL) model incorporated creative design. Our goal is to improve geometry teaching and help students to integrate and apply the learned knowledge. Recently, PBL method has been practiced broadly in the academic domain to facilitate self-learning when students are required to implement self-learning through questions. The core concepts of PBL are using professional knowledge, goal setting, problem resolution, and evaluation of the results. PBL is applicable to various scenarios, although slight adjustments of the PBL process are necessary. Moreover, PBL has the following characteristics:

Initiating learning with a real problem.

Connection between the cognition and professional knowledge of the learners and the problems.

Learning in small groups.

A self-oriented learning model.

Teachers or experts are considered helpers, not leaders.

Therefore, the main idea in this paper bases on students' cognition and focuses on students' creativity. Students are encouraged in classroom to construct their own work by the geometric concepts learned in mathematics curriculum.

However, although students have learned how to calculate the surface area, volume, and the trigonometric functions, conic sections with space vector concepts, they may be unable to integrate the mathematical knowledge to design the final geometric work. Therefore, it is necessary to develop learning activities to guide students in creative design. These activities have to meet students' learning interest as well as taking into account the effect of individual learning goals. This approach will first develop some problems by researchers and teachers to integrate geometric concepts, and then grouped students in learning activities to the construction of the different products involving geometric knowledge in the works.

II. Review of Problem-Based Learning Model

PBL can be defined from various perspectives. Barrows and Tamblyn [2] define PBL as the process in which learners learn knowledge by understanding or solving specific problems. Other studies (e.g., Fogarty [11]) have considered PBL as a course model that focuses on real-world problems. Trop and Sage [23] considered PBL as experiential learning because it can be employed as a curriculum organizer and a teaching strategy. Schmidt [20] and Walton and Matthews [25] indicated that PBL is a learning method and can be used to explain the process of learning and teaching. Numerous studies believed that PBL was initially developed as a teaching method for training medical students to discuss and solve clinical medical problems; and a student-centered and real problem solving apprenticeship-style contextualized teaching method or strategy that anchors learning and teaching to the problem itself. (Boud and Feletti [6]; Bridges and Hallinger [7]; Delisle [9]; Dods [10]; Hoffman and Ritchie [13]; Hmelo and Lin [12]; Norman [16]; Norman and Schmid [17]; Mokhtar et.al [15]; Pirinen [18]). Barrows [3] also indicated that PBL is a flexible teaching method, the definition of which may differ in accordance with the teaching design and the skill of teachers. Briefly, one can conclude that PBL has the following characteristics.

A. Using an ill-structured problem as the focus of organizing a curriculum and scenario of learning.

The most crucial characteristic of PBL is to focus on an ill-structured problem to organize a curriculum and learning scenario and to initiate teaching and learning processes, thereby inspiring students to learn, explore, and develop the necessary professional knowledge and problem-solving tools required in the future (Barrows [4]; Fogarty [11]). The primary feature of unstructured problems is that each problem may differ according to individual learner backgrounds; thus, the problem is difficult and may not have a single correct solution or formula. During the investigation of unstructured problems, students are able to develop skills of meta-cognition that allows them to monitor, critique, and direct their own inference skills through problem-solving (Barrows [3]). Therefore, an appropriately designed problem is the most crucial element in PBL.

B. Learners become stakeholder.

PBL is a student-centered teaching and learning model. Students can pursue meaning and understanding of matters through self-directed learning (Barrows [4]; Boud & Feletti [6]; Trop & Sage [23]). When solving a problem, a special role is given to students. This role contributes to the students' association to the old and new knowledge, understanding of the importance of special problem-solving strategies, and how to re-apply the problem-solving strategies in the future.

C. Teacher as a trainer in cognition and meta-cognition..

When teaching with a PBL approach, teachers must assume the role of a curriculum designer, learning or question-solving partner, supporter and director of learning, and evaluator of learning results (Barrows [3]; Bridges & Hallinger [7]; Delisle [9]; Stepien & Gallagher [22]; Trop & Sage [23]). Teachers manage the process of PBL by adjusting the PBL process, role-playing, and monitoring students' participation. In addition, evaluations are constantly performed throughout the process to determine the learning progress and results of students (Trop & Sage [23]).

D. Encouraging group cooperation and learning.

According to Barrows [3], group cooperation and learning is a necessary feature of PBL. Each group member must cooperate with each other as a learner and question-solver. Providing and perceiving different viewpoints that are shared among group members are useful for the clarification of complex matters, thereby improving the efficiency of cooperation. Group discussion is also useful for teaching individuals to cope with different viewpoints, and to facilitate high-level cognitive skills such as inference and knowledge building. Moreover, group discussion also assists students in expressing their own understanding and perspectives when they are unable to convince other group members.

E. Multiple evaluation methods.

PBL is a constructivism-based teaching and learning model. Therefore, the evaluation of PBL is based on constructivism, cognitive psychology, and situational learning, and meets the teaching and evaluation requirements in scientific education. Proponents of PBL typically advocate numerous evaluation methods, and perform authentic assessment using attractive, valuable, significant, and real questions that target specific evaluation standards, thereby fully demonstrating the learning process and results. Furthermore, students' abilities can be shown and provided as a feedback for course efficiency. Previous studies have tended to categorize the evaluation of PBL as "content," "process," and "outcomes" (Barrows & Tamblyn [2]). Inference skills, question-solving intelligence, group work, and communication skills can be evaluated by those conducting the three categories of PBL evaluation (Barrows [3]; Barrows & Tamblyn [2]; Walton & Matthews [25]).

Moreover, PBL has been considered by a number of higher educational institutions in many parts of the world as a method of delivery, and is a total pedagogical approach to education that focuses on helping students develops self-directed learning skills. It derives from the theory that learning is a process in which the learner actively constructs new knowledge on the basis of current knowledge. PBL provides students with the opportunity to gain theory and content knowledge and comprehension. PBL helps students develop advanced cognitive abilities such as creative thinking, problem solving and communication skills.

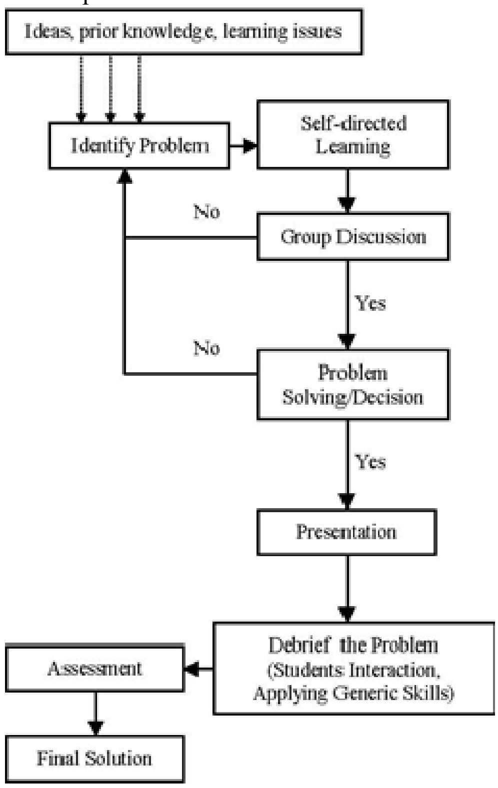


Fig. 1 Flowchart of Problem Solving Process in PBL

The above figure (Awang & Ramly [1]) shows the flowchart of problem solving process in Problem-based Learning approach. PBL exercises typically proceed through four phases – problem presentation, problem investigation, and problem solution and process evaluation. The problem would be a real-world situation, complex and open-ended that will challenge higher-order thinking, creativity and synthesis of knowledge (Steinemann [21]). Problem-Based Learning helps students develop creative thinking skills such as cooperative and interdisciplinary problem solving. Students learn to work both independently and collaboratively. Even though students engage in self-directed learning through PBL, they regularly convene to share, evaluate and critique each other's work during the group meeting. They deal with multiple and often conflicting goals and values, work with constraints and determine the most appropriate action to take.

In addition facilitates the acquisition of knowledge, group learning has another desirable attributes such as communication skills, teamwork, problem solving, independent responsibility for learning, sharing information and respect for others. Therefore PBL can be a small group teaching method that combines the acquisition of knowledge with the development of generic skills and attitudes. Presentation of technical material as the stimulus for learning enables students to understand the relevance of underlying scientific knowledge and principles in technical practice.

III. Creative Design

Torrance ([24]) defined creativity as

"*a process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficulty; searching for solutions, making guesses, or formulating hypotheses about the deficiencies: testing and retesting these hypotheses and possibly modifying and retesting them; and finally communicating the results.*"

Creativity may mean "bringing into being something that was not there before and has been brought into being [1]". It covers a wide range of different skills. Creative skills needed to change concepts and perceptions and are usually helpful to problems solving. However, creativity is poorly understood and difficult to teach. Edward de Bono notes in [8] that the creative techniques such as focus, challenge, alternatives, and concepts etc., which are positive techniques that everyone can learn. In teaching, creative thinking will make students to try different perceptions, different concepts, and different points of entry and then they can use various methods to solve the problems. Thus creative thinking is closed related to perception to put forward different viewpoints, which are not derived each from the other but are independently produced.

A widely used of creativity tests is Torrance Tests of Creative Thinking (TTCT) (Torrance [24]). There are three creative abilities measured by these tests, which are originality, fluency and flexibility. These abilities are defined as

Originality: the ability to produce uncommon or unique responses;

Fluency: ability to produce a large number of ideas;

Flexibility: the ability to produce a variety of ideational themes or categories.

Edward De Bono identified the uses of creativity. Creativity becomes part of normal thinking and can therefore be applied to any situations that require thinking (De Bono [7]). There are three aspects to for creativity [1]:

A. Defining the focus or creative task:

There may be problems that arise and identify themselves. Individuals make definite creative focuses. There may be an obvious creative need. All these are ways in which creative focuses can emerge.

B. Structure for the deliberate application of the systematic creative thinking tools:

Once the creative focus has been defined, it can be subjected to deliberate creative thinking. Groups or individuals or a combination of both in a discussion session among group members can do it. It often happens that the group that has the concern or problem will organize its own deliberate creative thinking session to tackle the problem. In (Pun [19]) the author presents a result to develop in students' creative thinking skills in solving design problems, and then show the effectiveness of this approach in making team-learning fun for the students who are from different majors.

C. Evaluation and implementation of the output of the creative thinking:

The group that has the creative focus may also be involved in evaluating the ideas that come out of the deliberate creative thinking. In such cases the process is continuous. If the "thinking" group is different from the "implementation" group, attention has to be paid to the transfer of ideas so that those expected to act on the idea are brought in at an early enough stage to feel some ownership in the new ideas.

Learning with creative thinking is important to be creative at each stage of discussion including the definition of thinking task, the structure for applying thinking process, the output of thinking effort, and the evaluation and implementation needs to be created.

IV. Apply to Conic Section Teaching

To apply PBL together with creative design to geometry teaching, teachers are encouraged to use methodologies to promote creative thinking and students are encouraged to be innovative and come up with creative products. Students can be encouraged to participate in this process by enabling them to become aware of the ways in which they think, learn and problem-solve. The way of thinking will also attempts to involve students in the teaching learning process through evaluations of what is taking place during learning and can provide a window into the student's thinking processes. This strategy directly addresses the core of the instructional challenge of allowing learners to actively learn about conic sections and their applications. By giving students a real-life situation to apply the mathematical content (conic sections and their applications) in, it will allow them to gain meaning from their learning. As the students apply the content in solving problems, they will come to a greater understanding of it. Moreover, working in a group to complete their creation will help lead students to work more vigorously because of their obligation to their learning team.

Here is an example of applying PBL together with creative design to geometry teaching. We wish to integrate the geometric concepts including space vector, lines and planes in the space, and conic sections etc. Students are encouraged to design a simple lamp involving these concepts. Its external appearance may be a polyhedron or a prism, and then the concepts about planes in space; dihedral, surface area and volume can be involved. Reflection properties of conic sections can be used to design the interior of the lamp. Group meeting and discussion is necessary to finish the final work.

We use the model consists of four phases; Search, Solve, Create and Share (Model SSCS) to introduce our example.

TABLE I. Model SSCS

Phase	Content	Example
Search	Brainstorming to identify problem, generate a list of ideas to explore, put into question format and focus on investigation.	**Problem:** What are the differences among an ellipse, a parabola and a hyperbola? How can you apply the knowledge to your daily life?
Solve	Generate and implement plans for finding a solution, develop critical and creative thinking	**Geometric concepts:** Definitions and equations of an ellipse, a parabola and a hyperbola, and their reflection properties. Integrate the geometric concepts including space

	skills, form hypothesis, select the method for solving the problem, collect data and analyze.	vector, lines and planes in the space, and conic sections etc.
Create	Students create a product in a small scale to the problem solution, reduce the data to simpler levels of explanation, and present the results as creatively as possible such using charts, poster or model.	**Design a product:** Students design a simple lamp with external appearance may be a polyhedron or a prism, and the interior may involve the reflection properties of conic sections. The concepts about planes in space, dihedral, surface area and volume can be can be used to design.
Share	Students communicate their findings, solution and conclusions with teacher and students, articulate their thinking, receive feedback and evaluate the solutions.	**Presentation:** Group meeting and discussion is necessary to finish the final work. Students present their works and receive feedback. Teachers can evaluate the effectiveness.

Following the completion of students' conic sections design project, our objectives include

1) *Mathematical Objectives:*
Identify the key concepts of conic sections (especially the reflection properties); demonstrate the ability to express the conic sections graphically and algebraically, and generate a design project that is applies the conic sections in a real-life situation.

2) *Collaborative Objectives:*
Demonstrate the ability to work collaboratively and cooperatively with others to produce a design project and the ability to present their project in an effective manner.

The ideal pedagogy is one that provides direct experience, namely object teaching. The conventional definition of object teaching is that teachers employ real objects to assist teaching delivery. These objects are effectively teaching aids that assist teachers in teaching and students. There are several characteristics of teaching aids

Facilitating interest in learning so that students become eager to learn.

Inspiring students to think so that students have less difficult in understanding abstract concepts.

Gaining experience and enhancing memory.

Coping with individual differences and needs among students.

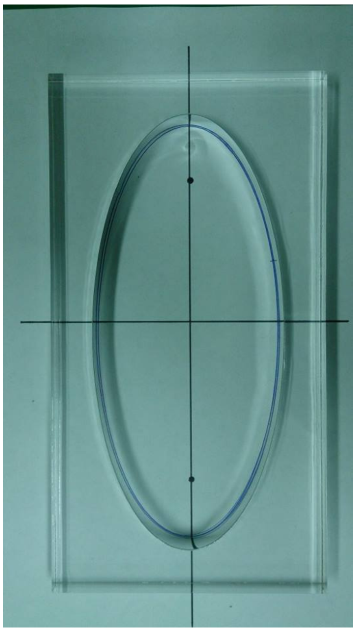
Here are students' works about presenting the reflection properties of conic sections:

A. *The Ellipse and its Reflection Property*

a) *Mathematics Equation:*

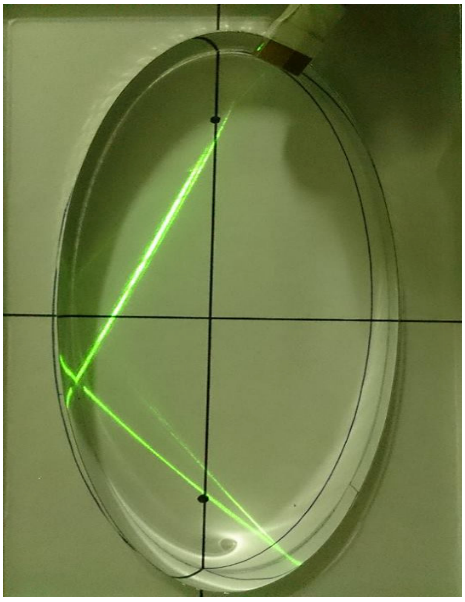
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, where\ b>a>0.$$

b) *A real model:*



c) *Reflection property*

Light which comes from one focus of an elliptical mirror is reflected at the ellipse to pass through the second focus.

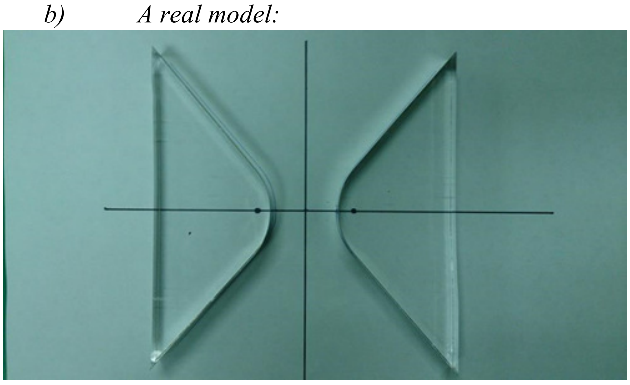


B. *The Hyperbola and its Reflection Property*

a) *Mathematics Equation:*

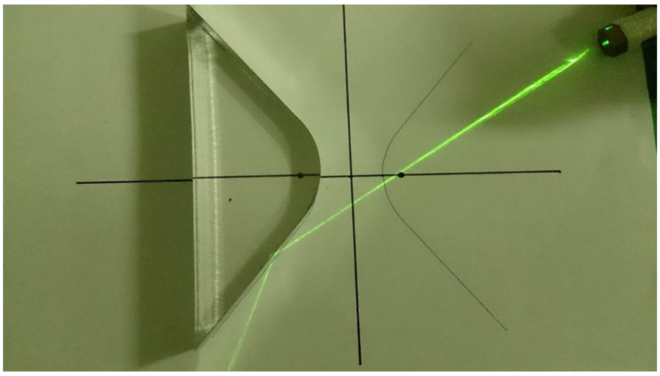
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, where\ b>0, a>0.$$

b) *A real model:*

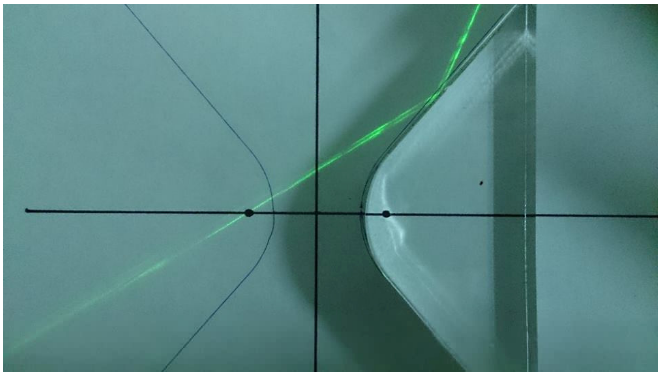


c) *Reflection property*

Light coming from one focus of a hyperbola mirror is reflected at the hyperbola in such a way that the light appears to have come from the other focus.



Also, light going towards one focus of a hyperbolic mirror is reflected at the mirror towards the other focus.

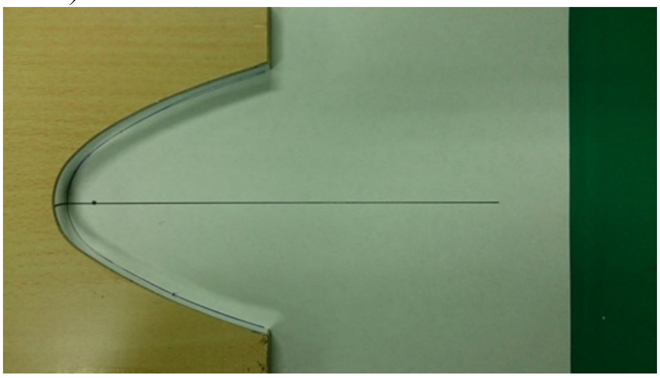


C. *The Parabola and its Reflection Property*

a) *Mathematics Equation:*

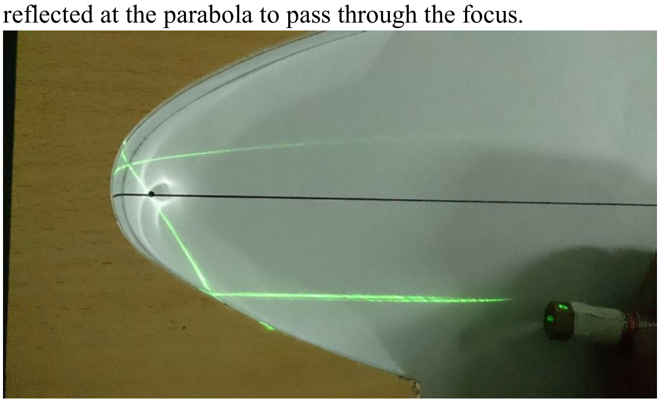
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, where\ b>a>0.$$

b) *A real model:*



c) *Reflection property*

Incoming light parallel to the axis of a parabola mirror is reflected at the parabola to pass through the focus.



Evaluation/Assessment

The teacher may evaluate this learning activity from the following categories:

1. Lamp Description: Explanation of the function and purpose of the designing lamp.

2. Design of the Appearance: The design looks professional and involves the concepts of space.

3. Design of the Internal: What kind of conic sections will be used?

4. Calculate the Conic Section: The calculation of elements of the conic section such as equation, focus and directrix etc.

5. Graphs: The graphs of the equations reflect an accurate representation of the equations.

6. Reflection property of the Conic Section: Investigate the result of lights comes from or toward the focus of the conic section.

Students will be asked to complete a peer assessment to evaluate their team. The questions may be
The teacher may evaluate this learning activity from the following categories:

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Design of the Internal: What kind of conic sections will be used?

Calculate the Conic Section: The calculation of elements of the conic section such as equation, focus and directory.

Graphs: The graphs of the equations reflect an accurate representation of the equations.

Reflection property of the Conic Section: Check that the result of lights comes from or toward the focus of the conic section fit the reflection theory.

Students will be asked to complete a peer assessment to evaluate their team. The questions may be

Are all members contributed equally to the project?

Do our group worked well together?

Were disagreements settled quickly and politely?

Did I feel encourage by my group members to work on the project?

Would I like to work with this group again?

V. Conclusion

We present an approach to enhance the effectiveness of geometry teaching through the problem-based learning model incorporated creative design. Our goal is to present PBL as an instructional model that could encourage the creative design during the geometry teaching learning process. We hope it will improve geometry teaching and help students to integrate and apply the learned knowledge.

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