Design of optimal water distribution systems

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Abstract**—** The paper approaches the optimization of water distribution networks supplied from one or more node sources, according to demand variation. Traditionally, in pipe optimization, the objective function is always focused on the cost criteria of network components. In this study an improved linear model is developed, which has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, operating expenses etc. The paper treats looped networks wich have concentrated outflows or uniform outflow along the length of each pipe. An improved model is developed for optimal desing of new or partially extended water distribution networks, which operate either by means of gravity or a pump system.The model is based on the method of linear programming and allows the determination of an optimal distribution of commercial diameters for each pipe in the network and the length of the pipes which correspond to these diameters. Also, it is possible to take into account the various functional situations characteristic found during operation. This paper compares linear optimization model to the some others, such as the classic model of average economical velocities and Moshnin optimization model. This shows the good performance of the new model. For different analyzed networks, the saving of electrical energy, due to diminishing pressure losses and operation costs when applying the developed model, represents about 10…35 %.

Keywords**—** Distribution, Linear optimization model, Looped networks, Water supply.

I. Introduction

DISTRIBUTION networks are an essential part of all water supply systems. The reliability of supply is much greater in the case of looped networks. Distribution system costs within any water supply scheme may be equal to or greater than 60 % of the entire cost of the project. Also, the energy consumed in a distribution network supplied by pumping may exceed 60 % of the total energy consumption of the system [20].

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimization in analysis and design. A water distribution network that includes bosster pumps mounted in the pipes, pressure reducing valves, and check–valves can be analyzed by several common methods such as Hardy–Cross, linear theory, and Newton–Raphson [27].

Traditionally, pipe diameters are chosen according to the average economical velocities (Hardy–Cross method) [6]. This procedure is cumbersome, uneconomical, and requires trials, seldom leading to an economical and technical op-timum.

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Nonlinear programming is one of the common methods that have been used to design water distribution systems, specially, in networks supplied by direct pumping [23].

Dynamic programming [29] is also used primarily to solve tree–shaped networks and could be extended to solve to looped systems [16].

For optimizing the design of pipe network with closed loops that is a nonlinear problem, the bulk transport function can be used as an objective function. Strictly, this will not be the optimum for nonlinear flow rate – cost relationships, since economy of scale is not introduced [27].

Dixit and Rao [9] have used a method in which only the cost of pipes is minimized. This method is only a tool to provide a good starting solution for the designer to further improve on the solution by using engineering judgement. There are other analytical and numerical models [1], [5], [18], [29] which make use of optimization of cost criteria, but have had relatively little succes. Some of these methods either require more feasible variants, or do not include the case of looped networks supplied by more sources and having bosster pmps installed in the pipes. On the other hand, all of these optimization models consider quadratic turblence flow, are based on the concentrated outflow and they permit the use of only one diameter for each pipe.

More recently, genetic algorithms (GA) and simulated annealing have been integrated with hydaulic network solvers for the optimization of water distribution systems [7], [25], [26]. One disadvantage is that the GA technique requires the large number of hydraulic simulation evaluation

This paper develops a linear model for optimal design of new and partially extended distribution systems supplied by pumping or gravitation. It is based on linear programming and allows for the determination of optimal distribution of commercial diameters along the length of each pipe and the length of pipe sectors corresponding to these diameters. It is possible to take into account various functional situations characteristic found during operation and uniform outflows along the length of each pipe. This model can serve as guidelines to supplement existing procedures of network design.

II. Basis of Hydraulic Calculation

A distribution network may be represented by orientation comprising a finite number of arcs (pipes, pumps, fittings) and a set of nodes as well as reservoirs and pumps or pipe intersections.

For a looped network with a simple topology, using the Euler’s equation which gives the cyclomatic number of the graph, the following expression may be established between the number of pipes T, nodes N and independent closed–loops (possibly containing bosster pmps installed in the pipes) M:

$$M = T - N + 1 \quad (1)$$

In the case of a complex topology, with reservoirs and pumps at the nodes, the number of open–loops (pseudoloops) N_{RP}–1 is added to the number of closed–loops given by (1), so that the total number of independent loops is determined from the equation:

$$M = T - N + N_{RP} \quad (2)$$

where N_{RP} is the total number of pressure generating fa-cilities.

The hydraulic calculation of a distribution networks involves in determining the diameters, discharges and head losses in pipes, in order to guarantee at each node the necessary discharge and pressure.

When performing the hydraulic calculation of a distri-bution network, the laws of water flow in all the pipes must be respected:

– discharge continuity at nodes:

$$f_j = \sum_{\substack{i=1 \\ i \neq j}}^{N} Q_{ij} + q_j = 0 \ (j = 1,..., N - N_{RP}) \quad (3)$$

in which: f_j is the residual discharge at the node j; Q_{ij} – discharge through pipe ij, with the sign (+) when entering node j and (–) when leaving it; q_j – con-sumption discharge (demand) at node j with the sign (+) for node inflow and (–) for node outflow.

– energy conservation in loops:

$$\Delta h_m = \sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 \ (m = 1,..., M) \quad (4)$$

in which: Δh_m is the residual head loss (divergence) in the loop m; h_{ij} – head loss of the pipe ij; ε_{ij} – orientation of flow through the pipe, having the values (+1) or (–1) as the water flow sense is the same or oposite to the path sense of the loop m, and (0) value if $ij \notin m$; f_m – pressure head introduced by the potential elements of the loop m, given by the relations:

- simple closed–loops:

$$f_m = 0 \quad (5)$$

- closed–loops containing bosster pmps installed in the pipes:

$$f_m = \sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} H_{p,ij} \quad (6)$$

- open–loops with pumps and/or reservoirs at nodes:

$$f_m = Z_I - Z_E \quad (7)$$

where: Z_I, Z_E are piezometric heads at pressure devices at the entrance or exit from the loop; $H_{p,ij}$ – pumping head of the bosster pump integrated on the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points:

$$H_{p,ij} = AQ_{ij}^2 + B\left|Q_{ij}\right| + C \quad (8)$$

the coefficients A, B, C can be determined from three points of operating data [20].

In the particular case of treee–shaped networks ($M = 0$, $T = N$–1), the number of N–1 node equations is sufficient for determining discharges as unknowns.

The design of a looped network with T pipes involves 2 kinds of unknowns: T values of discharges and T values of pipe diameters with a total of $2T$ unknowns. For computing there are T independent hydraulic equations: (3) and (4), being from the mathematical point of view an undetermined problem, with the undetermination degree equal to the cyclo-matic number in the graph system.

Classically, the undetermination is resolved by choosing diameters in a predesigned computation. In the method of optimization the undetermination must be removed by obtai-ning the other T equations, i.e, by equating the partial deriva-tives of the analytical optimization criterion with respect to pipe diameters to zero. Hence, theoretically there is a possi-bility to obtain an absolutely optimal solution, but mathema-tically it is very difficult to obtain a minimum, as is shown in the following.

III. NETWORK DESIGN OPTIMIZATION CRITERIA

Optimization of distribution network diameters considers a mono– or multicriterial objective function. Cost or energy cri-teria may be used, simple or complex, which considers the network cost, pumping energy cost, operating expenses, in-cluded energy, consumed energy, total expenses etc.

Newtork cost C_c is obtained by adding the costs of each compound pipe, by the relation:

$$C_c = \sum_{ij=1}^{T} (a + bD_{ij}^{\alpha}) L_{ij} \quad (9)$$

where: T is the number of pipes in a network; a, b, α – cost parameters depending on pipe material [21]; D_{ij}, L_{ij} – diameter and the length of pipe ij.

Pumping station cost C_p, proportional to the installed power, is given by:

$$C_p = \frac{9.81}{\eta} f \, \sigma Q_p \left(\sum h_{ij} + H_0 \right) \quad (10)$$

where: η is the efficiency of pump station; f – installation cost of unit power; σ – a factor greater than one which takes into account the installed reserve power; Q_p – pumped discharge; Σh_{ij} – sum of head losses along a path between the pump station and the critical node; H_0 – geodesic and utilization component of the pumping total dynamic head.

Pumping energy cost C_e is defined by:

$$C_e = W_e e = \frac{9.81}{\eta} 730 e \tau \sum_{1}^{12} \Phi_k Q_p \left(\sum h_{ij} + H_0 \right) \quad (11)$$

where: W_e is the pumping energy; e – cost of electrical energy; $\tau = T_p/8760$ – pumping coefficient, which takes into account the effective number T_p of pumping hours per year; Φ_κ – ratio between the average monthly discharge and the pumped discharge, having the value 1 for industrial enter-prises to which a constant discharge is delivered throughout the year, coresponding $\Sigma\Phi_k = 12$, while for population centres a sequence of 12 values can be taken, to which $\Sigma\Phi_k = 10.44$, corresponds.

Annual operating expenses C_{ex} are given by:

$$C_{ex} = p_1 C_c + p_2 C_p + C_e \tag{12}$$

where p_1 and p_2 are represented by repair, maintenance and periodic testing part for network pipes and pump stations respectively.

Annual total expenses C_{an} are defined by the multicriterial function:

$$C_{an} = \beta_o (C_c + C_p) + C_{ex} \tag{13}$$

where $\beta_0 = 1/T_r$ is the amortization part for the operation period T_r.

Total updated expenses C_{ac} are given by the multicriterial function:

$$C_{ac} = C_c + C_p + \frac{(1+\beta_0)^t - 1}{\beta_0 (1+\beta_0)^t} C_{ex} \tag{14}$$

and is considered during the whole operation period ($t = T_r$).

Network included e*nergy* W_c is defined by the binomial objective function of the form (1), where a, b, α parameters have statistically corresponding determined values [21].

Energetic consumption W_t represents the energy included in the pipes of the network and the energy consumed in net-work operation during one year and is expressed by:

$$W_t = (\beta_0 + p_1) W_c + W_e \tag{15}$$

where W_e is the pumping energy, having the expression determined from (3).

Taking into account (9) to (15) and denoting:

$$r_a = \frac{(1+\beta_0)^t - 1}{\beta_0 (1+\beta_0)^t} \tag{16}$$

$$\xi_1 = r_a p_1 + \frac{t}{T_r}; \quad \xi_2 = r_a p_2 + \frac{t}{T_r} \tag{17}$$

$$\psi = \frac{9.81}{\eta} (f \sigma \xi_2 + 730 r_a e \tau \sum_1^{12} \Phi_k) \tag{18}$$

a complex objective multicriterial function is determined, with the general form:

$$F_c = \xi_1 \sum_{ij=1}^{T} (a + b D_{ij}^{\alpha}) L_{ij} + \psi \sum_{j=1}^{NP} Q_{p,j} (\sum h_{ij} + H_0)_j \tag{19}$$

where: t is the period for which the optimization criterion expressed by the objective function is applied, having the value 1 or T_r; NP – number of pump stations.

For networks supplied by pumping, the literature [1], [5], [9], [28] suggests the use of *minimum annual total expenses criterion* (CAN), but choosing the optimal diameters obtained in this way, the networks become uneconomical at some time after construction, due to inflation.

Therefore, it is recommended the fore–mentioned criterion be subject to dynamization by using the *criterion of total updated minimum expenses* (CTA), the former being in fact a specific case of the latter when the investment is realised within a year; the operating expenses are the same from one year to another and the expected life-time of the distribution system is high. In particular, the use of energetical criteria different from cost criteria is recommendable. Thus, another way to approach the problem, with has a better validity in time and the homogenization of the objective function is network design according to *minimum energetic consumption* (WT).

The general function (19) enables us to obtain a particular objective function by particularization of the time parameter t and of the other economic and energetical parameters, cha-racteristic of the distribution system. For example, from $t = 1$, $r_a = 1$, $e = 1$, $f = 0$ the minimum energetic consumption criterion is obtained.

IV. Computational Model of Discharges Optimal Distribution

It is known that looped networks usually require a greater financial outlay than tree–shaped networks which supply the same nodes and have the same hydraulic load. The security of supply is much grea-ter in the case of looped networks.

The objective function (19), used for optimal design of looped networks, is a function of discharges Q_{ij} (by means of head losses h_{ij}) as well as diameters D_{ij} of each pipe. Therefore big problems arise when obtaining analytically its minimum. By equating zero with the partial derivatives of the form $\partial F_c/\partial D_{ij}$ and $\partial F_c/\partial Q_{ij}$ it is shown that the objective function F_c admits extrema, but second order derivatives indi-cate that the objective function has minimum values with res-pect to diameters D_{ij} and maximal values with respect to disharges Q_{ij}. This complicates the determination of the solution to the problem.

Also, knowing nodes discharges with respect to flow, pipes discharges could be calculated in a variety of ways for set of equations (3) to be satisfied, this however affects the security and technical and economic–energy conditions of the system. Thus, if exclusively quantitative criteria (CAN, CTA, WT) are used for the optimal design of looped systems, supply branches with very different values of diameters may result [20], [24].

Hence, computation of the optimal design of looped net-works must be performed in the following stages:

– establishment of optimal distribution for discharges through pipes, Q_{ij}, according to the minimum bulk transport criterion, which takes into account the network reliability;

– computation of optimal pipes diameters, $D_{k,ij}$, taking into account the optimized discharges.

In order to optimize discharges *the minimum transport work criterion* may be used, which is expressed analytically by M objective functions of the form:

$$F_t = \sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} Q_{ij}^{\gamma} \to \min, \quad (m = 1,...,M) \tag{20}$$

where pipe lengths L_{ij} are known and to which node–continuity equations (3) are added as constraints.

In the model described by the set of equations (20), (3) provides the supply of network nodes in the shortest way and with a minimum of transporation effort, and is solved by applying the itteration method, computing a circulation flow ΔQ_m for the loop m, from the condition:

$$F_t = \sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} (Q_{ij} + \Delta Q_m)^{\gamma} \to \min \tag{21}$$

which requires that the derivative $\partial F_t/\partial \Delta Q_m = f(\Delta Q_m)$ should equal zero, that is:

$$f(\Delta Q_m) = \gamma \sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} (Q_{ij} + \Delta Q_m)^{\gamma-1} = 0, \tag{22}$$

of which, only two terms of the Mac–Laurin series are retained:

$$f(\Delta Q_m) = f(0) + \Delta Q_m f'(0) = 0 \tag{23}$$

Performing the differentiation of the function (22) and, in particular for $\Delta Q_m = 0$ results in:

$$f'(0) = \gamma(\gamma-1) \sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} Q_{ij}^{\gamma-2} \tag{24}$$

Using this and (23) the following equations are obtained:

$$\Delta Q_m = -\frac{\sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} Q_{ij}^{\gamma-1}}{(\gamma-1) \sum_{\substack{ij \in m \\ ij=1}}^{T} L_{ij} \left| Q_{ij} \right|^{\gamma-2}} \quad (m = 1,...,M) \tag{25}$$

where discharges Q_{ij} are oriented quantities, having the sign (+) when their direction in pipes coincides with the loop path direction, and the sign (–) otherwise.

Initial distribution of the discharges is obtained by considering null discharges in the pipes which do not belong to the primary virtually tree–shaped network [21] and determining recurrently the other discharges, starting from the extreme nodes of this network. Starting from the initial solution, the discharges are itteratively corrected, until the precision given by the maximum admissable error is attained on a loop.

The effective discharge Q_{ij} is computed with (26) or (27) as the considered pipe is singular or common for the loops m and k:

$$Q_{ij} = Q_{ij}^{(0)} + \Delta Q_m \tag{26}$$

$$Q_{ij} = Q_{ij}^{(0)} + \Delta Q_m - \Delta Q_k \tag{27}$$

where: $Q_{ij}^{(0)}$ is the discharge through pipe ij at the previous approximation; ΔQ_m, ΔQ_k – circulation flow for the loops m and k respectively.

For 30 values of exponent γ between 0 and 4 were designed some looped networks with different geometry, contours, and inflow and outflow conditions using the Moshnin optimization model [1], where flow circulation model (25) is included and the optimization criteria CTA and WT are considered. From the exponent γ has resulted the optimum value of 1.8...2.2 [24] (for which the objective functions F_t and F_c have minimum value). Consequently, γ may be approximated to 2.

The implementation of the mathematical model (25) into a computer program for optimal design of looped networks, leads not only to optimal diameters, but also guarantee high security of supply in their function.

V. Linear Optimization Model

A. Generalization of relationship head loss–discharge for the pipe with uniform out-flow along their length

For the evaluation of the energy disipated in pipes with variable discharge, a complex computational relation has been established by specialised studies [8]. The relation takes into account the complete hydrodynamic effects including the secondary (branch) ones, in the zones of consumption nodes.

In particular, for pipes with uniform outflow along their length, the expression for head loss between extrem ends take the form:

$$h_{ij}^{*} = R_{ij} \left(Q_0^2 - Q_0 Q_c + \frac{Q_c^2}{3} \right) \frac{8\alpha_0 Q_c}{\pi^2 g D_{ij}^4} (2Q_0 - Q_c) \tag{28}$$

where: D_{ij}, R_{ij} are the diameters and hydraulic resistance of the pipe ij; Q_0 – inflow in the initial section of the pipe ij; Q_c – outflow along the length of the pipe ij; α_0 – nonuniformity coefficient of velocity distribution in the cross–sections of the pipe.

The second term of (28) represents the energy loss due to variation of outflow along the length of the pipe and determines a diminishing of the total head loss.

The discharge in the pipes of a network could be considered constant, equalizing head loss in a pipe with uniform outflow along its length with head loss in a simple pipe with concentrated outflow:

$$h_{ij}^{*} = R_{e,ij} \left(Q_0 - \frac{Q_c}{2} \right)^2 = R_{e,ij} Q_{ij}^2 \tag{29}$$

where: $R_{e,ij}$ is the equivalent hydraulic resistance of the pipe ij; Q_{ij} – discharge through pipe ij.

By equating relations (28) and (29), from elementary computations, the following expression of the equivalent hydraulic resistance is arrived at:

$$R_{e,ij} = R_{ij} \frac{\frac{1}{3}\theta_{ij}^2 - (1 - \omega_{ij})\theta_{ij} + 1 - 2\omega_{ij}}{\left(1 - \frac{\theta_{ij}}{2}\right)^2} \tag{30}$$

in which the following nondimensional characteristics have been used:

– *for outflow*: $$\theta_{ij} = \frac{Q_c}{Q_0} \tag{31}$$

– *for pipe:* $$\omega_{ij} = \frac{\alpha_0 D_{ij}}{\lambda_{ij} L_{ij}} \tag{32}$$

The total length of a pipe ij, with the discharge Q_{ij}, may be divided into s_{ij} partial length (sector), k, of $D_{k,ij}$ diameters and $X_{k,ij}$ lengths. Taking into account the Darcy–Weisbach's functional equation, the friction slope $J_{k,ij}$ for each sector k of the pipe ij can be calculated, in the hypothesis of concentrated outflow, with the equation:

$$J_{k,ij} = \frac{h_{k,ij}}{X_{k,ij}} = \frac{8}{\pi^2 g} \lambda_{k,ij} \frac{Q_{ij}^2}{D_{k,ij}^r} \tag{33}$$

where: r is an exponent having the value 5.0; g – gravi-tational acceleration; $\lambda_{k,ij}$ – friction factor of sector k in pipe ij, can be calculated using the Colebrook–White formula, or the explicit equation proposed in [2] for the transitory turbulence flow.

Since in real conditions the discharge decreases from one cross–section to another in the sense of outflow, an increase of pressure is accomplished at the outlet of the pipe, by a phenomenon similar to rebound, which has as the effect of diminishing the head loss. Thus, it is taked into account (30) and the notations (34), (35) are introduced in (33):

$$\Theta_{ij} = \frac{4}{3}\frac{\theta_{ij}^2 - 3\theta_{ij} + 3}{(2-\theta_{ij})^2} \tag{34}$$

$$\Omega_{k,ij} = \frac{4\alpha_0 D_{k,ij}}{\lambda_{k,ij}(2-\theta_{ij})} \tag{35}$$

where θ_{ij} has the value 1.00 if $Q_{ij} \le Q_{c,ij}/2$, and otherwise has the value given by (36) for the pipes of the primary tree–shaped network or by (37) for virtually supressed pipes from the given network, in the case of looped systems:

$$\theta_{ij} = \frac{Q_{c,ij} + q_j}{Q_{ij} + \dfrac{Q_{c,ij}}{2}} \tag{36}$$

$$\theta_{ij} = \frac{Q_{c,ij}}{Q_{ij} + \dfrac{Q_{c,ij}}{2}} \tag{37}$$

The expression of friction slope in each sector k of the pipe ij, for the uniform outflow along the length of the pipe, is rewritten as:

$$J^*_{k,ij} = \frac{h^*_{k,ij}}{X_{k,ij}} = J_{k,ij}\left(\Theta_{ij} - \frac{\Omega_{k,ij}}{X_{k,ij}}\right) \tag{38}$$

For Θ_{ij}=1 and $\Omega_{k,ij}$=0 the general equation (38) takes the particular form (33), valid for pipes with constant discharge. Specific consumption of energy for distribution of water w_{sd}, in kWh/m^3, is obtained by referring the hydraulic power dissi-pated in pipes to the sum of discharges:

$$w_{sd} = 0.00272\frac{\sum\limits_{ij=1}^{T} R_{ij}\left|Q_{ij}\right|^{\beta+1}}{\sum\limits_{\substack{j=1\\ q<0}}^{N}\left|q'_j\right|} \tag{39}$$

where q'_j is the outflow at the node j.

B. Development of the mathematical model

The discharges Q_{ij} are determined for each operating condition. The distribution of discharges is optimized by using model (25) with $\gamma = 2$.

The series of commercial diameters which can be used $D_{k,ij} \in [D_{max,ij}, D_{min,ij}]$ for each pipe ij are established using the limit values of optimal diameters $D_{max,ij}$ and $D_{min,ij}$, com-puted by (40) for pumping operation networks or by (41) for gravity networks:

$$D_{max(min),ij} = E_{max(min)}^{\frac{1}{\alpha+r}} Q_p^{\frac{1}{\alpha+r}} Q_{ij}^{\frac{\beta}{\alpha+r}} \tag{40}$$

$$D_{max(min),ij} = \sqrt{\frac{4Q_{ij}}{\pi V_{min(max),ij}}} \tag{41}$$

in which:

$$E = \frac{10.33\, n'^2 r\, \psi}{a b \xi_1} \tag{42}$$

where: Q_{ij} is the discharge of the pipe ij; $Q_p = \Sigma Q_{p,j}$ – pumped discharge; V_{min}, V_{max} – limits of the economic velocities; n' – Manning roughness coefficient of the pipes (Table 1); E – economy-energy factor of the pipes [1], [20], which has a maximum value and a minimum value, corresponding to the limit values of the variation of economy-energy parameters (p_1, p_2, η, f, σ, e, τ, $\Sigma\Phi_k$) for the distribution system, included in ψ and ξ_1. Table I contains E_{min} and E_{max} values, computed for conditions specific to Romania.

A penalty coefficient p_{ij} is used when optimizing diameters in the case of extending a network, which has the value equal to the value of corresponding imposed diameter, for pipes with fixed diameters, resulting in $D_{k,ij} = p_{ij}$.

Table I. E_{min} and E_{max} values of the economy–energy factor of the pipes

No.	Pipe material	n'	CAN		CTA		WT	
			E_{min}	E_{max}	E_{min}	E_{max}	E_{min}	E_{max}
0	1	2	3	4	5	6	7	8
1	Reinforced concrete	0.0120	0.46	2.28	0.21	1.46	0.34	1.38
2	Cast iron	0.0120	0.24	1.11	0.14	0.78	0.20	0.80
3	Steel	0.0120	0.46	2.28	0.21	1.46	0.34	1.38
4	PVC	0.0111	0.28	1.20	0.13	0.90	0.17	0.51
5	PE–HD	0.0111	0.28	1.22	0.13	0.95	0.17	0.55

Admitting that a pipe ij of length L_{ij} of a pumping ope-ration network made up of T pipes, can be divided into s_{ij} sectors k of diameters $D_{k,ij}$ and lengths $X_{k,ij}$ and taking into account the notations:

$$c^*_{k,ij} = \xi_1(a + bD^{\alpha}_{k,ij}) \tag{43}$$

$$Z_{IPP,j} = (\sum h_{ij} + H_0)_j \tag{44}$$

the objective function (19) takes the form:

$$F_c = \sum_{ij=1}^{T}\sum_{k=1}^{s_{ij}} c^*_{k,ij} X_{k,ij} + \psi \sum_{j=1}^{NP} Q_{p,j} Z_{IPP,j} \to \min \tag{45}$$

The unknowns of the objective function are variables $X_{k,ij}$ and $Z_{IPP,j}$, being $NP + \sum\limits_{ij=1}^{T} s_{ij}$ in number.

When the pressure device is comprised of one or more reservoirs ($\psi = 0$), the expresion (45) of the objective function becomes:

$$F_c = \sum_{ij=1}^{T}\sum_{k=1}^{s_{ij}} c^*_{k,ij} X_{k,ij} \to \min, \tag{46}$$

minimizing the included energy or the network cost and having as unknowns the variables $X_{k,ij}$.

Hence, the values of the variables must be determined in order to minimize the objective function F_c, provided the following constraints are satisfied:

– *constructive constraints:*

$$\sum_{k=1}^{s_{ij}} X_{k,ij} = L_{ij} \quad (ij = 1,...,T) \tag{47}$$

–*functional constraints* which are written for each operating situation, and which must provide the necessary pressure HN_o at the critical nodes, starting on different path from the pressure devices IPP_j (Fig. 1):

$$Z_{IPP,j} - \sum_{ij=1}^{NT_j}\sum_{k=1}^{s_{ij}} \varepsilon_{ij}\Theta_{ij} J_{k,ij} X_{k,ij} \geq ZT_o + HN_o - \\ - \sum_{ij=1}^{NT_j}\left(\sum_{k=1}^{s_{ij}} \varepsilon_{ij}\Omega_{k,ij} J_{k,ij} + H_{p,ij}\right) \tag{48}$$

where: NT_j is the pipes number of a path IPP_j – O; ZT_o – elevation head at the critical node O; Z_{IPPj} – available piezometric head at the pressure device j; $H_{p,ij}$ – pumping head of the bosster pump mounted in the pipe ij, having the expression (8).

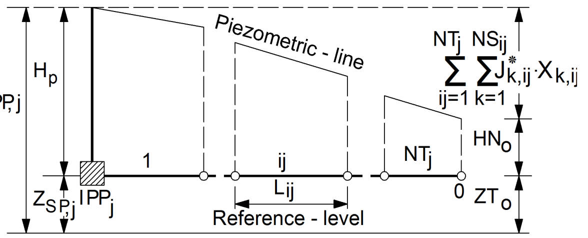


Fig. 1 Scheme of a path IPP_j – critical node O

– *hydraulic constraints* characteristic only for looped networks, expressing the energy conservation in loops:

$$\sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij}\Theta_{ij} J_{k,ij} X_{k,ij} = \sum_{\substack{ij\in m\\ m=1}}^{T} \varepsilon_{ij}\Omega_{k,ij} J_{k,ij} + f_m \quad (m = 1,...,M) \tag{49}$$

in which ε_{ij} is the orientation of the pipes and the pressure head f_m is given by (5), (6) and (7).

In the case that the available piezometric heads Z_{IPPj} are known, and it being unnecessary to determine them by optimization, the objective function (45) takes the form (46), while values Z_{IPPj} are contained in the free term of constraints (48) and (49).

As the objective function (45) or (46) and constraints (47), (48), (49) are linear with respect to the unknowns of system the optimal solution is determined according to the linear programming method, using the Simplex algorythm.

Computing the unknowns Z_{IPPj} by optimization, for pumping operation networks results in the corresponding pum-ping head:

$$H_{p,j} = Z_{IPP,j} - Z_{SP,j} \tag{50}$$

where Z_{SPj} is the water level in the suction basin of IPP_j.

Taking into account head loss H_{IPPj-n} on the path IPP_j – n:

$$H_{IPP,j-n} = \sum_{ij=1}^{NT_j}\sum_{k=1}^{s_{ij}} \Theta_{ij} J_{k,ij} X_{k,ij} - \\ - \sum_{ij=1}^{NT_j}\left(\sum_{k=1}^{s_{ij}} \Omega_{k,ij} J_{k,ij} + H_{p,ij}\right) \tag{51}$$

the piezometric head Z_n and the residual pressure head H_n at the node n are determined from the relations:

$$Z_n = Z_{IPP,j} - H_{IPP,j-n} \tag{52}$$

$$H_n = Z_n - ZT_n \tag{53}$$

where ZT_n is the elevation head at the node n.

For an optimal design, the piezometric line of a path of NT_j pipes, situated in the same pressure zone, must represent a polygonal line which resemble as closely as possible the optimal form expressed by the equation:

$$Z_n = Z_{IPP,j} - \left[1 - \left(1 - \frac{d}{\sum_{ij=1}^{NT_j} L_{ij}}\right)^{\frac{\beta\alpha}{\alpha+r}+1}\right] \sum_{ij=1}^{NT_j} h_{ij} \tag{54}$$

in which: Z_n is the piezometric head at the node n; d – distance between node n and the pressure device j.

The computer program OPLIRA has been elaborated based on the linear optimization model, in the FORTRAN programming language for IBM–PC compatible microsystems.

VI. Numerical Application

The looped distribution network with the topology from Fig. 2 is considered. It is made of cast iron and is supplied by pumping with a discharge of 0.23 m^3/s. The following data is known: pipes length L_{ij}, in m, elevation head ZT_j, in m, and necessary pressure HN_j = 24 m H_2O.

A comparative study of network dimensioning is performed using the classic model of average economical velocities (MVE), Moshnin optimization model (MOM) [1] and the linear optimization model (MOL) developed above, the last being applied in the hypothesis of concentrated outflow (MOL–N), as well as of uniform outflow along the lenght of the pipes (MOL–D).

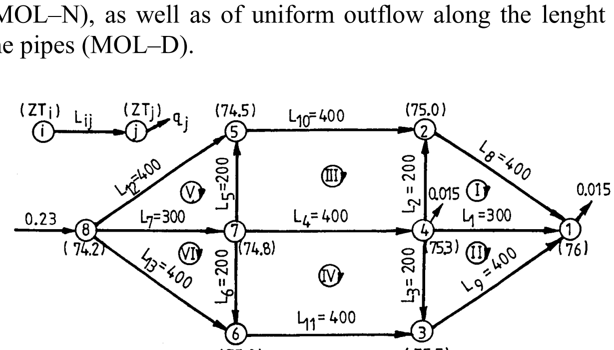


Fig. 2 Scheme of the designed distribution network

Calculus was performed considering a transitory turbulence regime of water flow and the optimization criterion used was that of minimum energetic consumption. Results of the numerical solution performed by means of an IBM–PC computer, referring to the hydraulic characteristics of the pipes are presented in Tables II and III.

The significance of (–) sign of discharges and head losses in Tables II and III is the change of flow sense in the res-pective pipes with respect to the initial sense considered in the Fig. 2.

In Fig. 3 there is a graphic representation, starting from the node source 8 to the control node 1, on the path 8–5–2–1, the piezometric lines being obtained using the three mentioned models of computation, evidencing their deviation from the optimal theoretical form. Fig. 3 also includes the corresponding values of the objective function F_c, the network included energy W_c, pumping energy W_e, as well as specific energy consumption for water distribution w_{sd}.

Table II. Hydraulic characteristics of the pipes determined with the models MVE and MOM

Pipe	L	MVE				MOM			
i - j	[m]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]
0	1	2	3	4	5	6	7	8	9
4-1	300	0.00782	100	4.009	1.00	0.00786	150	0.510	0.45
4-2	200	0.00512	100	1.177	0.65	-0.00174	100	-0.154	0.22
4-3	200	0.00512	100	1.177	0.65	-0.00174	100	-0.154	0.22
7-4	400	0.05924	300	0.963	0.84	0.04557	250	1.473	0.93
7-5	200	0.00517	100	1.199	0.66	0.00097	100	0.052	0.15
7-6	200	0.00517	100	1.199	0.66	0.00097	100	0.052	0.15
8-7	300	0.09576	350	0.833	1.00	0.07370	300	1.104	1.04
2-1	400	0.01669	150	2.886	0.94	0.01666	200	0.662	0.53
3-1	400	0.01669	150	2.886	0.94	0.01666	200	0.662	0.53
5-2	400	0.03538	250	0.902	0.72	0.04221	250	1.270	0.86
6-3	400	0.03538	250	0.902	0.72	0.04221	250	1.270	0.86
8-5	400	0.05403	250	2.051	1.10	0.06506	300	1.155	0.92
8-6	400	0.05403	250	2.051	1.10	0.06506	300	1.155	0.92

Table III. Hydraulic characteristics of the pipes determined with the models MOL–N and MOL–D

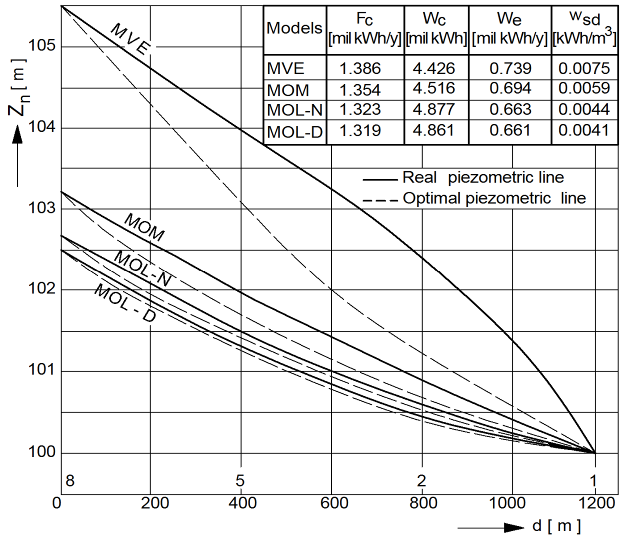
Pipe	MOL-N						MOL-D					
i - j	Q_{ij} [m^3/s]	k	$X_{k,ij}$ [m]	$D_{k,ij}$ [mm]	$h_{k,ij}$ [m]	$V_{k,ij}$ [m/s]	Q_{ij} [m^3/s]	k	$X_{k,ij}$ [m]	$D_{k,ij}$ [mm]	$h_{k,ij}$ [m]	$V_{k,ij}$ [m/s]
0	1	2	3	4	5	6	7	8	9	10	11	12
4-1	0.01462	1	107	250	0.045	0.30	0.01458	1	254	250	0.226	0.30
		2	193	200	0.249	0.47		2	46	200	0.070	0.46
4-2	-0.00484	1	200	150	-0.136	0.30	-0.00501	1	200	150	-0.176	0.28
4-3	-0.00484	1	200	150	-0.136	0.30	-0.00500	1	200	150	-0.176	0.28
7-4	0.04612	1	400	250	1.508	0.94	0.04575	1	400	250	1.442	0.93
7-5	0.00380	1	200	125	0.224	0.32	0.00383	1	200	150	0.108	0.25
7-6	0.00380	1	200	125	0.224	0.32	0,00383	1	200	150	0.108	0.25
8-7	0.08007	1	200	350	0.394	0.83	0.07962	1	82	350	0.075	0.82
		2	100	300	0.430	1.13		2	218	300	0.782	1.12
2-1	0.01329	1	400	200	0.429	0.42	0.01331	1	400	200	0.470	0.42
3-1	0.01329	1	400	200	0.429	0.42	0.01331	1	400	200	0.470	0.42
5-2	0.04194	1	56	300	0.069	0.59	0.04212	1	400	250	1.159	0.86
		2	344	250	1.078	0.85						
6-3	0.04194	1	56	300	0.069	0.59	0.04212	1	400	250	1.159	0.86
		2	344	250	1.078	0.85						
8-5	0.06187	1	400	300	1.048	0.88	0.06210	1	400	300	0.964	0.88
8-6	0.06187	1	400	300	1.048	0.88	0.06210	1	400	300	0.964	0.88

According to the performed study it was established that:

– all the pipes of the network are operating in a transitory turbulence regime of water flow;

– there is a general increase of pipes diameters obtained by optimization models (MOM, MOL) with respect to MVE, because the classical model does not take into account the minimum consumption of energy and the diversity of economical parameters;

– in comparison with the results obtained by MVE, the ones obtained by optimization models are more economical, a substantial reduction of specific energy consumption for water distribution is achieved (MOM – 21.3 %, MOL-N – 41.3 %, MOL-D – 45.3 %) as well as a reduction of pumping energy (MOM – 6,4%, MOL-N – 10.3%, MOL-D – 10.6%), at the same time the objective function has also smaller values (MOM – 2.3 %, MOL-N – 4.5 %, MOL-D – 4.8 %);



Models	Fc [mil kWh/y]	Wc [mil kWh]	We [mil kWh/y]	wsd [kWh/m^3]
MVE	1.386	4.426	0.739	0.0075
MOM	1.354	4.516	0.694	0.0059
MOL-N	1.323	4.877	0.663	0.0044
MOL-D	1.319	4.861	0.661	0.0041

Fig. 3 Piezometric lines along the path 8–5–2–1

– the optimal results obtained using MOL are superior energetically to those offered by MOM, leading to pumping energy savings of 5 %;

– also, the application of MOL for uniform outflow along the lenght of the pipes, has led to the minimum deviation from the optimal form of the piezometric line, especially to a more uniform distribution of the pumping energy, by elimi-nation of a high level of available pressure at some nodes even at maximum consumption. The smallest value of the specific energetic consumption, namely that of 0.0041 kWh/m^3, also supports this assertion;

– reduction of the pressure in the distribution network achieved in this way, is of major practical import, contri-buting to the diminishing of water losses from the system.

VII. Conclusions

The mathematical programming, as a fundamental proce-dure for optimizing the structures in general, together with graph theory and the increasing implication of computers in solving mathematical formulations have created conditions for solving efficiently some optimization problems of design of water distribution networks. The different types of pro-gramming which exist (linear, nonlinear, whole, geometric etc.) provide multiple possibilities for solving specific pro-blems.

The proposed optimization model, a very general and practical one, offers the possibility of optimal design of water supply networks using multiple optimization criteria and considers the transitory or quadratic turbulence flow. It has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, and other criteria can be expressed by simple options in the objective function (19).

The model of linear optimization could be applied to any looped or tree–shaped network, either when piezometric heads at pressure devices (pump stations or tanks) must be determined or when these heads are given. It permits the determination of an optimal distribution of comercial diameters along the length of each pipe of the network and the length of pipe sectors corresponding to these diameters. Also, this facilitates the consideration of uniform outflow along the length of the pipes network. A more uniform distribution of pumping energy is achieved so that head losses and para-meters of pump stations can be determined more precisely.

For different analized networks, the saving of electrical energy due to diminishing pressure losses and operation costs when applying the model of linear optimization represents about 10...35 %, which is of great importance, considering the general energy issues.

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