A New Method to Optimize Reactive Power with Hybrid Network Equations

Yi Tao, Wang Cheng-min, Sun Wei-qing, Gao Yang

***Abstract*—**Electric power network components can be simulated with π equivalent circuit. Establish the hybrid node voltage and branch current electricity network analysis model based on it, and then form the hybrid electric power network equations to research the reactive power optimization of power system. Firstly, define the node voltages and branch currents as the state variables in this model. The network flow is explicitly expressed along with the branch currents introduced, and they play a key role for the simplification of the calculation. Then, in the process of reactive optimization the optimal objective has been broken down into two sub-objectives: one is a network loss minimization objective augmented Lagrange function, forming the Kuhn-Tucker conditions, and the other is a linear equation. The calculated results of IEEE 30-bus system show that the complexity and high dimension of the model have been significantly reduced, the solving process becomes easier, and the solution is close to the global optimal solution. Compared with traditional optimal power flow algorithm, this algorithm can improve the computational efficiency of reactive power optimization.

***Keywords*—**Hybrid equation, Kuhn-Tucker condition, Power network flow, Reactive power optimization

I. INTRODUCTION

The status of electric power network can be enough reflected by physical quantities with nodal voltage, nodal injected active and reactive power, branch current. In addition, the branch active power is considered as variable only in simplified analysis with DC(direct current) power flow model of electric power network.

The electric power network is traditionally modeled by node-voltage-based equations when the nodal voltage, nodal injected active or reactive power are used as variables. Except the node-voltage-based model, the loop-current-based model is developed by Goswami[1~4] etc for load flow calculation of distribution network with better convergence and meshed modeling while the grounding admittance is ignored and the constant load impedance model is used, but it is improper and limited in the transmission network with such assumptions.

The method of reactive power optimization has been to conduct some research [5~8]. At present, the reactive power optimization problem is generally described by an optimal power flow mode

$$\min \quad f(x,u)$$
$$s.t.\begin{cases} g(x,u)=0 \\ h(x,u)\le 0 \end{cases} \tag{1}$$

Where, the x denotes state variable with nodal voltage and the u denotes control variable with nodal injected reactive power; the functions f, g, h respectively denote network losses, equality constraint and inequality constraint.

It has been proposed to solve problem (1) for various approaches including linear[9], quadric[10], non-linear programming algorithms[11~17], especially the interior point nonlinear programming algorithm[10, 12~17] and evolutionary algorithms[18~21] become the hot research topics in recent years. The all above mentioned algorithms are based on the node-voltage-based equations. Although the node analysis methods are very effective, but some questions and shortcomings have also emerged: 1) The flow which is a most remarkable character in electric power network is not directly exhibited, and many successful theories and arithmetic in network flow can be not used for electric power network analysis; 2) It is discommodious for the network losses that are generally represented by the sum of injective active powers at slack nodes or all nodes; 3) It is necessary to improve the computational efficiency of present reactive power optimization approaches due to the large numbers of inequality constraints.

Therefore, the branch current introduced as network flow should overcome above difficulties. In this paper, the enlarged electric power network equations are established by regarding the grounding branch as a current source with node voltage and branch current variables, so the objective function can be wrote as the product of line current and resistance, and the reactive power optimization problem can be decomposed into two sub-problems with a minimum cost flow model and a linear equations. It is solved to the minimum cost flow model by a quadric programming approach, therefore the computational efficiency is improved and the found optimal solution is closed to global. The case study is made at the IEEE-30 system and the better results are obtained.

The assumptions are introduced in this paper as following: 1) The reactive power optimization is aimed to the injective

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reactive power at all nodes; 2) The transformer's tap ratio optimization is ignored because it is not obvious to reduce the network losses; 3) The active power is considered as constant and the reactive power and node voltage at all nodes are regarded as variables except slack nodes; 4) The constant power load model is adopted; 5) The network conditions are considered as balanced three phase.

II. Electric Power Network Modeling

The electric network is generally simulated as an π equivalent circuit with an impedance branch and two grounding branches showed in figure 1, so the electric power network equations are composed with impedance branch equations and grounding branch equations.

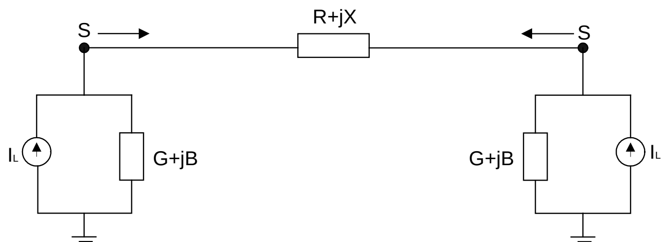

Fig. 1 The π equivalent circuit

A. Impedance branch equations

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Define the $U = \lfloor \cos\theta \rfloor V + j\lfloor \sin\theta \rfloor V$ as node voltage vector and the $Z = R + jX$ as branch impedance matrix, where the symbol $\lfloor \;\; \rfloor$ denotes diagonal matrix. The impedance branch equations can be described as:

$$ZI = A^T U \tag{2}$$

Where, the A denotes branch-node incidence matrix and the $I = I^a + jI^r$ is branch current vector. To define $Y = Z^{-1}$ and to multiply the matrix A along two sides of the formula (2), which is changed as:

$$AI = AYA^T U \tag{3}$$

Where, $Y = G + jB$ is branch admittance matrix; Define $Y^s = AYA^T = G^s + jB^s$, which is different from nodal admittance matrix, because the grounding branch admittance is not included in Y^s. It is obtained to extend the formula (3):

$$\begin{aligned} AI^a &= G^s \lfloor \cos\theta \rfloor V - B^s \lfloor \sin\theta \rfloor V \\ AI^r &= G^s \lfloor \sin\theta \rfloor V + B^s \lfloor \cos\theta \rfloor V \end{aligned} \tag{4}$$

i.e.

$$\begin{aligned} AI^a &= (G^s \lfloor \cos\theta \rfloor - B^s \lfloor \sin\theta \rfloor) V \\ AI^r &= (G^s \lfloor \sin\theta \rfloor + B^s \lfloor \cos\theta \rfloor) V \end{aligned} \tag{5}$$

B. Grounding branch equations

The equivalent current source can be used to simulate the grounding branch as in figure 2. The I_L denotes the nodal injective load or generation current and the I_G denotes the equivalent current source of the grounding branch. To define the nodal injective power vector as $S = P + jQ$ and the grounding branch admittance matrix as $Y^g = G^g + jB^g$, so $I_G = Y^g U$ and $I_L = \left\lfloor \overset{*}{U} \right\rfloor^{-1} S$, where $\left\lfloor \overset{*}{U} \right\rfloor = \lfloor V \rfloor \lfloor \cos\theta \rfloor - j \lfloor V \rfloor \lfloor \sin\theta \rfloor$ is the conjugate of node voltage vector U. The nodal injective current is as following:

$$I_n = I_L + I_G = Y^g U + \left\lfloor \overset{*}{U} \right\rfloor^{-1} S \tag{6}$$

because of:

$$\begin{aligned} \left\lfloor \overset{*}{U} \right\rfloor^{-1} &= (\lfloor V \rfloor \lfloor \cos\theta \rfloor + j \lfloor V \rfloor \lfloor \sin\theta \rfloor)(\lfloor V \rfloor \lfloor V \rfloor)^{-1} \\ &= \lfloor \cos\theta \rfloor \lfloor V \rfloor^{-1} + j \lfloor \sin\theta \rfloor \lfloor V \rfloor^{-1} \end{aligned} \tag{7}$$

It is obtained to extend the formula (6) and ignore the grounding branch conductance:

$$\begin{aligned} I_n^a + jI_n^r = {} & jB^g (\lfloor \cos\theta \rfloor V + j \lfloor \sin\theta \rfloor V) + \\ & (\lfloor \cos\theta \rfloor \lfloor V \rfloor^{-1} + j \lfloor \sin\theta \rfloor \lfloor V \rfloor^{-1})(P + jQ) \end{aligned} \tag{8}$$

i.e.

$$\begin{aligned} I_n^a &= \lfloor \cos\theta \rfloor \lfloor V \rfloor^{-1} P - \lfloor \sin\theta \rfloor \lfloor V \rfloor^{-1} Q - B^g \lfloor \sin\theta \rfloor V \\ I_n^r &= \lfloor \cos\theta \rfloor \lfloor V \rfloor^{-1} Q + \lfloor \sin\theta \rfloor \lfloor V \rfloor^{-1} P + B^g \lfloor \cos\theta \rfloor V \end{aligned} \tag{9}$$

Where, the I_n^a, I_n^r respectively are real and imaginary parts of the nodal injected current, then:

$$P = \lfloor V \rfloor \lfloor \cos\theta \rfloor^{-1} (I_n^a + B^g \lfloor \sin\theta \rfloor V + \lfloor \sin\theta \rfloor \lfloor V \rfloor^{-1} Q) \tag{10}$$

It is obtained to introduce the above formula into (9):

$$\begin{aligned} Q &= \lfloor \cos\theta \rfloor \lfloor V \rfloor I_n^r - \lfloor \sin\theta \rfloor \lfloor V \rfloor I_n^a - \lfloor V \rfloor B^g V \\ &= \lfloor \cos\theta \rfloor \lfloor V \rfloor AI^r - \lfloor \sin\theta \rfloor \lfloor V \rfloor AI^a - \lfloor V \rfloor B^g V \end{aligned} \tag{11}$$

$$\begin{aligned} P = {} & \lfloor V \rfloor \lfloor \cos\theta \rfloor I_n^a + \lfloor \sin\theta \rfloor \lfloor V \rfloor I_n^r = \\ & \lfloor V \rfloor \lfloor \cos\theta \rfloor AI^a + \lfloor \sin\theta \rfloor \lfloor V \rfloor AI^r \end{aligned} \tag{12}$$

So the hybrid form of electric power network equations composing with formula (5), (11) and (12) is obtained by regarding the node voltage and branch current as state variables. There are characteristics that network equations are linear functions of branch current and non-linear functions of node voltage. The traditional power flow equations just with node voltage variables can be performed by introducing the formula (5) into (11) and (12).

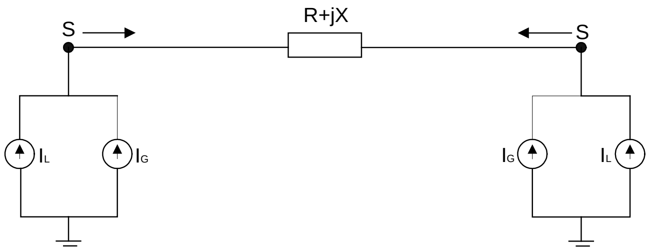

Fig. 2 The π equivalent circuit II

III. Reactive power optimization problem

A. Mathematical model

The network losses minimization and the objective function of reactive power optimization problem can be written as:

$$z = \min \sum_{l=1}^{M} \left[(i_l^a)^2 + (i_l^r)^2 \right] R_l = \min \, (I^a)^T R I^a + (I^r)^T R I^r \tag{13}$$

Subject to equality constraints (5), (11), (12) and inequality constraints as following:

$$V^{\min} \le V \le V^{\max} \tag{14}$$

$$Q^{\min} \le Q \le Q^{\max} \tag{15}$$

Where, $l = 1, 2, \cdots, M$ is the line set. The node voltage can be represented as the function of branch current from formula (5):

$$\begin{aligned} &(G\lfloor \cos\theta \rfloor - B_s \lfloor \sin\theta \rfloor)^{-1} A I^a = V \\ &(G\lfloor \sin\theta \rfloor + B_s \lfloor \cos\theta \rfloor)^{-1} A I^r = V \end{aligned} \tag{16}$$

so the formula (14) is changed as:

$$\begin{aligned} &V^{\min} \le (G\lfloor \cos\theta \rfloor - B_s \lfloor \sin\theta \rfloor)^{-1} A I^a \le V^{\max} \\ &V^{\min} \le (G\lfloor \sin\theta \rfloor + B_s \lfloor \cos\theta \rfloor)^{-1} A I^r \le V^{\max} \end{aligned} \tag{17}$$

The formula (15) is also changed as:

$$\begin{aligned} &Q^{\min} \le \lfloor \cos\theta \rfloor \lfloor V \rfloor A I^r - \lfloor \sin\theta \rfloor \lfloor V \rfloor A I^a \\ &- \lfloor V \rfloor B^g V \le Q^{\max} \end{aligned} \tag{18}$$

The mathematical model of reactive power optimization problem is composed with the formulas (13), (5), (11), (12), (17) and (18) and following characteristics: 1) The objective function is quadric function of branch current variables; 2) The constraints are linear function of branch current variables.

B. The variable classification

The nodal injective active power is constant at all nodes for the reactive power optimization problem. Therefore, there are three-kinds of variables with branch current, node voltage and nodal injective reactive power. The branch current and node voltage representing the network states are considered as state variables and the nodal injective reactive power is considered as control variable in the above model. The augmented Lagrangian function of above reactive power optimization model with formulas (13), (5), (11), (12), (17) and (18) is performed as following:

$$\begin{aligned} &L(U, I, Q) = (I^a)^T R I^a + (I^r)^T R I^r \\ &+ \alpha^a (A I^a - G^s \lfloor V \rfloor \cos\theta + B^s \lfloor V \rfloor \sin\theta) \\ &+ \alpha^r (A I^r - G^s \lfloor V \rfloor \sin\theta - B^s \lfloor V \rfloor \cos\theta) \\ &+ \beta^p (P - \lfloor V \rfloor \lfloor \cos\theta \rfloor A I^a - \lfloor \sin\theta \rfloor \lfloor V \rfloor A I^r) \\ &+ \beta^q (Q - \lfloor \cos\theta \rfloor \lfloor V \rfloor A I^r + \lfloor \sin\theta \rfloor \lfloor V \rfloor A I^a \\ &+ \lfloor V \rfloor B^g V) \\ &+ \gamma^{u1} \left[(G\lfloor \cos\theta \rfloor - B_s \lfloor \sin\theta \rfloor)^{-1} A I^a - V^{\max} \right] \\ &+ \gamma^{l1} \left[V^{\min} - (G\lfloor \cos\theta \rfloor - B_s \lfloor \sin\theta \rfloor)^{-1} A I^a \right] \\ &+ \gamma^{u2} \left[V^{\min} - (G\lfloor \sin\theta \rfloor + B_s \lfloor \cos\theta \rfloor)^{-1} A I^r \right] \\ &+ \gamma^{l2} \left[(G\lfloor \sin\theta \rfloor + B_s \lfloor \cos\theta \rfloor)^{-1} A I^r - V^{\max} \right] \\ &+ \lambda^u (Q^{\min} - \lfloor \cos\theta \rfloor \lfloor V \rfloor A I^r \\ &+ \lfloor \sin\theta \rfloor \lfloor V \rfloor A I^a + \lfloor V \rfloor B^g V) \\ &+ \lambda^l (\lfloor \cos\theta \rfloor \lfloor V \rfloor A I^r - \lfloor \sin\theta \rfloor \lfloor V \rfloor A I^a \\ &- \lfloor V \rfloor B^g V - Q^{\max}) \end{aligned} \tag{19}$$

The derivative of control variable Q is:

$$\partial L / \partial Q = \beta^q = 0 \tag{20}$$

It is indicated that formula (11) is unnecessary because the control variable Q is only appeared in formula (11), and the optimal solution is not affected by Q. The simplified reactive power optimization model is composed with formulas (13), (5), (12), (17) and (18), and the optimization is only implemented aiming to state variables $[I, U]^T$. The final optimal nodal injective reactive power Q can be obtained from formula (11) while the state variables are determined in order to minimize the network losses.

IV. Calculations

It is can be obtained to define $E = \lfloor \cos\theta \rfloor V, F = \lfloor \sin\theta \rfloor V$ as the real and imaginary parts of node voltage:

$$\begin{bmatrix} E \\ F \end{bmatrix} = \begin{bmatrix} G^s & -B_s \\ B_s & G^s \end{bmatrix}^{-1} \begin{bmatrix} A I^a \\ A I^r \end{bmatrix} \tag{21}$$

The reactive power optimization model called as problem A is composed with formulas (13), (21), (12), (17) and (18) by replacing the formula (5) with (21).

The sub-problem called as problem S is defined with formulas (13), (12), (17) and (18). It is divided into two steps to solve the problem A, which are to solve the problem S and linear equations (21) in turn. The iteration steps are as following:

1) To define k =0 and set the initial value $U^{(k)}$ of node voltage;

2) The branch current $I^{(k)}$ is obtained by solving quadric sub-problem S regarding the branch current as state variable with $U^{(k)}$;

3) The iterations are ended if $\left\| I^{(k+1)} - I^{(k)} \right\| \le \varepsilon$ (ε is a small positive number) and it is returned to step 5), otherwise to continue;

4) It is to solve node voltage by formula (21), define $k = k+1$ and return to step 2);

5) The nodal injective reactive power is obtained by formula (11).

V. Computational efficiency

The computational costs are obviously reduced while the problem A is divided into two sub-problems. The problem A can be described as following normal mathematical programming form:

$$\text{Problem A:} \quad \begin{cases} \min \quad f(x,u) \\ h(x,u) \le 0 \\ g(x,u) = 0 \end{cases} \tag{22}$$

Where, the u is the branch current vector with its real and imaginary parts; x is the node voltage vector with its magnitude and single. Because there is no node voltage vector in the objective function, so it can be changed as following:

$$\min \quad f(u) \tag{23}$$

The formula $h(x,u) \le 0$ denotes the equality and inequality constraints as formulas (12), (17) and (18). The formula $g(x,u) = 0$ denotes equality constraint as formula (21). The Kuhn-Tucker optimal conditions of problem A can be wrote as following:

$$f_x + h_x^T \alpha + g_x^T \beta = 0 \tag{24-1}$$

$$f_u + h_u^T \alpha + g_u^T \beta = 0 \tag{24-2}$$

$$h(x,u) \le 0 \tag{24-3}$$

$$\alpha \ge 0 \tag{24-4}$$

$$h(x,u)\alpha^T = 0 \tag{24-5}$$

$$g(x,u) = 0 \tag{24-6}$$

Where, $f_x = 0$; α, β are Lagrangian multipliers corresponding the constraints $h(x,u) \le 0$ and $g(x,u) = 0$. It is obtained by formula (24-1):

$$\beta = -(g_x^T)^{-1} h_x^T \alpha \tag{25}$$

To introduce formula (25) into (24-2), it is changed as:

$$f_u + h_u^T \alpha - g_u^T (g_x^T)^{-1} h_x^T \alpha = 0 \tag{26}$$

The Kuhn-Tucker optimal conditions of problem A can be rewrote as:

$$f_u + h_u^T \alpha - g_u^T (g_x^T)^{-1} h_x^T \alpha = 0 \tag{27-1}$$

$$h(x,u) \le 0 \tag{27-2}$$

$$\alpha \ge 0 \tag{27-3}$$

$$h(x,u)\alpha^T = 0 \tag{27-4}$$

$$g(x,u) = 0 \tag{27-5}$$

In the same way, the normal mathematical programming form of sub-problem S can be described as following:

$$\text{Sub-problem S:} \quad \begin{cases} \min \quad f(x,u) \\ h(x,u) \le 0 \end{cases} \tag{28}$$

If the u is considered as variable, the Kuhn-Tucker optimal conditions are:

$$f_u + h_u^T \alpha = 0 \tag{29-1}$$

$$h(x,u) \le 0 \tag{29-2}$$

$$\alpha \ge 0 \tag{29-3}$$

$$h(x,u)\alpha^T = 0 \tag{29-4}$$

It can be seen that differences between formulas (27) and (29) with $g(x,u) = 0$ are exhibited in following formula:

$$\Delta u' = -g_u^T (g_x^T)^{-1} h_x^T \alpha \tag{30}$$

The simultaneous solution of the sub-problem S and linear equations (21) is different from the solution of problem A with $\Delta u'$. In order to obtain the accurate solution of reactive power optimization, it must be corrected for variable u according to formula (30) after the sub-problem S is resolved.

The solution of sub-problem S is global optimal due to it is a convex quadric programming problem with line resistance $R_l \ge 0$, and there is unique solution for the formula (21) because it is linear. Therefore, it is possible to cause multi-solutions of reactive power optimization problem A just in transforming the real and imaginary parts of node voltage as magnitude and single. In practice, the magnitude and single of node voltage is taken to approach the normal operation conditions of power system, so the obtained optimal solution is enough closed to global.

VI. Case study

The case study is made at IEEE-30 system with active set arithmetic solved the sub-problem S. The upper limit of voltage at node 10 is set as 1.0421 and the upper limit of voltage at node 24 is set as 1.0261 while upper limit of voltage is set as 1.1 and lower limit of voltage is set as 0.95 at all nodes.

The final calculating results are listed in table 1, and the reactive powers at node 10 and 24 are 0.144558 and 0.0914809 calculated by the nodal injective current.

Tab. 1 Node Data of IEEE-30 System (After Calculation)

Node Number	Voltage Magnitude	Voltage Single	Real Part of Nodal Injective Current	Imaginary Part of Nodal Injective Current
1	1.001	-12.867	0.0994	-0.0422
2	1.034	-2.773	-0.3471	0.0024
3	1.031	-4.746	0.0243	0.0118
4	1.026	-5.686	0.0778	-0.0470
5	1.006	-9.054	0.6936	-0.0466

6	1.022	-6.561	0.0075	-0.1056
7	1.007	-8.112	0.2114	-0.1204
8	1.023	-6.565	-0.0430	0.0520
9	1.054	-8.392	-0.0030	0.0770
10	1.039	-10.181	0.0914	0.1781
11	1.091	-6.854	-0.1456	0.1647
12	1.053	-9.537	0.1044	0.0277
13	1.089	-8.830	-0.1198	0.2407
14	1.038	-10.440	0.0560	-0.0260
15	1.034	-10.550	0.0736	-0.0382
16	1.040	-10.084	0.0301	-0.0229
17	1.034	-10.361	0.0754	-0.0708
18	1.024	-11.121	0.0290	-0.0146
19	1.021	-11.267	0.0848	-0.0508
20	1.024	-11.053	0.0197	-0.0108
21	1.028	-10.674	0.1471	-0.1386
22	1.030	-10.675	-4.9789×10^{-5}	8.0926×10^{-5}
23	1.027	-11.021	0.0277	-0.0213
24	1.026	-11.299	0.0787	0.0085
25	1.024	-11.141	-6.6480×10^{-6}	1.9351×10^{-5}
26	1.006	-11.557	0.0296	-0.0294
27	1.032	-10.783	0.0090	0.1032
28	1.018	-6.980	0.0011	-0.0788
29	1.012	-11.996	0.0214	-0.0136
30	1.050	0	-1.3347	0.0185

The thirteen iterations are needed with step 3.24 listed in tables 2. The upper limit of voltage at node 10 is violated in iteration 4, then the search direction is changed and the violation is eliminated with Lagrangian multiplier 0.0178555. Up to iteration 12, the upper limit of voltage at 24 is violated and it is eliminated with Lagrangian multiplier 0.00792428. The final network losses are reduced from 0.0879016 to 0.0863921.

Tab. 2 Calculating Process Information

Iterations	Network losses	Lagrangian Multipliers	Voltage Magnitude at Node 10	Voltage Magnitude at Node 24
1	0.0879016	0	1.04077	1.02017
2	0.0878057	0	1.04146	1.02195
3	0.0877322	0	1.04190	1.02334
4	0.0876737	0.0178555	1.04214	1.02441
5	0.0876258	0	1.03724	1.01923
6	0.0876102	0	1.03812	1.02109
7	0.0875532	0	1.03873	1.02254
8	0.0875103	0	1.03913	1.02367
9	0.0874768	0	1.03936	1.02454
10	0.0874499	0	1.03947	1.02521
11	0.0874275	0	1.03949	1.02573
12	0.0874085	0.00792428	1.03943	1.02612
13	0.0873921	0	1.03662	1.02269

The comparing results of proposed approach with Newton method and reduced gradient algorithm are listed in table 3 while the inequality constraints are ignored. It can be seen that optimization effect of proposed approach is better than Newton method and reduced gradient algorithm.

Tab. 3 Comparing results with others

Arithmetic	Proposed in This Paper	Newton Method	Gradient Method
Network Losses	0.0852368	0.0854694	0.0854816
Injective Reactive Power at Node 10	0.1746810	0.2937780	0.2872100
Injective Reactive Power at Node 24	0.0157019	0.1408180	0.1534660

VII. Discussion and Outlook

The hybrid electric power network equations composed of node voltage and loop current can also provide useful ideas to solve practical problems of power system in several other areas in addition to the excellent performance in the reactive power optimization.

A. *Explicit expression of node voltage high and low solution*

The equation (31) can be deduced by the equation (11) and (12), node i, for example:

$$\begin{aligned} & e_i \sum_{l\in i} i_{li}^a + f_i \sum_{l\in i} i_{li}^r = p_i \\ & e_i \sum_{l\in i} i_{li}^r - f_i \sum_{l\in i} i_{li}^a - (e_i^2 + f_i^2) B^g = -q_i \end{aligned} \tag{31}$$

where : $p_i - jq_i$ is the load of node i, $\sum_{l\in i} i_{li} = \sum_{l\in i} i^a{}_{li} + j\sum_{l\in i} i^r{}_{li}$ is the sum of the injection current of node i, $\dot{u}_i = e_i + jf_i$ is the voltages of node i.

Suppose: $x_i = \sum_{l\in i} i_{li}^a$, $y_i = \sum_{l\in i} i_{li}^r$. To the PQ node, by the formula (31), the node voltage can be derived as:

$$\begin{cases} e_i = (p_i - f_i y_i)/x_i \\ f_i = \dfrac{\left\{ \begin{aligned} & \left[2B^g p_i y_i - x_i (x_i^2 + y_i^2) \right] \pm \\ & x_i \sqrt{(x_i^2 + y_i^2)^2 + 4B^g q_i (x_i^2 + y_i^2) - 4B^{g2} p_i^2} \end{aligned} \right\}}{2B^g (x_i^2 + y_i^2)} \end{cases} \tag{32}$$

The equation (32) is the node voltage analytical expression represented by the branch current. It represents the high and low solutions of node voltage.

B. *Voltage instability region (unstable round)*

If the equation (32) has solutions the following condition must be met:

$$\begin{aligned} & (x_i^2 + y_i^2)^2 + 4B^g q_i (x_i^2 + y_i^2) - 4B^{g2} p_i^2 \geq 0 \\ & \Rightarrow \left[(x_i^2 + y_i^2) + 2B^g q_i \right]^2 \geq (2B^g \sqrt{p_i^2 + q_i^2})^2 \\ & \Rightarrow x_i^2 + y_i^2 \geq -2B^g q_i + 2B^g \sqrt{p_i^2 + q_i^2} \end{aligned} \tag{33}$$

The meaning of equation (33) is: 1) When the square of the amplitude of the injection current of nodes is out of the circle of which the center is $2B^g q_i$ and the radius is $2B^g \sqrt{q_i^2 + p_i^2}$, that is only '>' condition has been met in the equation (33), the high and low solutions of equation (32) exist, and the system is stable. 2) When the square of the amplitude of the injection current of nodes is in the circle, that is only '<' condition has been met in the equation (33), no solutions of equation (32) exist, so the system is unstable. 3) Equation (32) has a unique solution and the solution is on the circle when '=' condition has been met in the equation (33), so the unique solution is the stable margin of system. System voltage collapse point can be found if calculate the power flow equations under this conditions.

Node types also include PU node and balance node in power system analysis, the voltage of balance node is considered to be known, and so do not need to be calculated. The voltage expression of PU node is similar with PQ node, and the difference is that the voltage equation of PU node is instead of reactive power equation of PQ node.

$$\begin{cases} e_i = (p_i - f_i y_i)/x_i \\ f_i = \dfrac{p_i y_i \pm \sqrt{p_i^2 y_i^2 - (x_i^2 + y_i^2)(p_i^2 - V_i^2 x_i^2)}}{x_i^2 + y_i^2} \end{cases} \quad (34)$$

also：

$$\begin{aligned} & p_i^2 y_i^2 - (x_i^2 + y_i^2)(p_i^2 - U_i^2 x_i^2) \geq 0 \\ & \Rightarrow U_i^2 x_i^2 - p_i^2 + U_i^2 y_i^2 \geq 0 \\ & \Rightarrow x_i^2 + y_i^2 \geq \frac{p_i^2}{U_i^2} \end{aligned} \quad (35)$$

The equation (34) has solutions if the equation (35) has been met, and there is a voltage instability circle, too. The circle's center is origin and the radius is p_i / U_i, the unstable region is in the circle, stability margin is on the circle.

C. Voltage stability critical condition

When the Equality of equation (33) and (35) meet the two solution curves intersect, and reach the voltage stability critical point. So, the voltage stability critical condition is:

$$(x_i^2 + y_i^2) = 2B_{i0}\gamma_i \quad (36)$$

or：

$$(x_i^2 + y_i^2) = \frac{p_i^2}{U_i^2} \quad (37)$$

Where: $\gamma_i = q_i + \sqrt{p_i^2 + q_i^2}$. The corresponding node voltage changes as follows:

$$\begin{cases} e_i = \dfrac{p_i x_i - y_i \gamma_i}{2B_{i0}\gamma_i} \\ f_i = \dfrac{p_i y_i + x_i \gamma_i}{2B_{i0}\gamma_i} \end{cases} \quad (38)$$

or：

$$\begin{cases} e_i = U_i x_i \\ f_i = U_i y_i \end{cases} \quad (39)$$

The formula (36) and (37) are called the characteristics of the voltage stability critical point. When formula (36) or (37) was met in any node the voltage instability will occur.

When the loop current and node voltage are as variables the expressive information is more abundant, and can solve the problem difficult to resolve in the past. So the hybrid electric network equations can be applied to research in different fields of power system.

VIII. CONCLUSIONS

It can be seen that electric power network flow is similar to the traffic network flow while the branch current is taken into the network equations, and the computational complexity is just increased because the limitations of power system operation to the node voltage and nodal injective powers are required.

It is easy to solve for the sub-problems divided by the reactive power optimization problem, so the computational efficiency is improved. The following conclusions are obtained: 1) The electric power network equations can be described as node-voltage-based and branch-current-based hybrid form; 2) The obtained solution is closed to global optimal while the error of node voltage is small.

It is indicated that mathematical model based on node-voltage-based and branch-current-based equations can be also applied in other problems of electric power network optimization in order to improve calculation efficiency.

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