

Comparative characteristics, analysis, mathematical model and simulation of selected energy-electronic converters of modern aircraft in line with the concept of a more electric aircraft MEA/ AEA

L. Setlak, R. Kowalik

Abstract— The work deals with the issues of modern architecture power supply compatible with the trend of more electric aircraft in the field of energy-electronic power electronics systems PES (Power Electronics Systems), both civil aircraft "classical" companies Airbus and Boeing aircraft (A-320, B-767) and military Lockheed Martin (F-16) as well as a more civil aircraft/ fully electric MEA/ AEA (A-380 and A-350XWB, B-787) and military JSF (Joint Strike Fighter) F-35 and F-22 Raptor. The subject, and also the aim of this paper is to make a comparative characteristics of selected elements of power electronic converters, which are multi-pulse rectifiers (6- and 12-pulse) or (18- and 24- pulse) used on modern aircraft with 48- pulse rectifier, presenting them analysis and mathematical models, and based on the above, the analysis and draw practical conclusions in the field of advanced power electronic power systems PES according to the concept aircraft more/ fully electric (MEA/ AEA). The idea behind this concept is to replace conventional key air devices, circuits and systems, electrical power, pneumatic, hydraulic and mechanical - devices, circuits and systems fed one kind of energy, namely electricity.

Keywords— mathematical model, power electronic converters, multi-pulse rectifiers, power electronic supply systems (PES), the plane more/ fully electric (MEA/ AEA)

I. INTRODUCTION

NOW, taking into account both civil aviation (Airbus, Boeing), the aircrafts 'classic' (A-320, B-767), with particular emphasis on aircraft compatible with the modern concept of the power system more/ fully electric MEA/ AEA (A-380 and A-350XWB, B-787), as well as for military aircraft (Lockheed Martin), and in aircrafts 'conventional' (F-16) and in line with the modern trend MEA/ AEA (JSF F-35, F-22 Raptor), made many significant changes. Among other things, you can see the dynamic development of on-board power architectures in the field of power electronics systems PES, which are key components of the multi-pulse rectifiers (6- , 12- and 18- and 24- pulse) [1]. At the beginning, in a short introduction to selected topics in the field of power electronics, connected with the modern power systems PES, their key components, i.e. transmitter

electronics and their essential components, which are rectifiers multi- pulse, it should be noted that these systems are among some of the most important systems in addition to electrical power systems EPS (Electric power systems). In addition, they are an essential component of the so-called. on-board autonomous power systems ASE (Autonomous Electric Power Systems), known in the literature, among others, as integrated power systems IPS (Integrated Power Systems) [2]. Based on the above, a comparative analysis of selected types of the multi- pulse rectifiers, analysis and mathematical models have been carried out, among others, according to certain criteria, namely: types of systems production/ processing of electricity, their evolution and key sources in the field produced by them under the advanced systems used on modern aircraft, both civilian and military (Fig. 1).

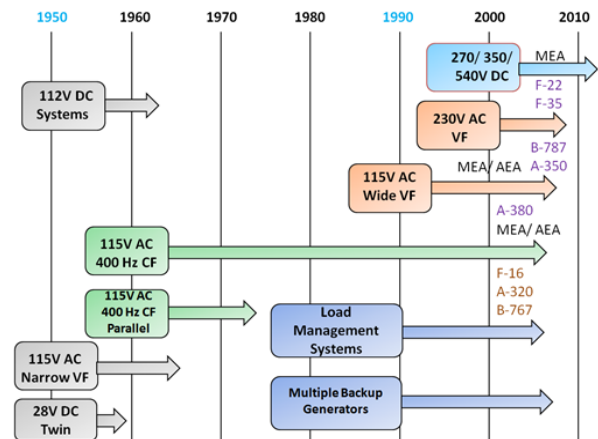


Fig. 1 Evolution of ASE (EPS, PES) aircraft 'classic' and 'MEA/ AEA' [6], [7]

Today, on advanced aircraft (civil, military), in particular in the field of electric power more MEA/ AEA, there is a tendency in the direction of the architecture of the power supply system voltage alternating current as the leading. A key component of on-board systems EPS and also a source of electricity production team starter/ generator AC variable frequency AS/ G VF (Alternating Starter/ Generator Variable Frequency) company (Thales), whose main function is

primarily to provide electricity to the subsystem, based on the supply, transmission and distribution of electricity PMAD (Power Management and Distribution) resistant to any kind of damage [3], [4]. Another very important trend, differentiating planes "classic" of aircraft in line with the modern concept of the plane more electrical MEA/ AEA, and therefore the main components of the system PES, which are rectifiers multi-pulse, is to replace the traditional types of energy (electric, pneumatic, hydraulic and mechanical), one of its kind - in electricity, which is the domain of advanced aircraft. Moreover, later in the paper they were considered the most important, and what goes with it important in the context of flight safety advantages of this arrangement created by the dynamic development of energy-power electronic systems (PES) and their key components, i.e.: rectifiers multi-pulse, transducers cycles, transformers, etc. blocks amplifiers, generally referred to as electronic systems PE (Power Electronics) [8]. The following figures (Fig. 2-3) illustrated the key elements of PES in the context of the distribution of electricity PDS (Power Distribution System), used on modern transport aircraft, in particular civil (Airbus, Boeing).

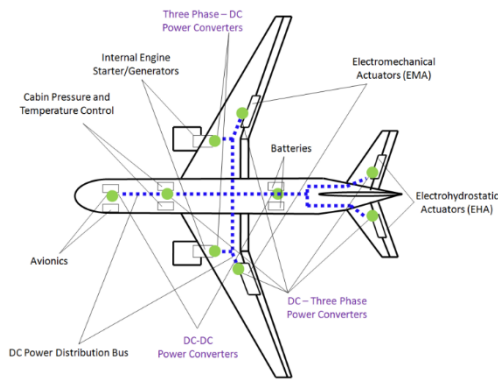


Fig. 2 Power Distribution System (PDS) of a Transport Aircraft [14]

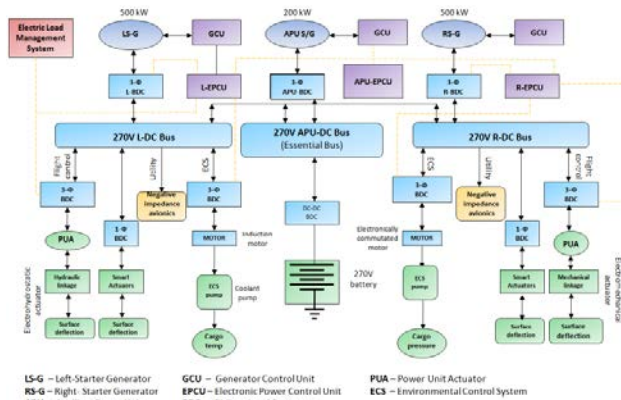


Fig. 3 Power Distribution System (PDS) Architecture [14]

Therefore, pursuant to gradually intro-jana concept MEA/ AEA, and even more AEA, appeared in the literature of expression such as optimizing energy plane POA (Power Optimized Aircraft), and with it a more open technology in the field of electricity MOET (More Open Electrical Technology),

developed by the aircraft concern Airbus, and what goes with it also created innovative solutions for power electronic power systems, i.e. advanced technology PES (Power Electronics Systems) [5]. According to this technology, an essential system in terms of the autonomous power supply system ASE (above the EPS) in modern airplanes is the second component thereof, namely PES energy-electronic supply system and its main components - multi-pulse rectifiers (6-, 12- and 18- and 24-pulse) or even (48- and 60-pulse), and in particular their benchmarks, which are the subject of detailed analysis later in this article.

II. COMPARATIVE ANALYSIS AND MATHEMATICAL MODEL SELECTED RECTIFIERS MULTI-PULSE IN THE FIELD OF ENERGY-ELECTRONIC POWER ELECTRONICS SYSTEM (PES)

The basic components of advanced power electronic power systems PES (Power Electronics Systems) for modern aircraft, widely used in both military aviation as well in civil aviation are called. multi-pulse rectifiers, also referred to as a device transformer-rectifier TRU (Transformer Rectifier Unit). These chargers, as a type (type) transducers (converters) energy-electronic are supplied with on-board electricity network. In the case where the function was to process the AC voltage to a DC voltage, are called transducers (converters), and when processing the DC voltage to AC voltage, are referred to as inverters (inverters). It should also be noted that the power converters are devices for electric energy transfer between the circuits of various types of current, the use of electric valves, in other words, these are devices for direct conversion of the time course of electric current. For example, converters AC called cycloconverters (AC/ AC) or otherwise direct frequency converters are among the so-called. converters AC voltage to the AC voltage AC, without intermediate DC circuit. While the rectifiers are called power electronic, used to convert alternating voltage in constant voltage (one-way). They are usually supplied with a voltage sinusoidal single or three phase. The time course of the output voltage one-way, also referred to as rectified voltage comprised of the appropriate pieces of sinusoidal supply voltage rectifier. The number of pulses q voltage and current rectified per period of the alternating voltage power line rectifier derived names rectifiers. For example, a rectifier, a voltage rectified includes the period of tension power line three pulses ($q = 3$) is referred to as a three-pulse rectifier. The following table (Tab. 1) illustrates one with divisions (due to the nature of the waveforms of input and output) and also by making a literature review of the key elements of PES, there are two types of multi-pulse transducers: transducers power sources CSC (Current Source Converters), and transducers source voltage VSC (Voltage Source Converters). It should be noted that CSC pickups are typical converters, the applicable filter high-power necessary to eliminate the harmonic AC, in addition to filters, DC.

Input waveform	Output waveform	a) Term (name)
AC Alternating	Constant DC	b) Inverter AC/ DC
Constant DC	Constant DC	Converter DC/ DC (Chopper)
Constant DC	AC Alternating	Inverter DC/ AC
AC Alternating	AC Alternating	AC Inverter AC/ AC (Cycloconverter)

Tab. 1 Distribution of power electronic converters by criterion of waveforms (I/ O)

A. The mathematical model in the field of energy-electronic power system PES according to the concept of a more electric aircraft MEA/ AEA

Comparative analysis of the sample and the mathematical models were performed for multi- pulse rectifiers 12- and 24-pulse, selected from the group of 6- to 12- and 18-, 24- pulse used for advanced aircraft in line with the trend of the aircraft more advanced electrical MEA/ AEA Airbus, Boeing (A-380 and A-350XWB, B-787) in the case of civil aircraft, and Lockheed Martin (JSF F-35 and F-22 Raptor) for military aircraft. Then, the selected rectifiers and analysis with mathematical models were compared with the rectifier 48-pulse rectifier selected from the group of 48- and 60- pulse. The figure below (Fig. 4) shows the block diagram of the power of modern architecture in the field of PES, which is a key component of independent on-board autonomous power systems ASE, compatible with the trend of more electric aircraft. A main power supply 3-phase AC this architecture is an electric motor PMSM.

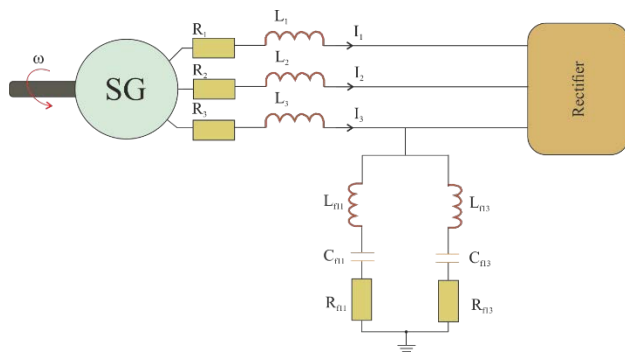


Fig. 4 Block diagram of the power architecture with regard to the system PES, using multi- pulse rectifier more electric aircraft MEA/ AEA

It should be noted that the 3-phase AC voltage, in that the individual harmonics, in addition to the desired signals, also contains undesired signals as mutual interference, noise, etc., which must be eliminated or reduced. To this end, the structure of the rectifier system employing multi- pulse rectifiers (12- and 24- pulse), applied set of filters, implemented in the form of two parallel-connected elements RLC. This kind of solution in the final form will eliminate

some unwanted signals, affecting the final shape of the waveform of the output voltage of the rectifier multi-pulse. Therefore, on the basis of the analysis block diagram architecture with regard to the system PES, mathematical notation physical phenomena (model) aircraft power system more electric voltage at the input transducer assembly energy-electronic AC/ DC is as follows [10], [12]:

$$\begin{aligned}
 u_1 &= U_m \sin\left(\omega t + \beta + \frac{\pi}{2}\right) \\
 u_2 &= U_m \sin\left(\omega t + \beta - \frac{\pi}{6}\right) \\
 u_3 &= U_m \sin\left(\omega t + \beta - \frac{5\pi}{6}\right)
 \end{aligned}
 \tag{1}$$

Based on the above expression (1), we see that the waveform shape (waveform), which shows the input voltage of the same amplitude, U_m , but different phase shift. For the voltage flowing through the filter assembly, mathematical notation takes the following form [9]:

$$\begin{aligned}
 u_{f1} &= u_1 - R_1 i_1 + L_1 \frac{di_1}{dt} \\
 u_{f2} &= u_2 - R_2 i_2 + L_2 \frac{di_2}{dt} \\
 u_{f3} &= u_3 - R_3 i_3 + L_3 \frac{di_3}{dt}
 \end{aligned}
 \tag{2}$$

Assuming that the three-phase voltage u_1 , u_2 and u_3 are generated by an open circuit at the terminals of the electric motor with a maximum value of amplitude U_m . Values u_{f1} , u_{f2} and u_{f3} define the voltage value of the RLC filter circuit, while the ω - indicates the angular velocity of the electric motor. The total resistance in the analyzed circuit is expressed as the sum of the resistance of resistance elements, comprising the motor and the resistance of the cables connecting the various components of the power system supplies the PES in the trend more electric aircraft MEA/ AEA. Mathematical notation occurring phenomena is as follows:

$$R_1 = R_s - R_C \tag{3}$$

A similar figure assumes a record for the event of the total inductance of power electronic power system PES:

$$L_1 = L_s - L_C \tag{4}$$

The electric current in the three-phase system appropriately labeled for the branch from 1 to 3 by i_1 , i_2 , and i_3 , while the electric current at input unit converters, which is responsible for the performance of key functions, which is to transform the voltage and AC on the DC power - determined by size I_p . This operation is performed by a unit transformer-rectifier TRU (Transformer Rectifier Unit). With above derived mathematical relations and properly defined objectives for electrical systems can be determined equations describing all physical phenomena occurring in the field of electricity considered in the circuit shown in the figure above (Fig. 4). Therefore [13], [14]:

$$L_{f11} \frac{d^2 i_{f11}}{dt^2} + R_{f11} \frac{di_{f11}}{dt} + \frac{1}{C_{f11}} i_{f11} = \frac{du_f}{dt} = L_{f13} \frac{d^2 i_{f13}}{dt^2} + R_{f13} \frac{di_{f13}}{dt} + \frac{1}{C_{f13}} i_{f13} \quad (5)$$

$$i_{f11} + i_{f13} = I_f = I_p' - I_p$$

where: L_{f11} , C_{f11} and L_{f13} , C_{f13} are respectively the inductance and capacitance on the respective elements L, C filter circuit in each branch. It should also be noted that the filter whose primary function is the filtration of harmonic components, in this case the order of the harmonic filters 11 and 13. Similarly, as in the case of EPS power supply system for an electric motor, including in the case of a system PES is the resistance a filter circuit, which is determined by R_{f11} oraz R_{f13} . The individual harmonic voltage and current are generated by the rectifier 12- pulse rectifier selected from the group multi- pulse 6- , 12- and 18- , 24- pulse, which is more detailed analysis will be submitted in the following part of the paper.

B. Transmitters (6- and 12- pulse) and (18- and 24- pulse)

Multi- pulse rectifier AC/ DC in the form of one of the energy-electronic transmitter, i.e. 12- pulse rectifier is widely used for civil aircraft of various types, in particular, it is used on airplanes compatible with the concept of an airplane electrical MEA/ AEA. Rectifiers Systems of this type are designed for power supply DC with all the necessary electrical receivers DC, on board an airplane 3-phase alternating current (similar to 6- pulse rectifier), which is applied to the primary winding of the transformer. Joining analysis and mathematical description of physical phenomena occurring at the output rectifier (define a mathematical model) assumes that the electric currents flowing through the rectifier assembly configuration was determined by I_1 , I_2 and I_3 for the three branches of three-phase system. Analyzed the system for which they were carried out mathematical considerations are illustrated in the following diagram (Fig. 5).

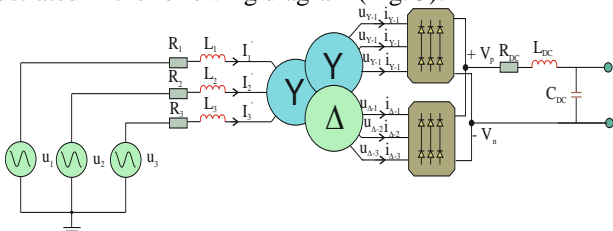


Fig. 5 Wiring diagram 12- pulse rectifier AC/ DC

The structure of such a system rectifier is realized on the basis of two rectifiers (after 6 LEDs), in which a transformation of the AC to DC current (Fig. 5). In this device follows the processing of the AC voltage, then the voltage is via a 2 secondary windings is served to the rectifier. At the output of the rectifier were applicable-type filter LPF (Low-pass filter) eliminates signals (in this case harmonics of voltage-cut below the cut-off frequency), and suppresses those above the cutoff frequency. In other words, it is a low-pass filter, which is a system which transmits only the signal frequency below the cutoff frequency. This system consisting of a "network" of interconnected 12 rectifier diodes, bridge rectifiers creating 2 converts the voltage and alternating current AC at their appropriate values into DC power.

Summary of the operation of this system lies in the fact that the input of the rectifier circuit reaches the value of the alternating AC and receive the output of DC power. The process carried out by the bridge rectifier consisting of 12 LEDs can be saved mathematically by means of non-linear differential equations, as illustrated as follows [11], [14]:

$$\begin{aligned} u_{f1} &= U_{mp} \sin\left(\omega t + \frac{\pi}{2}\right) \\ u_{f2} &= U_{mp} \sin\left(\omega t - \frac{\pi}{6}\right) \\ u_{f3} &= U_{mp} \sin\left(\omega t - \frac{5\pi}{6}\right) \end{aligned} \quad (6)$$

where: U_{mp} - it is the maximum amplitude of harmonic frequencies voltages generated locally on the 12-pulse rectifier. Assuming that the efficiency of the electric motor PMSM is equal to unity, and assuming further step considerations that the converter converts the voltage of both the lower and the upper voltage value of all the harmonics, which can respectively written respond as follows:

For the electric motor running in star pattern [4]:

$$\begin{aligned} u_{Y-1} &= u_{f1} = U_{mp} \sin\left(\omega t + \frac{\pi}{2}\right) \\ u_{Y-2} &= u_{f2} = U_{mp} \sin\left(\omega t - \frac{\pi}{6}\right) \\ u_{Y-3} &= u_{f3} = U_{mp} \sin\left(\omega t - \frac{5\pi}{6}\right) \end{aligned} \quad (7)$$

For the motor operating in a triangular pattern [6]:

$$\begin{aligned} u_{\Delta-1} &= U_{mp} \sin\left(\omega t + \frac{\pi}{2} + \frac{\pi}{6}\right) = U_{mp} \sin\left(\omega t + \frac{2\pi}{3}\right) \\ u_{\Delta-2} &= U_{mp} \sin\left(\omega t - \frac{\pi}{6} + \frac{\pi}{6}\right) = U_{mp} \sin(\omega t) \\ u_{\Delta-3} &= U_{mp} \sin\left(\omega t - \frac{5\pi}{6} + \frac{\pi}{6}\right) = U_{mp} \sin\left(\omega t + \frac{2\pi}{3}\right) \end{aligned} \quad (8)$$

Thus, for rated power generated from the electric motor 90 [kW], DC current is relatively low. In such circumstances, the phase angle φ of the carrier harmonic voltage is smaller than $\frac{\pi}{6}$, the harmonic frequencies for the low and high range do not overlap, hence the equations for the electric circuit of the multi burst rectifier take the following form [12], [24]:

For the case of the phase angle $\alpha \leq \theta < \varphi$

$$\begin{aligned} L_{ac} \frac{di_{\Delta-1}}{dt} &= -R_{ac} i_{\Delta-1} + u_{\Delta-1} - V_p \\ L_{ac} \frac{di_{\Delta-2}}{dt} &= -R_{ac} i_{\Delta-2} + u_{\Delta-2} - V_n \\ L_{ac} \frac{di_{\Delta-3}}{dt} &= -R_{ac} i_{\Delta-3} + u_{\Delta-3} - V_n \\ L_{ac} \frac{di_{Y-1}}{dt} &= -R_{ac} i_{Y-1} + u_{Y-1} - V_p \\ L_{ac} \frac{di_{Y-2}}{dt} &= -R_{ac} i_{Y-2} + u_{Y-2} - V_p \end{aligned} \quad (9)$$

Subsequently obtained:

$$\begin{aligned} i_{\Delta-1} &= -(i_{\Delta-2} + i_{\Delta-3}) \\ i_{Y-1} &= -i_{Y-2} \\ i_{Y-3} &= 0 \\ i_{Y-1} + i_{\Delta-1} &= I_{DC} \end{aligned} \quad (10)$$

where: $\theta = \omega t$, α - ką the phase angle of the initial signal voltage alternating, i_{Δ} , i_Y - alternating current measured at the input of the rectifier, L_{ac} and R_{ac} - are respectively the inductance and resistance occurring in a rectifier. On the other hand, V_p and V_n - determined value of the negative or positive DC voltage node.

For the case of the phase angle $\varphi \leq \theta < \alpha + \frac{\pi}{6}$

$$\begin{aligned} L_{ac} \frac{di_{\Delta-1}}{dt} &= -R_{ac}i_{\Delta-1} + u_{\Delta-1} - V_p \\ L_{ac} \frac{di_{\Delta-3}}{dt} &= -R_{ac}i_{\Delta-3} + u_{\Delta-3} - V_n \\ L_{ac} \frac{di_{Y-1}}{dt} &= -R_{ac}i_{Y-1} + u_{Y-1} - V_p \\ L_{ac} \frac{di_{Y-2}}{dt} &= -R_{ac}i_{Y-2} + u_{Y-2} - V_n \end{aligned} \quad (11)$$

Hence, after appropriate transformations were obtained:

$$\begin{aligned} i_{\Delta-1} &= -i_{\Delta-3} \\ i_{\Delta-2} &= 0 \\ i_{Y-1} &= -i_{Y-2} \\ i_{Y-1} + i_{\Delta-1} &= I_{DC} \\ i_{Y-1} = i_{\Delta-1} &= \frac{1}{2}I_{DC} \\ i_{Y-3} &= 0 \end{aligned} \quad (12)$$

Comparing each of equation (9), (10) the mathematical relations (11) and (12) were prepared:

$$\begin{aligned} V_p - V_n &= \frac{6}{7}u_{\Delta-1} + \frac{3}{7}(u_{Y-1} - u_{Y-2}) - \frac{6}{7}R_{ac}I_{DC} - \frac{6}{7}L_{ac} \frac{dI_{DC}}{dt} \quad \alpha \leq \theta < \varphi \\ V_p - V_n &= \frac{1}{2}(u_{\Delta-1} - u_{\Delta-3}) + \frac{1}{2}(u_{Y-1} - u_{Y-2}) - R_{ac}I_{DC} - L_{ac} \frac{dI_{DC}}{dt} \quad \varphi \leq \theta < \alpha + \frac{\pi}{6} \end{aligned} \quad (13)$$

Substituting equation (7) and (8) to the mathematical equation (13) was obtained:

$$\begin{aligned} V_p - V_n &= \frac{6 + \sqrt{3}}{7}U_{mp} \sin\left(\theta + \frac{2\pi}{3}\right) - \frac{6}{7}R_{ac}I_{DC} - \frac{6}{7}L_{ac} \frac{dI_{DC}}{dt} \quad \alpha \leq \theta < \varphi \\ V_p - V_n &= \frac{1}{2}[\sqrt{3}U_{mp} \cos \theta + \sqrt{3}U_{mp} \sin\left(\theta + \frac{2\pi}{3}\right)] - R_{ac}I_{DC} - L_{ac} \frac{dI_{DC}}{dt} \quad \varphi \leq \theta < \alpha + \frac{\pi}{6} \end{aligned} \quad (14)$$

wherein: I_{DC} - the DC output side of the rectifier circuit.

Thus, the voltage and current can be expressed as:

$$\begin{aligned} R_{DC}I_{DC} + L_{DC} \frac{dI_{DC}}{dt} &= (V_p - V_n) - U_{DC} \\ I_{DC} &= C_{DC} \frac{dU_{DC}}{dt} + I_{Load} \end{aligned} \quad (15)$$

wherein: R_{DC} - it is the total resistance of the electronic components included in the electrical system of the

rectifier so. load, while the well L_{DC} and C_{DC} - marked inductance and capacitance in the circuit of the rectifier. Substituting expression (15) to (14) were obtained [24]:

$$\begin{aligned} U_{DC} + R_1C_{DC} \frac{dU_{DC}}{dt} + L_1 \frac{d^2U_{DC}}{dt^2} &= \frac{6 + \sqrt{3}}{7}U_{mp} \sin\left(\theta + \frac{2\pi}{3}\right) - R_1I_{Load} - L_1 \frac{dI_{Load}}{dt} \quad -\frac{\pi}{4} \leq \theta < \varphi - \frac{\pi}{4} \\ V_{DC} + R_2C_{DC} \frac{dU_{DC}}{dt} + L_2 \frac{d^2U_{DC}}{dt^2} &= \frac{1}{2}[\sqrt{3}U_{mp} \cos \theta + \sqrt{3}U_{mp} \sin\left(\theta + \frac{2\pi}{3}\right)] - R_2I_{DC} - L_2 \frac{dI_{DC}}{dt} \quad \varphi - \frac{\pi}{4} \leq \theta < -\frac{\pi}{12} \end{aligned} \quad (16)$$

where:

$$\begin{aligned} R_1 &= R_{DC} + \frac{6}{7}R_{AC} \\ L_1 &= L_{DC} + \frac{6}{7}L_{AC} \\ R_2 &= R_{DC} + R_{AC} \\ L_2 &= L_{DC} + L_{AC} \end{aligned} \quad (17)$$

C. The mathematical model of the transducer 24- pulse power electronic power system PES in accordance with the trend of MEA/ AEA

Joining the analysis and determination of a mathematical model of the rectifier 24- and the pulse should be noted that the mathematical model presented earlier pulse rectifier 12 and AC / DC has one major drawback namely, voltage signals or current in the form of a wave-border harmonization can not be reduced (to alleviate) harmonics. Reduction of harmonics that occur in the drive currents rectifiers of AC/ DC is thus a very important problem. This task is performed by the rectifiers higher orders: 24- , 36- , and 48- or even 60- pulse. Reducing the proportion of higher harmonics is achieved with using filters of active working with high switching frequency, which, however, are not suitable for use in high power systems. In addition, the rectifier 24- pulse allows for higher currents of distorted power in relation to the 36- pulse rectifier. The figure below (Fig. 6) shows an example of a solution containing two three-phase rectifier bridge circuits, showing the behavior of the 24- pulse rectifier. The envisaged system 24- pulse consists of two circuits rectifier 12- pulse, creating a network bridge 1 and 2 [9].

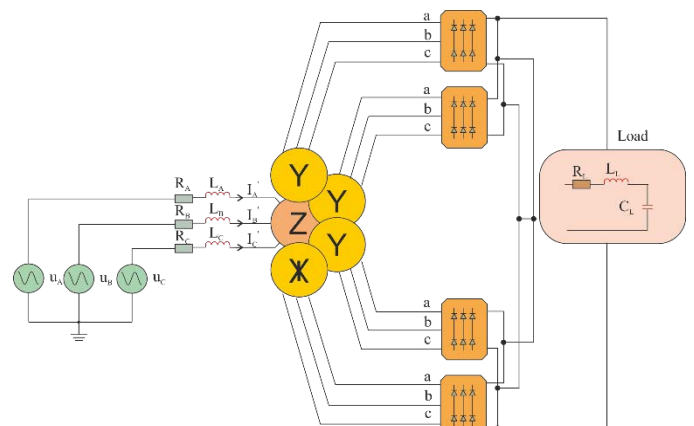


Fig. 6 Wiring diagram 24- pulse rectifier AC/ DC

The system is powered by a three-phase electric motor, and the current at the input of the 24- pulse rectifier can be described as [23]:

$$i_{a1} = \sum_{n=1,3,5} b_n \cdot \sin n(\omega t + \varphi) = \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t + \frac{\pi}{24} \right) \quad (18)$$

Example:

$$\begin{aligned} i_{a2} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t + \frac{\pi}{8} \right) \\ i_{a3} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t - \frac{\pi}{24} \right) \\ i_{a4} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t - \frac{\pi}{8} \right) \end{aligned} \quad (19)$$

Depending on the opposite side take the form of:

$$\begin{aligned} i_{c1} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t + \frac{17\pi}{24} \right) \\ i_{c2} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t + \frac{19\pi}{24} \right) \\ i_{b4} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t - \frac{19\pi}{24} \right) \\ i_{b3} &= \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin n \left(\omega t - \frac{17\pi}{24} \right) \end{aligned} \quad (20)$$

Each phase angle of the harmonic wave AC consists of a phase angle power the main branch and the phase angle performance-overuse in the lateral branch whose mathematical notation as follows:

$$i_{a0} = (i_{c1}k_2 + i_{b3}k_2) \cdot \cos 52,5^\circ + (i_{c2}k_4 + i_{b4}k_4) \cdot \cos 37,5^\circ \quad (21)$$

Thus, the current in the first phase A is:

$$i_{ax} = (i_{c1} + i_{c2})k_1 \cdot \cos 7,5^\circ + (i_{c2} + i_{c4})k_3 \cdot \cos 22,5^\circ \quad (22)$$

Wherein said input current of the electric motor is expressed by the formula:

$$i_a = i_{ax} - i_{a0} \quad (23)$$

Further, by making a Fourier transform of equation (23) were made [7]:

$$i_a = 2 \cdot \sum_{n=1,3,5} \left[\frac{4I_{DC}}{n\pi} \cdot \cos \frac{n\pi}{6} \right] \sin(n\omega t) \cdot \left[\cos \frac{1}{24}n\pi k_1 \cdot \cos \frac{\pi}{24} - \cos \frac{1}{8}n\pi k_2 \cdot \cos \frac{3\pi}{24} - \cos \frac{17}{24}n\pi k_2 \cdot \cos \frac{7\pi}{24} - \cos \frac{19}{24}n\pi k_4 \cdot \cos \frac{5\pi}{24} \right] \quad (24)$$

where: I_{DC} - a DC in the 6-pulse rectifier and amounts $I_{DC} = \frac{I_{DC24}}{4}$, and I_{DC24} - this output current 24 pulse rectifier.

The amplitude of the n-th harmonic signal is defined as [19], [20], [21], [23], [24]:

$$A_n = \frac{2I_{DC24}}{n\pi} \cdot \left(\cos \frac{n\pi}{6} \right) \cdot \left[\cos \frac{1}{24}n\pi k_1 + \cos \frac{1}{8}n\pi k_2 - \cos \frac{17}{24}n\pi k_2 - \cos \frac{19}{24}n\pi k_4 \right] \quad (25)$$

In the case of 24- pulse rectifier total distortion factor THD harmonic 23 and 25 amounts to 5.09%.

A. 48-Pulse Transmitter of Power Electronic Power System PES in Accordance with the Trend of MEA/ AEA

For the currently used power electronics supply systems PES elements responsible for the conversion of the AC to DC power, multi-pulse transmitters are in most cases used in 12-pulse or 24- pulse rectifiers. However, in the literature [14], [22] developed models of 48-pulse rectifiers can be found. One such characteristic of the sensor is the ability to produce significantly less harmonic distortion of the output voltages, and less likelihood of loss of power in the system, as was in the case of 6- , 12- and 18- and 24-pulse rectifiers. The results presented in the literature [13], [22] from the contained simulation it can be seen that instability of 48- pulse rectifier is possible resulting from overloading the system and the possibility of changes in the voltage harmonics. Emerging incompatibilities indirectly can be eliminated by using suitably phased voltage three-phase system of 7.5°, this value will provide valuable full operation of 48- pulse rectifier. Also, be sure to make symmetrical movements of all three voltages produced by the transformer windings. Transmitter model shown in Figures 7-8 consists of four identical 12-pulse rectifiers, associated with four 12- pulse transformer windings with offset. Generally speaking, the 48- pulse converter phase shifts are implemented in the following way, two voltage transformers are offset by the value of 3.75° is connected directly to the 24- pulse rectifiers. Same situation is in the next two transformers. The basic element of the power supply is AC transformer. The transformer at 400 Hz (the frequency used in aviation) under load drops occur in the output voltage caused by the resistance of the primary winding and a secondary resistance of the core losses and leakage inductance of the primary winding and secondary (particularly for high power transformers). Transformer manufacturers usually provide only the basic parameters of the transformer: the power output on the secondary winding, rated primary voltage, secondary voltage at the maximum load resistance, the secondary winding current at maximum load.

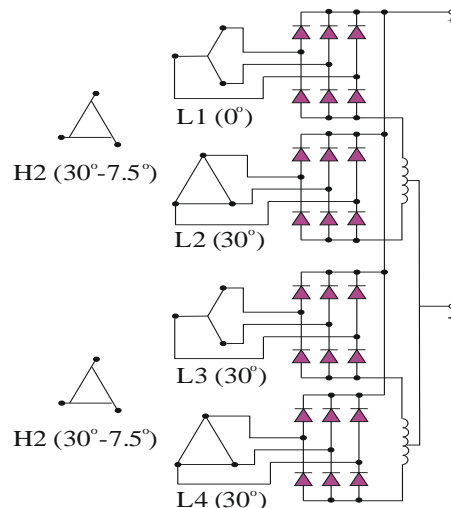


Fig. 7 Block diagram of the 48- pulse rectifier in supply system of F-16 electrical installation (part with the generator harmonic signal with a phase of 0° and 30°)

In the presented system, you can see the connectors and transformer circuits 24- pulse rectifiers in the final result from the 48- pulse rectifier. Modeled system can be used in devices or systems supplying high voltage applications with high

power without the use of filtering systems due to the treatment of its very low harmonic distortion arising from the side that produces alternating current. It should be noted that the output voltages from the system are characterized by standard values of the harmonics of, respectively, $n = 48r + 1$, where $r = 0, 1, 2, 3$, i.e. 47, 49, 95, 98 with typical sizes of $1/47$, $1/49$ and so on respectively as the system that produces DC voltage.

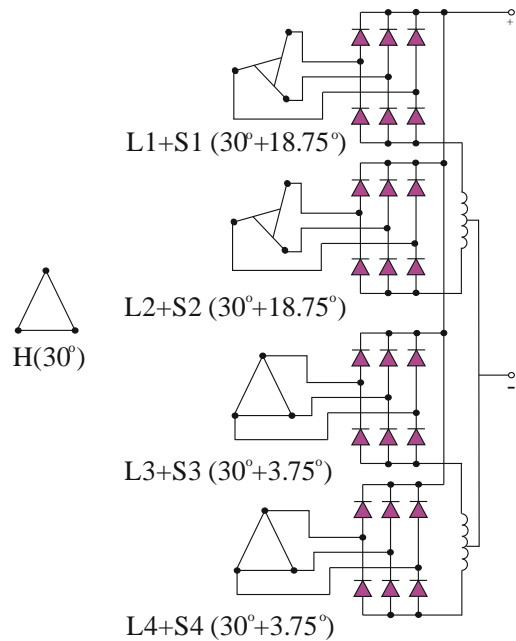


Fig. 8 Block diagram of the 48-pulse rectifier supply system of F-16 electrical installation (part with the generator of harmonic signal with a phase of 30° , 18.75° and 3.75°)

The lower range of the harmonic voltage is 48. With this information in mind the authors attempted to test how 48-pulse rectifier behaves in the electrical power supply system of more electric aircraft. In addition, in the development of a simulation model in Matlab/ Simulink it was tested how a 48-pulse rectifier reacted to the phase shift change in the voltage transformer.

D. The Mathematical Model of The 48- Pulse Converter in Power System PES Based on Example of TRU Rectifier Transformer

Description of occurring physical phenomena and mechanisms of action (mathematical model) 48- pulse transmitter can be summarized as follows [14], [18]. The system is divided into four main components, wherein each unit for the 12- pulse converter and the main attention was focused on the mathematical description (Fig. 7-8).

The Mathematical Model of the 48- Pulse Converter in Power System PES Based on Example of TRU Rectifier Transformer is given in the Appendix A.

III. ANALYSIS AND SAMPLE SIMULATION OF SELECTED MULTI-PULSE TRANSUCERS ASE MODERN POWER SYSTEMS (EPS, PES), USED FOR ADVANCED AIRCRAFT IN LINE WITH THE CONCEPT OF MEA/ AEA

Current technological development in the field of electrical machines [1], [4] and its derivatives disciplines (electronics, power electronics), allows for systems EPS to reduce their numbers to a single system, which is the electro-energetic system power supply EPS power system AC, leaving emergency power source, such as batteries (rechargeable

batteries), air turbine RAT (Ram Air Turbine) auxiliary power unit APU (Auxiliary Power Unit) and fuel cells (fuel cells) [4], [5]. On the other hand, in the case of advanced systems PES, whose key elements of modern electronic systems PE (Power Electronics), their intensive development fully meets the requirements of the functions carried out by converters (LNBS), whose primary purpose is to fit one type of system, electro-energy (mains) power AC for all kinds of receivers on board modern aircraft, regardless of the value and type of DC and AC. The condition for this is the realization of a number of studies relating to obtaining systems PES answer-snowfalls parameters, namely the value of PF (Power Factor) and the ratio of the signal THD (Total Harmonic Distortion), which perform essential functions, i.e. Power factor (PF) is part of the economic effects and the THD is an element of compatibility electro-magnetic [9], [15], [17]. The next figure (Fig. 9) were imaged possibilities of using transducers multi-pulse in the implementation of the project by MOET aircraft concern Airbus, according to the concept aircraft more/ fully electric (MEA/ AEA). The topology of power electronic systems (converters, devices) take into account a wide range of controllable elements, of which the most important are: IGBT (Insulated Gate Bipolar Transistor), field power transistors (power MOSFETs), thyristors GTO (Gate Turn Off) and others. The use of electronics in modern systems available items fully steerable resulted in great progress in the construction mainly electro-energy system (mains) AC. The inverter (DC/ AC, AC/ DC/ AC) and DC/ DC (hopper). In addition, it became possible to use known from other fields of modulation techniques such as PWM (Pulse Width Modulation) or PFM (Pulse Frequency Modulation), where the elements are switched frequencies from a few [kHz] to several [MHz], so that the output waveforms of currents or voltages are very close to sine waves.

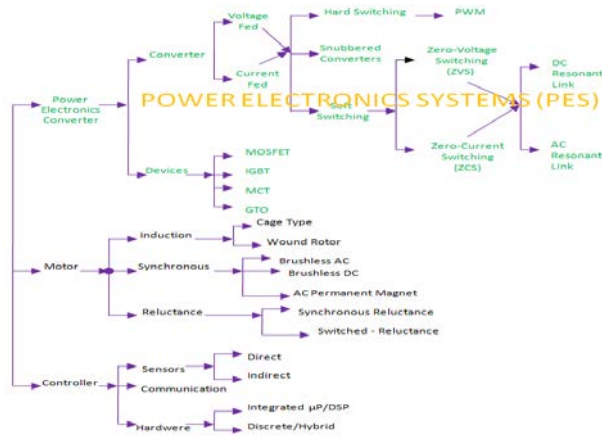


Fig. 9 Options for 'More Electric' Technology (MET) [16]

In addition, a variety of methods for controlling the electronic power elements with regard to the system PES, namely the so-called traditional method. "Hard switching" (hard switching) and later called. "Soft switching" (soft switching). The main idea of this method is the requirement of switching the zero-current ZCS (Zero Current Switching) or at zero voltage ZVS (Zero Voltage Switching). The same topology variety of equipment, methods and elements for the EPS system (motor, controller) is also illustrated.

A. Energy-electronic transducers (12-, 24- and 48- pulse)

The main objective of carrying out computer simulations of selected components, which included a synchronous motor and 48- pulse converter, was to assess the performance of modern power systems (EPS, PES), used in advanced aircraft in use today, both civilian and military. Therefore, the authors have made an exemplary analysis of the operation of the electric drive in modern power systems (EPS, PES) aircraft in accordance with the trend of More Electric Aircraft and their comparison. Given the complexity of both these power systems, particular attention is focused on two components that play an important role in modern electrical systems of the aircraft. These are three-phase synchronous motor PMSM and the elements responsible for the processing of AC to DC current (multi-pulse converters). Computer simulations were carried out in the computer program Matlab/ Simulink. At the beginning fixed parameters are defined that will be used in computer simulations performed. Thus, one used relative units related to the rating of the three-phase synchronous motor with permanent magnets $R_s = 0,2 [\Omega]$, $L_d = L_q = L_s = 0,0085 [H]$, $\Psi_f = 0.175 [Vs]$, $p = 4$.

The following illustrations show the models of power 3-phase synchronous motor AC (Fig. 10) and the unit transformer-rectifier (Fig. 11) TRU (Transformer Rectifier Unit), made in Matlab/ Simulink.

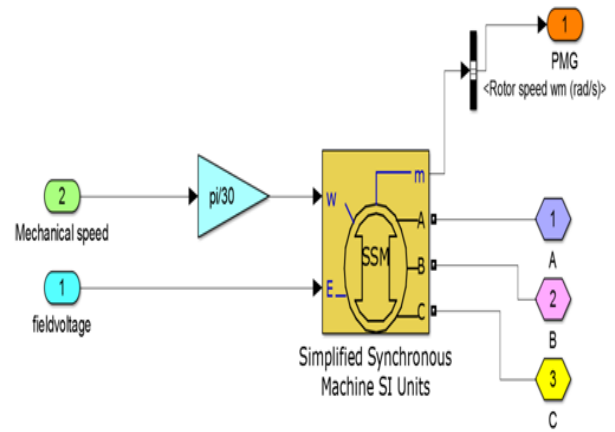


Fig. 10 PMSM motor model in Matlab/ Simulink

In the pad motor the following values of parameters were defined characterizing his work: Power output (Power Generation) 100 [kVA], operating frequency $f = 400 [Hz]$, line-to-line voltage of 200 [V].

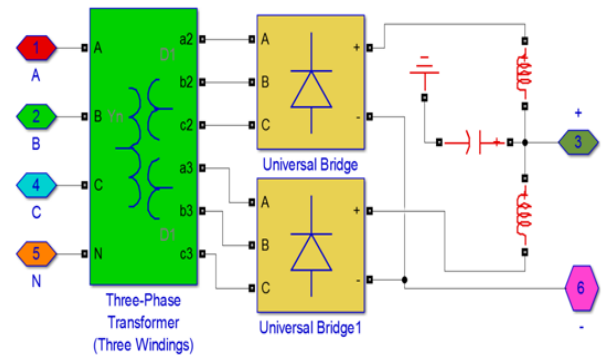


Fig. 11 TRU model component in Matlab/ Simulink

B. Comparative analysis, mathematical model and simulation of energy-electronic transducers (12-, 24- and 48-pulse)

All models of multi- pulse rectifiers, which are one type of power electronic converters have been modeled in the programming environment Matlab/ Simulink using the library SimPowerSystems, because all the necessary elements models included in the electro-energy supply system according to PES the concept of a more electric aircraft MEA/ AEA, are now available and ready for use. Conducted computer simulations for all types of rectifiers realized for the technical parameters presented in the following table (Tab. 2).

Tab. 2 Specifications rectifiers multi- pulse (12-, 24- and 48- pulse)

Parameter	Value
The load RL	R= 50 [Ω], L=0.65 [H]
Frequency	400 [Hz]
The input voltage received from electric motor	300 [V]

A key goal of this computer simulation was to observe waveforms harmonic voltages and currents, taking into account the inductance and resistance, which tests serve as load.

C. Mathematical model of energy-electronic transducers (12- and 24- pulse)

In the following figures (Fig. 12-13) show block diagrams of rectifiers 12- and 24- made in the pulse Simulink.

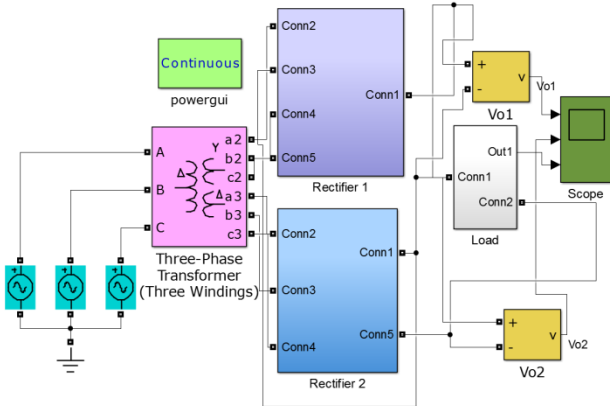


Fig. 12 Block diagram of the pulse rectifier 12- made in the Simulink

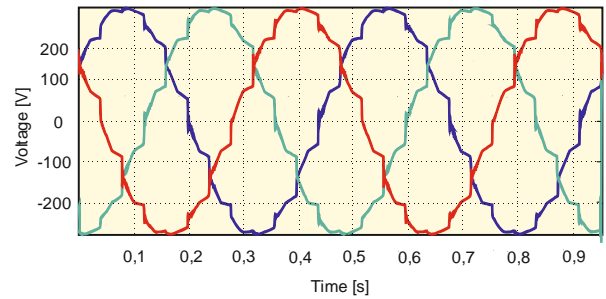


Fig. 14 Course of harmonic voltage rectifier 12- pulse

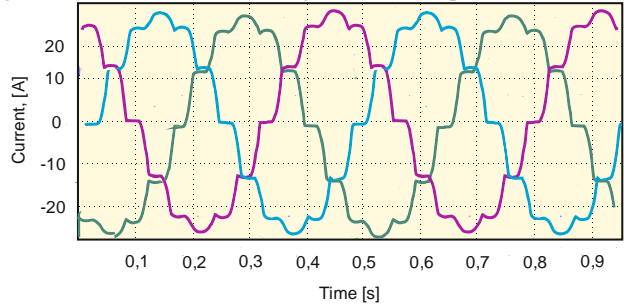


Fig. 15 Course of harmonic current rectifier 12- pulse

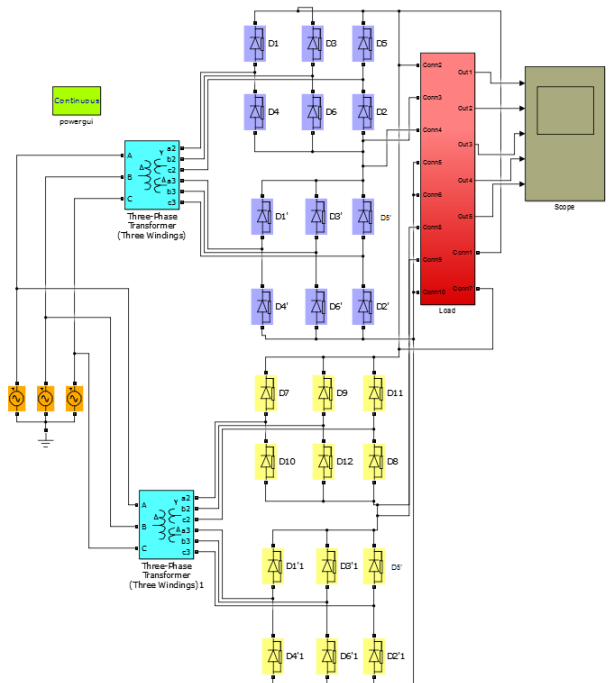


Fig. 13 Block diagram of the pulse rectifier 24- made in the Simulink

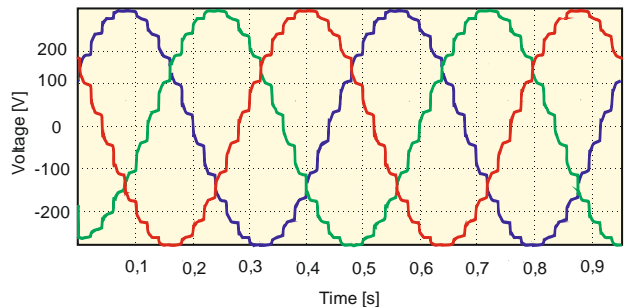


Fig. 16 Course of harmonic voltage rectifier 24- pulse

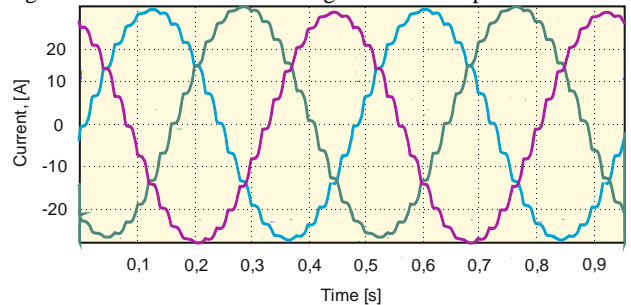


Fig. 17 Course of harmonic current rectifier 24- pulse

D. Comparative analysis and simulation of power electronic converters (12- and 24- pulse)

The results obtained from computer simulations for transmitters (12- and 24- pulse) in the nature of harmonics of voltages and currents illustrated in the graphs below (fig. 14-17).

It should be noted that the multi- pulse converters AC/ DC, which is one type of transducer electronics generate odd harmonic current government - see. formula (2). When making a comparative analysis of these charts (fig. 18-21), and the analysis presented earlier models, as follows the charts above and the previously mentioned patterns and relationships, increasing the number of pulses q is the primary method that allows to eliminate harmonics the drive currents. It should also be noted that in classical solutions to increase the number of pulses q obtained by serial or parallel connection of p- phase bridge circuits powered by transformer a symmetrical arrangement of the secondary voltages are offset by an angle of 120°.

E. Comparative analysis and simulation of power electronic converters 48- pulse

The next stage of the analysis was simulation of synchronous motor to assess the performance of the power supply system in accordance with the concept of PES MEA/ AEA. The test values in this case were the voltage and current waveforms on the three-phase TRU (48- pulse rectifier system), and the observations of the effect of the starter was carried out at the time of switching. The results obtained at this stage of the study are presented below (Fig. 18-21).

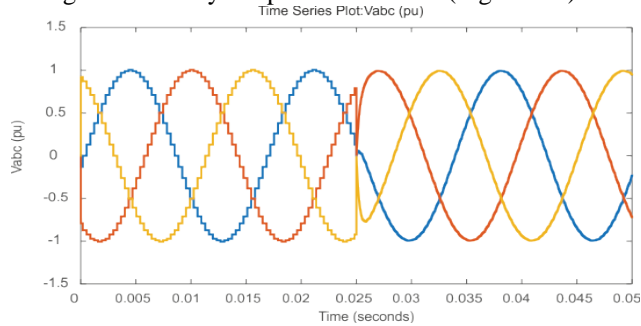


Fig. 18 The course of tension in the TRU in accordance with the concept of MEA/ AEA for the angle of the voltage harmonic 7.5°

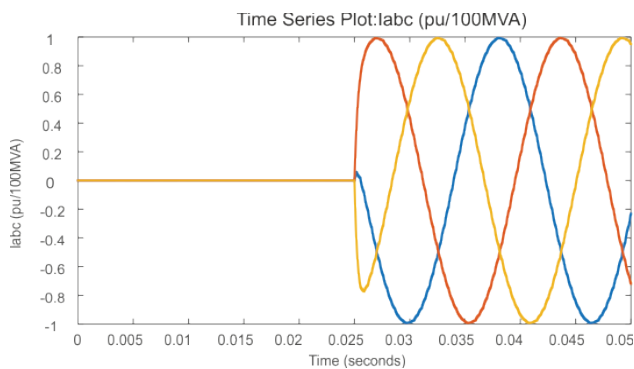


Fig. 19 The course of the current in the TRU in accordance with the concept of MEA/ AEA for the angle of the voltage harmonic 7.5°

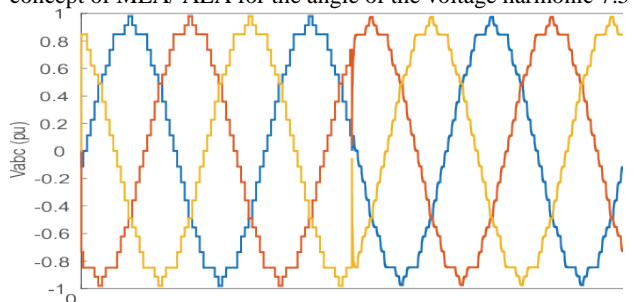


Fig. 20 The course of tension in the TRU in accordance with the concept of MEA/ AEA for the angle of the voltage harmonic 30°

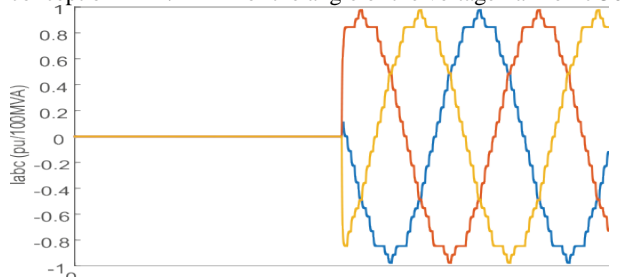


Fig. 21 The course of the current in the TRU in accordance with the concept of MEA/ AEA for the angle of the voltage harmonic 30°

IV. CONCLUSION

The concept of more aircraft/ fully electric (MEA/ AEA) was determined (identified) in the form of a forward-looking trend for modern aircraft, including commercial aircraft in particular. The main idea of it comes down to replace traditional types of energy (electric, pneumatic, hydraulic and mechanical), used by the devices, systems, installations and systems for conventional aircraft, in one form of energy - electricity. As a result, a wide range of functions performed by each device driven by pneumatic energy, mechanical and hydraulic components have been replaced by electrical. Therefore, the key role was played in this respect the autonomous on-board power supply systems ASE (EPS, PES) including in particular power electronic power systems PES (Power Electronics Systems) and their constituent components, which include: systems and power electronic converters, multi- pulse rectifiers, converters, etc. Based on the analysis on the basis of comparative characteristics of the individual multi- pulse rectifiers (6- and 12- pulse) or (18- and 24- pulse) with a 48- pulse rectifier and on the basis of mathematical models of harmonics of voltages and currents during the simulation can be seen that the greater the number pulse of the power electronic converter (rectifier multi burst), the voltage on the output is more stabilized. Assuming that one of the criteria eg., the structure of the rectifier circuit (starting transformer design and ending with the number of used converters 6-chip) it can be seen that the multi- pulse rectifier with a higher number of pulses (18-, 48- and 24-pulse) are a much more complex structure and consist of a much larger number of elements, in comparison with systems with a smaller number of pulses (6-, 12- pulses). On the other hand, the multi- pulse rectifiers, in particular those with a higher number of pulses have a much lower ripple, and the current after rectification is more susceptible to adjustment. These features of the rectifier circuits 18-, 24- and 48- pulse largely translate into high efficiency in converting the 3-phase AC to DC current and this is directly linked also to the high efficiency of the transmitter. These features rectifiers multi-pulse are among their key advantages. Therefore, this complicated and complex structure is associated with increased susceptibility to all kinds of damages and the costs of the purchase of equipment and its operation, which are called. operating costs and with a complex structure are considered to be defects in multi- pulse rectifiers.

A very important power system, used in aviation (except EPS system) is an energy-electronic power system PES, which is a leading component of the multi- pulse converters (eg. TRU). From the presented simulation performed in Matlab/ Simulink it is clear that the use of 48- pulse converter in a power supply system of modern aircraft from the user point of view is very attractive and effective in the configuration intended for processing high-voltage AC to DC.

In modern power systems (EPS, PES) every step of the AC voltage AC power into DC power is carried out by sub-12-pulse rectifiers, which are the proper configuration of 24-

pulse rectifier circuit with its own voltage response. Two sets of 24- pulse connection harmonic transform of phase by 7.5° from each other, which in turn ensures proper operation of the 48- pulse converter (Fig. 18-19). Furthermore, in this system in a situation where we are dealing in various types of loads, supply system will alleviate the problem of voltage instability. On the other hand, changing the phase angle of the voltage to a value of 30° makes the end result a waveform voltage and current is significantly distorted than in the previously discussed case, which could mean unstable operation of all electrical devices, with which a particular aircraft is supplied (Figure 20-21). This effect is an effect of mutual harmonic voltages generated by the generator.

In summary, the stabilization voltage transformer rectifier subassembly TRU using a 48- pulse rectifier systems using 12-chip connected in common with each other when the voltage phase of the harmonic 7.5° , produced by the AC power as a result of simulations proceeded positively. The results confirmed the validity of established concepts, with the proviso that when the voltage phase of 30° respectively, the results were unsatisfactory, and the voltage waveform revealed a lot of distortion that adversely affect the power system TRU.

In addition, the inverter based on the concept of multi-pulse operation (eg. rectifiers 12-, 24-, 36- pulse), which are high-power supply of the reducing higher harmonics of the drive currents to affect eliminate the drawbacks, consisting in the fact that the voltage or current signals in the form of harmonic waves are not able to reduce (mitigate) harmonics and meets the current standards of quality electricity.

In summary, the simplest 6- pulse rectifier comprising six diodes, producing considerable harmonic distortion, while the pulse circuit 12 constructed of the two circuits 6- pulse connected in series and the input AC voltage connected with transformer, which has a phase shift between the outputs of 30° . Such a structure provides the lower distortion rectified voltage. While rectifiers (18- and 24- pulse) or (48- and 60-pulse) used in advanced architectures power aircraft, in line with the trend of MEA/ AEA show a much smaller ripple.

REFERENCES

- [1] Setlak L., Ruda E., Przegląd, analiza i symulacja wybranych komponentów elektro-energetycznego systemu zasilania EPS samolotu zgodnych z trendem samolotu zelektryfikowanego MEA. Instytut Napędów i Maszyn Elektrycznych KOMEL. Zeszyty problemowe ZP-ME nr 3/ 2015 (107), str. 139-144, Katowice 2015.
- [2] Moir I., Seabridge A., Design and Development of Aircraft Systems. Second Edition, 2013 John Wiley & Sons, Ltd.
- [3] Pfahler D., Air Force Power Requirements, Standard Form 298 (Rev. 8-98), 2006.
- [4] Setlak L., Ruda E., Review, Analysis and Simulation of Advanced Technology Solutions in Power Electronics Systems (PES) of More Electric Aircraft. World Academy of Science, Engineering and Technology, Vol: 9, No: 10, 2015.
- [5] Editors Abu-Rub Haitham, Malinowski Mariusz, Al-Haddad Kamal, Power Electronics for Renewable Energy Systems, Transportation, and Industrial Applications, First Edition, 2014 John Wiley & Sons Ltd.
- [6] Moir I., Seabridge A., Aircraft Systems: Mechanical, Electrical, and Avionics Subsystems Integration. Third Edition, 2008 John Wiley & Sons, Ltd.
- [7] Setlak L., Ruda E., Przegląd, analiza porównawcza i symulacja wybranych komponentów architektury elektroenergetycznego systemu zasilania samolotów "konwencjonalnych" i "More Electric Aircraft" (MEA). Instytut Napędów i Maszyn Elektrycznych KOMEL. Zeszyty problemowe ZP-ME nr 1/ 2016 (109), str. 139-146, Katowice 2016.
- [8] Moir I., Seabridge A., Military Avionics Systems, 2006 John Wiley & Sons, Ltd.
- [9] Cheng R., Zhao W., Deng H., and Jiang X., "Modeling and Optimization Control for Aircraft AC Generator Brushless Excitation System Based on Improved Adaptive PSO", The Open Automation and Control Systems Journal, 2015, 7, 21-30.
- [10] Yang Z., Qu J., Ma J., and Shi X., "Modeling and Simulation of Power Distribution System in More Electric Aircraft", Hindawi Publishing Corporation Journal of Electrical and Computer Engineering Volume 2015, Article ID 847624, 7 pages.
- [11] Abdel-Fadil R., Eid A., Abdel-Salam M., "Electrical distribution power systems of modern civil aircrafts", 2 nd International Conference on Energy Systems and Technologies 18-21 Feb. 2013, Cairo, Egypt.
- [12] Singh B., Murthy S.S., and Gupta S., "Analysis and design of STATCOM-based voltage regulator for self-excited induction generators," IEEE Trans. Energy Convers., vol. 19, no. 4, pp. 783-790, Dec, 2004.
- [13] Singh B., Gairola S., Singh B. N., Chandra A., and Haddad K.A., Multi-pulse AC-DC Converter for Improving Power Quality: A Review IEEE Transactions, On Power Delivery, Vol. 23, No. 1 January 2008.
- [14] Raghuvanshi S., Singh N., Comparative analysis of 36, 48, 60 pulse AC-DC Controlled Multipulse Converter for Harmonic Mitigation, International Journal of Advanced Research in Computer Engineering & Technology (IJARCET) Volume 3 Issue 4, April 2014.
- [15] Ronkowski M., Michna M., Kostro G., Kutt F., Maszyny elektryczne wokół nas. Politechnika Gdańska 2009/ 2010.
- [16] Gong G., Drofenik U., Kolar J.W., 12-Pulse Rectifier for More Electric Aircraft Applications. ETH Zurich, Power Electronic Systems Laboratory, ICIT 2003.
- [17] Zhao X., Guerro J.M., Wu Xiaohao, Review of Aircraft Electric Power Systems and Architectures. IEEE 2014, International Energy Conference ENERGYCON).
- [18] Nya B.H., Brombach J., Schulz D., Benefits of higher voltage levels in Aircraft Electrical Power Systems. Railway and Ship Propulsion (ESARS), pp. 1-5, Oct. 2012.
- [19] Skvarenina T.L., Pekarek S., Wasynczuk O., Krause P.C., Simulation of a More-Electric Aircraft Power System using an automated state model approach. Proceedings of the Intersociety Energy Conversion Engineering Conference, 1996, pp. 133-136.
- [20] Cheng R., Zhao W., Deng H. and Jiang X., "Modeling and Optimization Control for Aircraft AC Generator Brushless Excitation System Based on Improved Adaptive PSO", The Open Automation and Control Systems Journal, 2015, 7, 21-30.
- [21] Vinod Vashram Vadher, "Mathematical Modelling of Aircraft Electrical Power Systems", Doctoral Thesis, Loughborough University of Technology 1981.
- [22] El-Moursi M. S., Sharaf A. M., "Novel Controllers for the 48-Pulse VSC STATCOM and SSSC for Voltage Regulation and Reactive Power Compensation", IEEE Transactions On Power Systems, vol. 20, no. 4, November 2005.
- [23] Monroy A. O., H. Le-Huy, C. Lavoie, Modeling and Simulation of a 24-pulse Transformer Rectifier Unit for More Electric Aircraft Power System; Electrical Systems for Aircraft, Railway and Ship Propulsion (ESARS), 2012
- [24] Hernadi A., Taufik, and M. Anwari, "Modeling and Simulation of 6-Pulse and 12-Pulse Rectifiers under Balanced and Unbalanced Conditions with Impacts to Input Current Harmonic", Second Asia International Conference on Modelling & Simulation
- [25] Setlak L., Kowalik R., Comparative Analysis and Simulation of Selected Components of Modern on-board Autonomous Power Systems (ASE) of Modern Aircraft in line with the Concept of MEA/ AEA. World Congress on Engineering and Computer Science 2016 Vol I WCECS 2016, USA, pp.321-326.

APPENDIX A

The Mathematical Model of the 48- Pulse Converter in Power System PES Based on Example of TRU Rectifier Transformer

Description of occurring physical phenomena and mechanisms of action (mathematical model) 48- pulse transmitter can be summarized as follows [14], [18]. The system is divided into four main components, wherein each unit for the 12- pulse converter and the main attention was focused on the mathematical description (Fig. 7-8). Thus, the voltage generated by the first 12- pulse rectifier is described by the equation:

$$u_{12}(t)_1 = 2[V_{AB1} \sin(\omega t + 30^\circ) + V_{AB11} \sin(11\omega t + 195^\circ) + V_{AB13} \sin(13\omega t + 255^\circ) + V_{AB23} \sin(23\omega t + 60^\circ) + V_{AB25} \sin(25\omega t + 120^\circ) + \dots] \quad (26)$$

In a second 12- pulse rectifier output voltage switches as follows:

$$U_{AB12}(t)_2 = 2[V_{AB1} \sin(\omega t + 30^\circ) + V_{AB11} \sin(11\omega t + 15^\circ) + V_{AB13} \sin(13\omega t + 75^\circ) + V_{AB23} \sin(23\omega t + 60^\circ) + V_{AB25} \sin(25\omega t + 120^\circ) + \dots] \quad (27)$$

For the third rectifier unit equation voltage at its output is shown as:

$$U_{AB12}(t)_3 = 2[V_{AB1} \sin(\omega t + 30^\circ) + V_{AB11} \sin(11\omega t + 285^\circ) + V_{AB13} \sin(13\omega t + 345^\circ) + V_{AB23} \sin(23\omega t + 240^\circ) + V_{AB25} \sin(25\omega t + 300^\circ) + \dots] \quad (28)$$

The last equation defining the output voltage of the rectifier 12- pulse of the fourth pulse component of the 48- pulse converter is as follows:

$$u_{12}(t)_4 = 2[V_{AB1} \sin(\omega t + 30^\circ) + V_{AB11} \sin(11\omega t + 105^\circ) + V_{AB13} \sin(13\omega t + 165^\circ) + V_{AB23} \sin(23\omega t + 240^\circ) + V_{AB25} \sin(25\omega t + 300^\circ) + \dots] \quad (29)$$

The four mathematical equations for recording of the physical phenomena occurring in a 12- pulse rectifier in particular relates to shift the AC voltage at the output of the system described by equations (3)-(6) [25] is inserted in series to the respective secondary windings of the transformers. As a result, the total value of the AC voltage at the 48- pulse rectifier output is:

$$U_{AB12}(t)_{48} = U_{AB12}(t)_1 + U_{AB12}(t)_2 + U_{AB12}(t)_3 + U_{AB12}(t)_4 \quad (30)$$

After the summation operation obtained:

$$B_{12}(t)_{48} = 8[V_{AB1} \sin(\omega t + 30^\circ) + V_{AB47} \sin(47\omega t + 150^\circ) + V_{AB49} \sin(49\omega t + 210^\circ) + V_{AB95} \sin(95\omega t + 330^\circ) + V_{AB97} \sin(97\omega t + 30^\circ) + \dots] \quad (31)$$

Thus, ultimately the output voltage generated by the neutral 48- pulse rectifier can be written as:

$$U_{AB12}(t)_{48} = \frac{8}{\sqrt{3}} \sum_{n=1}^{\infty} V_{AB1} \sin(n\omega t + 18.75^\circ n - 18.75^\circ i) + \dots \quad (32)$$

where: $n = (48r \pm 1)$, $r = 0, 1, 2, \dots$. A voltage $U_{BN12}(t)_{48}$ and $U_{CN12}(t)_{48}$ having a sine wave-like shape, are offset with an angle of 120° and 240° degrees relative to the voltage $U_{AB12}(t)_{48}$.