Impact of the Sampling Period on the Design of Digital PID Controllers

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Abstract— **The impact of the sampling period on the parameterization of a digital PID controller in the frequency domain is attempted using three different digital approximations of the integral action. The controller is implemented in the industrial process of regulation of the cement sulphates in the cement mill outlet. The maximum sensitivity, M_s, has been utilized as a main robustness criterion. For the same M_s, proportional and differential gain, a rise of the sampling period leads to a decrease of the integral gain k_i for all the three approximations. For the same sampling period, the function between proportional and integral gain differs for the three approximations studied. If the design satisfies two criteria simultaneously, maximum sensitivity and phase margin in the current study, then the permissible PID gains zone becomes narrower.**

Keywords—digital PID; sampling; frequency; cement; robustness

I. INTRODUCTION

Digital controllers are implemented nowadays in numerous applications in order to provide feedback control on a continuous time process, whose outputs are sampled in discrete time instances. As it has been noticed by Astrom et al. [1] in the industrial process control the PID type controllers constitute a percentage more than 95% of the installed ones. Thus, the study of the discrete PID controllers remains a challenging field of research due to its large field of application. Various techniques have been developed for the design of digital controllers. The first attempt to analyse and design discrete time control systems was described in a paper by Kalman [2]. Madsen et al. [3] refers two main approaches for designing a digital controller: The direct sampled – data approach and a second one called digital redesign. The redesign method involves an initial development of an analog controller meeting the system requirements. Then this controller is transformed to a digital one through a series of redesign procedures. Seborg et al. [4] describes two basic methodologies for digital controller design: The direct synthesis (Dahlin's method, Vogel – Edgar algorithm, internal model control) and minimum variance control. Das et al. [5] has been utilized the continuous and discrete time Linear Quadratic Regulator (LQR) theory in designing of optimal analog and discrete PID controllers. Yousefzadeh et al. [6] after an initial design in the frequency domain of an analog PID, then applies the pole-zero mapping method to tune its digital form. Peretz et al. [7] refers that the most popular approach in designing digital compensators is the frequency domain based method. Okuyama [8] has been presented a designing problem of discrete-time PID algorithm solved in the frequency domain. During the procedure, a modified Nichols diagram is applied. Papadopoulos et al. [9] utilized also the frequency domain to describe a general process model. Then the transition to the z domain in conjunction with a symmetrical optimum criterion has been implemented to generate tuning rules of digital PID controllers for integrating processes.

This study aims at investigating of the parameterization of a digital PID controller in the frequency domain by applying it to an actual industrial process of high importance as concerns the quality of the manufactured goods: The regulation of the cement sulphates in the cement mill outlet. Due to the significance of the sampling period in the controller design, an extensive search has been attempted to be carried out.

II. PROCESS MODEL

The SO_3 control and regulation is performed by sampling cement in the mill outlet, measuring the sulphates and changing the gypsum percentage in the cement composition. Gypsum is the main source of SO_3. Another source is the clinker which constitutes the main cement component and the fly ash also in the case of pozzolanic cements.
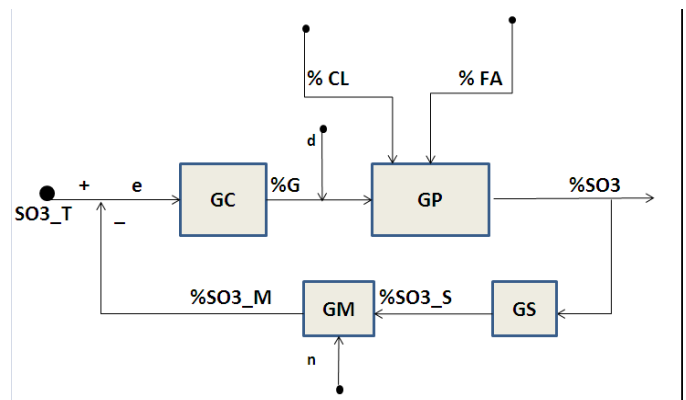

Fig.1.   Block diagram of the SO_3 regulation feedback loop.

The variation of the SO_3 content of the raw materials represents disturbances that have to be eliminated. The block diagram and the transfer functions of the SO_3 regulation process are shown in Figure 1. The blocks represent the subsequent sub-processes: GP = mixing of materials inside the milling installation; GS = cement sampling; GM = SO_3 measurement; GC = control law. The signals of the feedback loop are denoted with the following symbols: %G, %CL, %FA = the percentage of gypsum, clinker and fly ash respectively. %SO3 = the cement sulphates. %SO3_S, %SO3_M = SO_3 of the sampled and measured cement correspondingly. %SO3_T = sulphates target. e = %SO3_T - %SO3_M is the error of the regulation. d = the feeders disturbances. n = noise of the SO_3 measurement. The scheme shown in Figure 1 comprises all the basic components of the feedback control loop.

A. Transfer Functions of the Control Loop

To model the control loop, the transfer functions and their Laplace transform has been considered. The determination of the dynamics between the gypsum content of the material fed to the mill and the SO_3 of the cement in the mill circuit outlet has been achieved with a step response test in an actual closed circuit cement mill. A first order with time delay (FOTD) model describes this dynamics which constitutes the process transfer function GP. The Laplace transform of this model is given by equation (1):

$$GP = \frac{\%SO3}{\%G} = \frac{k_g \cdot exp(-T_D \cdot s)}{1 + T_0 \cdot s} \tag{1}$$

Where k_g = the dynamics gain (%SO_3/%Gypsum), T_D, T_0 = the delay time and first order time constant respectively (min). During the sampling period an average sample is collected. Thus, the variables %SO3 and %SO3_S in the time domain are connected with the relation (2):

$$\%SO3\_S(t) = \frac{1}{T_s} \int_{t}^{t+T_s} \%SO3 \, dt \tag{2}$$

The Laplace transform of (2) is given by (3):

$$GS = \frac{\%SO3\_S}{\%SO3} = \frac{1}{T_s \cdot s}(1 - e^{-T_s s}) \tag{3}$$

Where T_s = the sampling period (min). The sample transfer from the sampling point to the quality laboratory, the measurement procedure of SO3, the calculation of the new gypsum content and its transfer to the mill feeder lasts T_M minutes. This delay is expressed by (4).

$$GM = \frac{\%SO3\_M}{\%SO3\_S} = e^{-T_M s} \tag{4}$$

The PID controller has as input the measured values of SO_3, %SO3_M and produces the new percentage of gypsum, %G. Its analog implementation is described by (5):

$$\%G(t) = k_p \cdot e + k_i \int_{0}^{t} e \, dt + k_d \frac{de}{dt} \tag{5}$$

Where k_p, k_i, k_d are the proportional, integral and derivative gain of the controller respectively. To derive an expression for the digital form of the controller, the analog formula is used in the instances $t = k \cdot T_s$ and $t = (k-1) \cdot T_s$ where k is an integer. By subtracting the two equations the digital expression of PID is derived, described by (6) in time domain. Its Laplace transform is given by (7). For comparison reasons the Laplace form of the analog PID is provided also by the formula (8).

$$\%G_k - \%G_{k-1} = k_p(e_k - e_{k-1}) + k_i e_k T_s$$
$$+ \frac{k_d}{T_s}(e_k + e_{k-2} - 2e_k) \tag{6}$$

$$GC = \frac{\%G}{e} = k_p + \frac{k_i \cdot T_s}{1 - exp(-T_s \cdot s)} + \frac{k_d}{T_s}$$
$$\cdot (1 - exp(-T_s \cdot s)) \tag{7}$$

$$GC = \frac{\%G}{e} = k_p + \frac{k_i}{s} + k_d \cdot s \tag{8}$$

Probably the best criterion of robustness is the sensitivity function determined by the Laplace equation (9):

$$S = \frac{1}{1 + GC \cdot GP \cdot GS \cdot GM} \tag{9}$$

The variable $1/M_s$ can be interpreted as the shortest distance between the open loop Nyquist curve and the critical point (-1,0) shown in the Figure 2. In the same figure other system properties, characterizing the system stability are also shown:

- The gain margin, g_m
- The gain crossover frequency, ω_{gc}
- The phase margin, φ_m
- The sensitivity crossover frequency, ω_{sc}
- The maximum sensitivity crossover frequency, ω_{mc}.
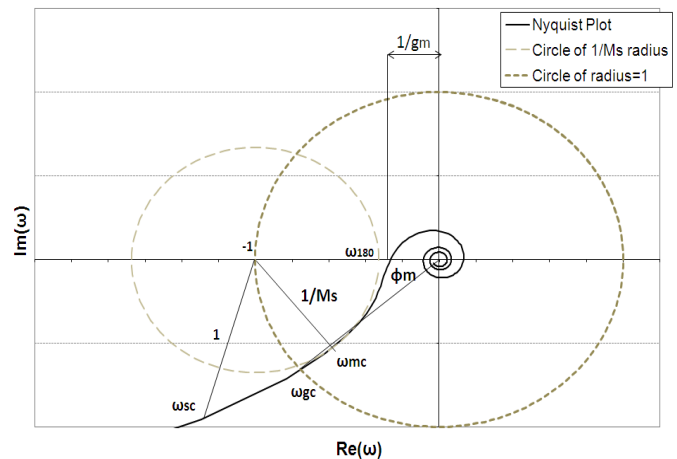
Figure 2. Maximum sensitivity, phase margin and crossover frequencies

Analog PID, k_d=0, Ts=1 min

Digital PID, k_d=0, Ts=1 min

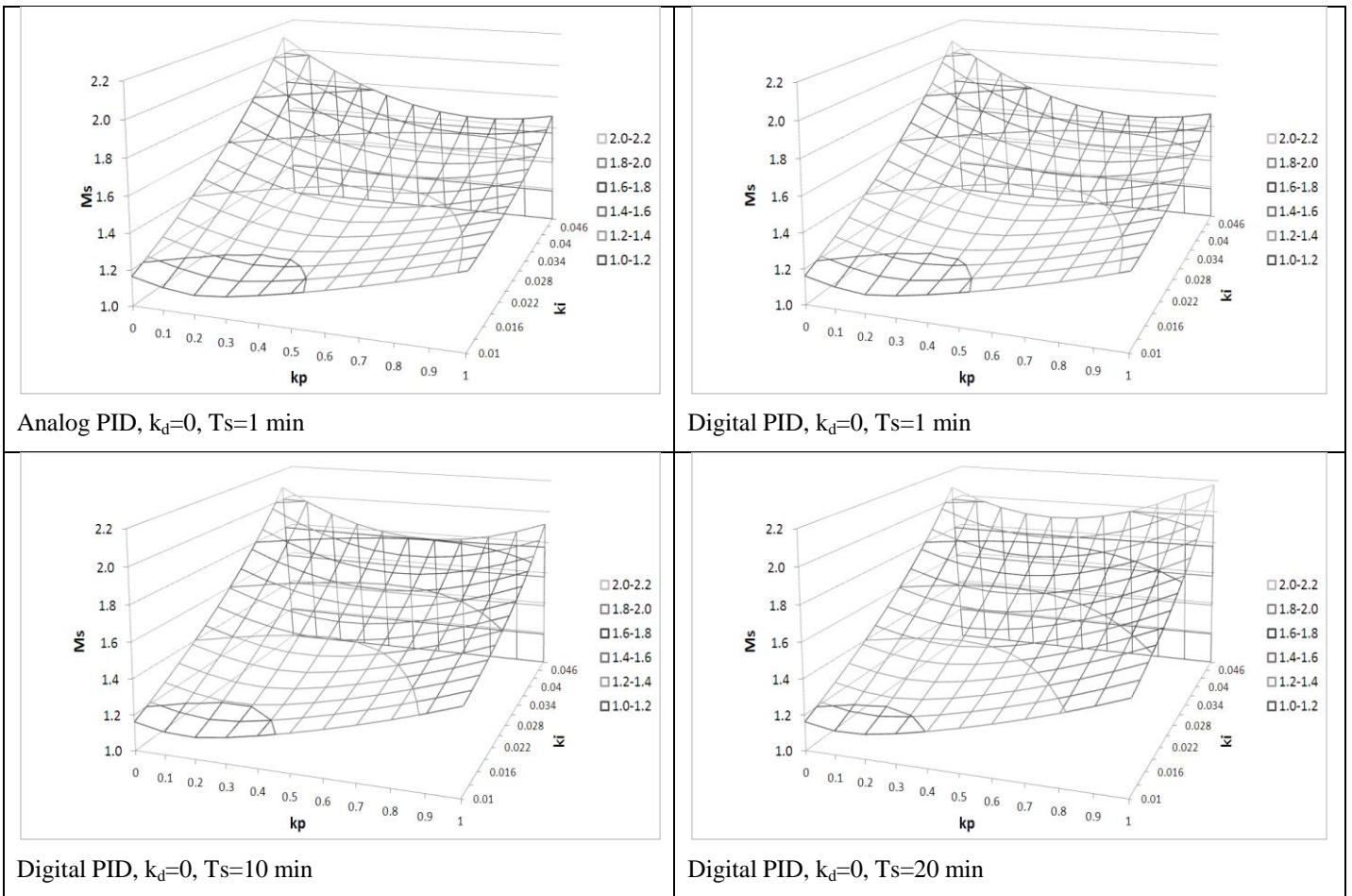Digital PID, k_d=0, Ts=10 min

Digital PID, k_d=0, Ts=20 min

Figure 3. Maximum sensitivity as function of k_p, k_i and T_s for analog and digital PID implementation

III. RESULTS AND DISCUSSION

To investigate the performance of the digital PID in the regulation of the given process, for different values of k_d in %G/%SO3·min, the subsequent area of the other two parameters has been selected: $0 \leq k_p \leq 1$ in %G/%SO3, $0.01 \leq k_i \leq 0.046$ in %G/%SO3/min. The process parameters are the following: k_g=0.42 %G/%SO3, T_0=14 min, T_D=8 min, T_M=20 min. The sampling period T_s is a variable under investigation.

A. Comparisons of Analog and Digital PID

Initially the performances of analog and digital controllers are compared using M_s as a criterion. For k_d=0, the maximum sensitivity as a function of k_p, k_i for analog and digital controllers is shown in Figure 3. For fast sampling frequency, T_s=1 min, the performance of both implementations is the same. As the sampling period augments for the same pair (k_p, k_i) the M_s of digital controller increases too. The above result is very significant: The tuning of a digital PID by using its analog implementation and fast sampling period, leads to an inaccurate M_s if the actual sampling period applied in the process is high enough.

To study the impact of the sampling period on the digital controller performance for various (k_p, k_i, k_d) sets the next procedure has been followed: (i) Two sets (k_p, k_i)$_1$=(0.5, 0.031) and (k_p, k_i)$_2$=(0.2, 0.04) have been selected; (ii) a fast sampling period T_s=1 min and then periods from 10 to 120 min with a step of 10 min; (iii) derivative gains from 0 to 5 with a step of 1; (iv) The M_s values have been computed. The results are shown in Figures 4 and 5. As T_s increases, M_s increases too, by tending in an asymptotic value for each k_d. The impact of k_d is not in one direction: for lower T_s an increase of k_d causes a decrease of M_s, while for high values of T_s, the relation between k_d and M_s is the reverse.
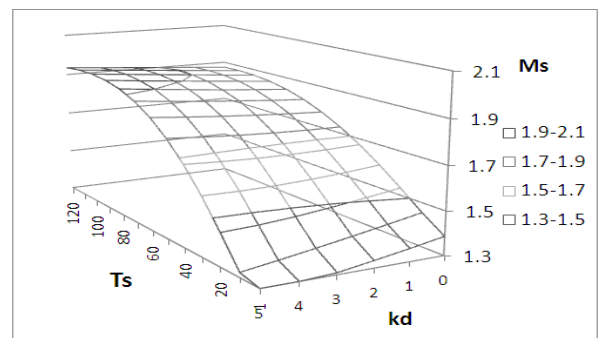

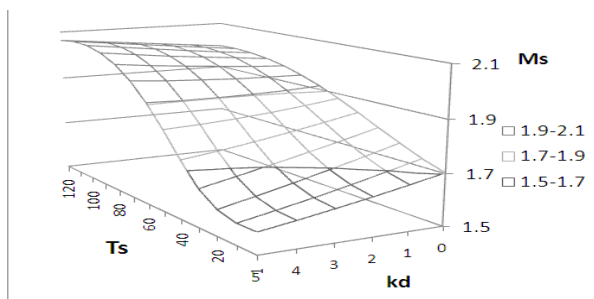Figure 4. Maximum sensitivity as function of T_s, k_d for (k_p,k_i)=(0.5, 0.031)

Figure 5. Maximum sensitivity as function of T_s, k_d for $(k_p,k_i)=(0.2, 0.04)$

B. Digital PID Design Satisfying One Criterion

To investigate the impact of the sampling period on the digital PID design, the maximum sensitivity has been selected as the criterion. The requirement placed in the design is $1.45 \leq M_s \leq 1.55$. PID sets satisfying this constraint have been determined for sampling periods ranging from 1 min to 120 min. The PID regions for $k_d = 0, 2, 4$ are shown in Figures 6, 7, 8 respectively.
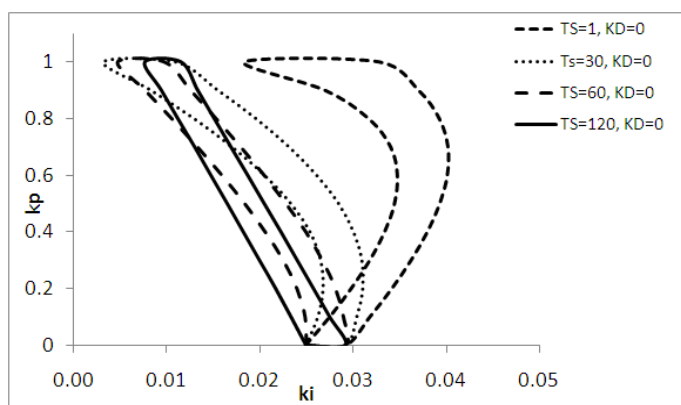
Figure 6. Plots of (k_p, k_i) for $M_s \in [1.45,1.55]$, various T_s and $k_d=0$.
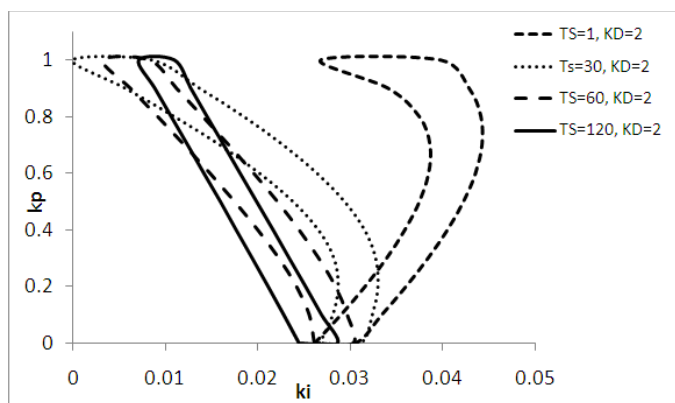

Figure 7. Plots of (k_p, k_i) for $M_s \in [1.45,1.55]$, various T_s and $k_d=2$.

From these Figures the following conclusions can be extracted:

- The magnitude of T_s has strong function on the sets (k_p, k_i, k_d) deriving $M_s \in [1.45, 1.55]$. Generally, when T_s increases, for the same k_p, k_d, the integral gain is moving to lower values.

- For small sampling period – $T_s=1$min- for the same proportional gain, when k_d increases, the integral gain increases too. The reverse happens to $T_s=30$ min

- For higher sampling periods, there is negligible impact of the differential gain on the function between k_p, k_i.
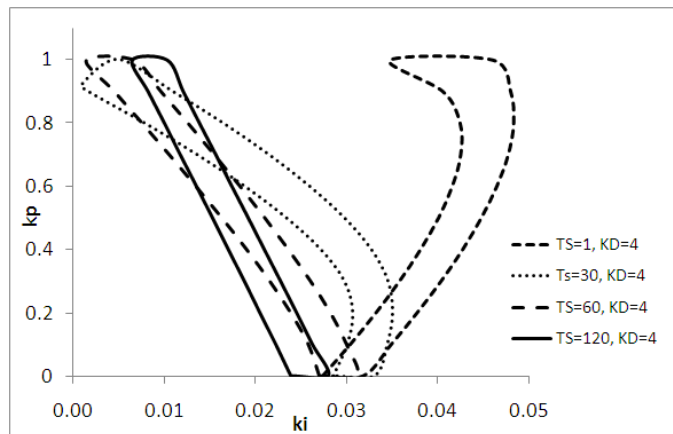

Figure 8. Plots of (k_p, k_i) for $M_s \in [1.45,1.55]$, various T_s and $k_d=4$.

C. Digital PID Design Satisfying Two Criteria

In this case two criteria of performance and robustness have been combined: (a) The maximum sensitivity M_s and (b) the phase margin φ_m. The design is applied for $k_d=0$, $1.45 \leq M_s \leq 1.55$ and $60° \leq \varphi_m \leq 70°$. As in the previous case, T_s varied from 30 min to 120 min. The design is performed for each criterion separately and the corresponding regions (k_p, k_i) are found. The common region satisfies both criteria. The results are shown in Figures 9 to 11. From these Figures it is concluded that for each T_s the common area satisfying both criteria is much narrower compared with the one satisfying each criterion. Additionally this common segment is located to small values of k_p. Generally an increase of T_s results in a decrease of both k_p and k_i.
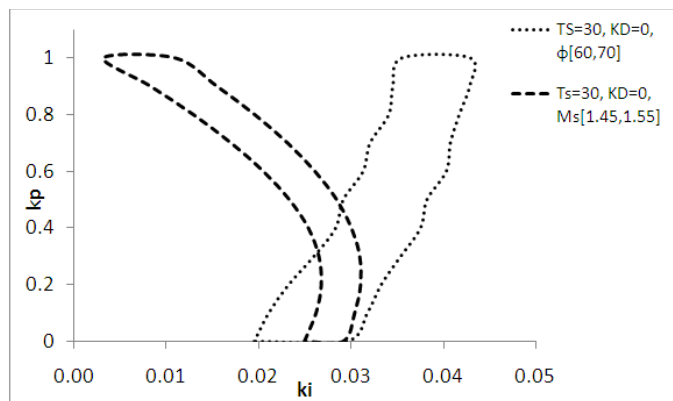

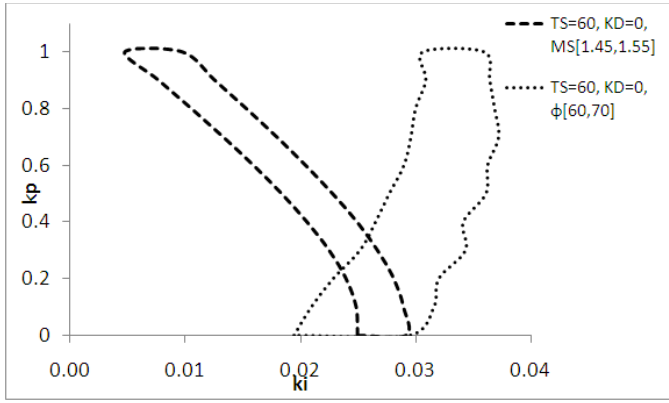Figure 9. Plots of (k_p, k_i) for $T_s = 30$ min and $k_d=0$.

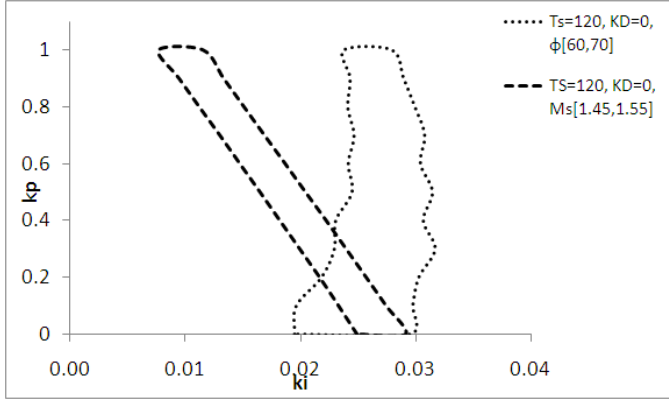Figure 10. Plots of (k_p, k_i) for T_s = 60 min and k_d=0.

Figure 11. Plots of (k_p, k_i) for T_s = 120 min and k_d=0.

D. Other Digital Implementations of the PID Controller

The digital implementation of the analog PID described by [6] includes backward differences of both integral and derivative actions. There are several other digital implementations as Astrom et al. analyzed [1]. Two of them have been investigated as concerns the impact of the sampling period on the controller robustness and performance: (i) forward differences (ii) Tustin approximation which is also known as bilinear or trapezoidal approximation. Both implementations have been applied to the integral term of the PID. The derivative action continued to be approximated by backward difference.

The digital PID with the forward difference of the integral term is described by (9) in time domain. Its Laplace transform is provided by (10):

$$\%G_k - \%G_{k-1} = k_p(e_k - e_{k-1}) + k_i e_{k-1} T_s$$
$$+ \frac{k_d}{T_s}(e_k + e_{k-2} - 2e_k) \tag{9}$$

$$GC = \frac{\%G}{e} = k_p + \frac{k_i \cdot T_s \cdot exp(-T_s \cdot s)}{1 - exp(-T_s \cdot s)} + \frac{k_d}{T_s}$$
$$\cdot \left(1 - exp(-T_s \cdot s)\right) \tag{10}$$

The corresponding expressions of the Tustin approximation are given by (11), (12).

$$\%G_k - \%G_{k-1} = k_p(e_k - e_{k-1}) + k_i \frac{(e_k + e_{k-1})}{2}T_s$$
$$+ \frac{k_d}{T_s}(e_k + e_{k-2} - 2e_k) \tag{11}$$

$$GC = \frac{\%G}{e} = k_p + \frac{k_i \cdot T_s}{2} \cdot \frac{1 + exp(-T_s \cdot s)}{1 - exp(-T_s \cdot s)} + \frac{k_d}{T_s}$$
$$\cdot \left(1 - exp(-T_s \cdot s)\right) \tag{12}$$

Using these two implementations the controller has been designed satisfying the maximum sensitivity criterion, for $1.45 \le M_s \le 1.55$, k_d=0 and T_s ranging from 1 min to 120 min. The results are shown in Figures 12, 13 and the comparison has to be made with the ones shown in Figure 6.
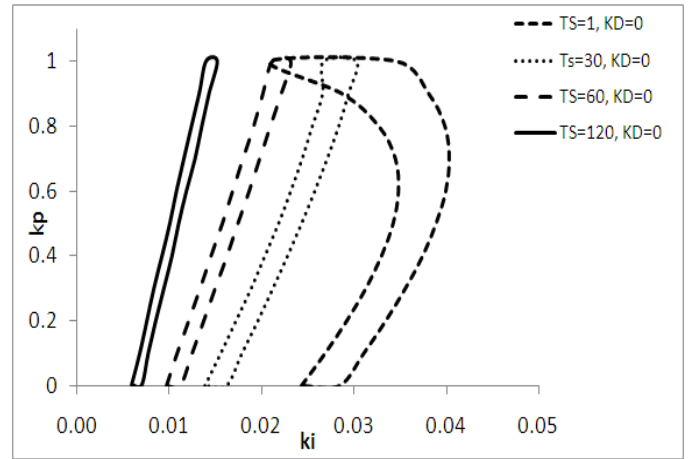

Figure 12. Plots of (k_p, k_i) for forward difference of integral part
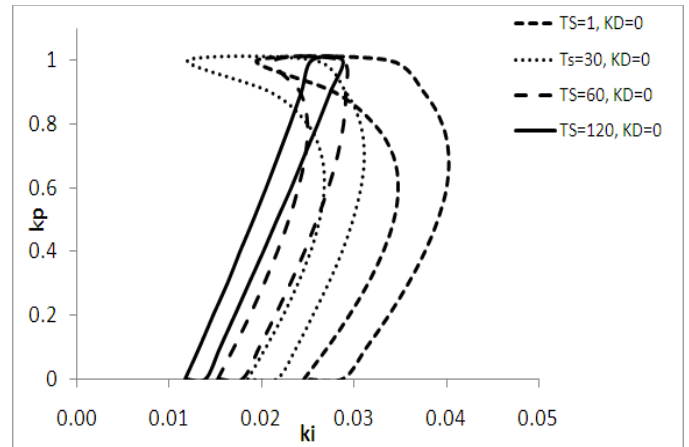

Figure 13. Plots of (k_p, k_i) for Tustin approximation of integral part

The following conclusions can be extracted from Figures 6, 12, 13.

- For high frequency sampling all the three approximations give similar (k_p, k_i) zones of equal M_s

- The general trend is that an increase of T_s, causes a decrease k_i for the same k_p.

- The behavior of the three approximations differs significantly, i.e. if a PID has been parameterized using

a certain digital form, the robustness does not remain the same in case the same gains are applied to another digital PID implementation.

- For $T_s \geq 30$ min, the function between k_p, k_i is not the same for the three approximations. In the backward approximations a rise of k_p, causes a decrease of k_i. The reverse function appears in the forward approximation. The trapezoidal rule shows an intermediate behaviour, between the other two.

CONCUSIONS

An investigation of the impact of the sampling period on the design and parameterization of a digital PID controller has been attempted using the frequency domain. An industrial process of high importance as concerns the quality of the products has been chosen: The regulation of the cement sulphates in the cement mill outlet. Three different approximations have been utilized for the integral action of the controller: The backward, forward and Tustin approximations.

In case the design should satisfy one robustness constraint, the maximum sensitivity, M_s, has been chosen as such. In this case for the same M_s and differential gain, a rise of the sampling period leads to a decrease of the integral gain for all the three approximations. An increase of the differential gain has not strong impact on the (k_p, k_i) region satisfying the M_s requirement when the backward difference is utilized and $T_s \geq 60$ min. The function between k_p and k_i, for the same sampling period differs for the three approximations studied. Thus the way the PID is implemented digitally plays an important role too.

If the design satisfies two criteria simultaneously, maximum sensitivity and phase margin in the current study, then the (k_p, k_i) zone becomes more narrow. An increase of T_s, has not a severe influence on the k_i values but causes a slight drop of the proportional gains.

A further future study using the z-transform and process simulations can extend the results of the current study and generalize them to a variety of other processes.

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