Passive Fault Tolerant Control of motorboat system

Yosra Hammi, Nadia Zanzouri, Mekki Ksouri

Analysis conception and control of systems laboratory , university EL Manar
National School of Engineering of Tunis BP, 37, Le Belvédère, 1002 Tunisia
Yosrahammi@yahoo.fr, Nadia.Zanzouri@enit.rnu.tn, Mekki.ksouri@yahoo.fr

Abstract— A hybrid passive control strategy is developed for a class of hybrid systems modeled by Mixed Logical Dynamical (MLD) approach. It allows to model different operating modes of the system and constraints. We proposed using the MPC for control system .The passive controller is used to take in account the actuator failure and the optimization problem is transformed into a mixed-integer quadratic programming problem (MIQP). The considering fault-tolerance capabilities are developed and discussed. The proposed method is illustrated by a motorboat system.

Index Terms—**Hybrid System, MLD, MPC controller, Passive Tolerant Control, MIQP.**

I. INTRODUCTION

Every engineering system can malfunction because faults in its component ,in modern technological systems, there is a high demand on performance, safety, and reliability of systems. It is desired that if a fault happens, the control system can automatically detect the fault and moderate its effect on the system such that it can continue working while providing an acceptable performance. If an acceptable performance is not possible, it should be able to preserve the overall functionality and stability of the systems while allowing some degradation in the performance of the system. In any case, it is important to avoid dangerous areas to prevent damages to the system. Therefore, Fault Tolerant Control (FTC) is very important for modern technological systems. The area of FTC has attracted a considerable attention in recent years. It is a relatively new idea recently introduced in the research literature, which allows having a control loop that fulfills its objectives when faults in system components (instrumentation, actuators and/or plant) appear. In fact, the fault tolerant systems in literature can be derived into two main groups: active and passive techniques. On the one hand, the passive technique is designed, such that it is robust, within performance range, to fault occurrences. On the other hand, the active fault tolerant system aims at changing the control operation when the fault is detected. These changes can comprise reconfiguration of the controller scheme, modification of controller parameters or alternative set point trajectories.

In recent decades there exists an emerging area of research working in fault tolerant control of hybrid systems, for a survey one can look at [1] [2], [3].

A class of approaches for diagnosis of hybrid systems discrete/temporal abstraction of the continuous dynamics is presented in [4]. In [5], the diagnoser uses a discrete event abstraction of the system and the continuous dynamics information is taken in consideration when it becomes necessary. In [6], the authors use a Petri net abstraction for dealing with continuous behaviors of hybrid systems. In [3] a model based diagnosis method on a hybrid bond graph modelling framework is proposed. Particle filtering methods are another class of methods for diagnosis of hybrid systems [7], [8].

Motivated by different capabilities and applications of the mixed logical dynamical (MLD) modeling of hybrid system, many approaches have been reported in [9], [10], [11].

In this paper Mixed Logical Dynamical (MLD) framework is used for modeling of hybrid system.

This formalism covers important classes of hybrid system. In addition, by using the MLD framework, the optimization problem used for FTC will be transformed to a mixed integer linear or quadratic problem for which there are many efficient solvers.

The main propose consists in embedding the passive fault-tolerant design of controllers based on model predictive control (MPC) within the hybrid system framework. In this context a new methodology is developed. The goal is to verify the fault tolerant MPC control and computational aspects of MLD framework to deal with hybrid systems modeling and control problem.

The paper is organized as follows: in section 2, we presented the MLD formalism. The passive fault tolerant system based on MPC is developed and the optimization problem is transformed into a mixed-integer quadratic programming problem (MIQP) in section 3. An illustrative example of motorboat system and simulation results are presented and discussed in section 4.

II. MIXED LOGICAL DYNAMICAL SYSTEMS

Mixed logical dynamical formalism is a powerful modeling approach in hybrid systems theory. It transforms dynamics, logic and constraints of a complex system into an integrated model. Logical and dynamical constraints are translated to mixed-integer inequalities (see[11] for more details). Mixed logical dynamical modelling allows the state and control inputs to be continuous or discrete. A basic principle of MLD modelling is the interaction between logical and dynamical variables.

It can be proved that $[\delta = 1] \leftrightarrow [f(x) \leq 0]$ is true if

$$\begin{cases} f(x) \leq M - M\delta \\ f(x) \geq \varepsilon + (m - \varepsilon)\delta \end{cases} \tag{1}$$

Where M (m) is maximum (minimum) or an upper (lower) bound of f and ε is a small positive number. This equivalence permits the assignment of binary variables to dynamical constraints which may define the different operation modes of hybrid system. Another useful equivalence that deals with the interaction of logical and dynamical variables is as follows: The equality relation $z = \delta f(x)$ regardless of the relation between δ and $f(x)$ could be translated to the following four mixed integer inequalities:

$$\begin{cases} z \leq M\delta \\ z \geq m\delta \\ z \leq f(x) - m(1 - \delta) \\ z \geq f(x) - M(1 - \delta) \end{cases} \tag{2}$$

The MLD modeling framework is based on the idea of translating logic relations, discrete/logic dynamics, A/D (analog to digital (logic)), D/A conversion and logic constraints into mixed integer linear inequalities. These inequalities are combined with the continuous dynamical part, which are described by linear difference equations. The resulting MLD system is described by the following relations:

$$x(k+1) = Ax(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k)$$
$$y(k) = Cx(k) + D_1 u(k) + D_2 \delta(k) + D_3 z(k) \tag{3}$$
$$E_2 \delta(k) + E_3 z(k) \leq E_1 u(k) + E_4 x(k) + E_5$$

Where:
A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4 and E_5 are matrices of appropriate dimension.and the continuous and binary (mixed) variables x, y and u are respectively the state, inputs and outputs of MLD system which are defined as follows:

$$x = \begin{bmatrix} x_c \\ x_l \end{bmatrix}, x_c \in \Re^{n_c}, x_l \in \{0,1\}^{n_l}, x \in \Re^n, n = n_c + n_l$$

$$y = \begin{bmatrix} y_c \\ y_l \end{bmatrix}, y_c \in \Re^{p_c}, y_l \in \{0,1\}^{p_l}, y \in \Re^p, p = p_c + p_l$$

$$u = \begin{bmatrix} u_c \\ u_l \end{bmatrix}, u_c \in \Re^{m_c}, u_l \in \{0,1\}^{m_l}, u \in \Re^m, m = m_c + m_l$$

$\delta \in \{0,1\}^{r_l}$ are the auxiliary binary variables and

$z \in \Re^{r_c}$ are the auxiliary continuous variables.

The variables δ and z are introduced when translating logic propositions into linear inequalities. There are used to define the relations between continuous and discrete parts
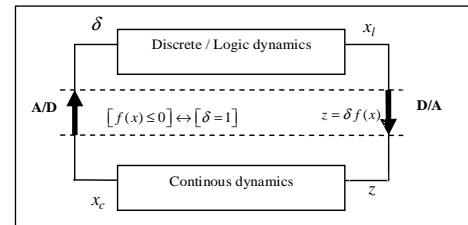

Fig 1. MLD structure

III. PASSIVE PREDICTIVE CONTROL BASED ON MLD FAULT MODEL

A. Predictive control of MLD formalism

In [11], Bemporad and Morari are introduced a model predictive control (MPC) of hybrid MLD system description and a mixed integer linear program solver.

The main idea of MPC based model is to predict the future evolution of the system in a fixed prediction horizon with the measurements of the system.

Consider the MLD system (3) and an equilibrium state, input and output variable (x_{eq}, u_{eq}, y_{eq}) and let (δ_{eq}, z_{eq}) be the corresponding pair of auxiliary variables.

for a MLD system of the form Eqs.(3), consider the following problem:

Problem 1. Given an initial state x_0 and a Horizon prediction N, find (if it exists) the control sequence

$$u_k^* \triangleq \left\{ u^*(0|k),...,u^*(k+N-1|k) \right\}),$$

which transfers the state from x_0 to x_{eq} and minimizes the

cost function, for the MLD model, the optimization has the following form:

$$J(k) = \sum_{j=1}^{N} \left\| u(k+j-1) - u_{eq} \right\|_{Q_u}^2 + \left\| \delta(k+j-1/k) - \delta_{eq} \right\|_{Q_\delta}^2 +$$

$$\left\| z(k+j-1/k) - z_{eq} \right\|_{Q_z}^2 + \left\| x(k+j/k) - x_{eq} \right\|_{Q_x}^2$$

$$+ \left\| y(k+j/k) - y_{eq} \right\|_{Q_y}^2 \qquad (4)$$

Where Q_u, Q_x are positive definite matrices and Q_y, Q_δ, Q_z are nonnegative definite matrices. Furthermore, the MLD system equations have the end-point condition (stability constraint):

$$x(k+N \mid k) = x_{eq} \qquad (5)$$

The optimal MPC minimizes the objective function J(k) subject to constraints (3) and (5) .This is able to stabilize the MLD on desired reference trajectories

Problem 1 can be solved as a mixed-integer quadratic programming (MIQP) problem. By using the Eq. (3), for time-invariant systems we have the solution formula:

$$x(k+j) = A^j x(k) + \sum_{i=0}^{j-1} (A^i (B_1 u(k+j-1-i)$$

$$+ B_2 \delta(k+j-1-i) + B_3 z(k+j-1-i))) \qquad (6)$$

by plugging Eq.(3) and Eqs. (6)

$$x(k+j) = A^j x(k) + (A^{j-1} B_1 ... AB_1 \; B_1 \; 0...0)U$$

$$+ (A^{j-1} B_2 ... AB_2 \; B_2 \; 0...0)\Delta$$

$$+ (A^{j-1} B_3 ... AB_3 \; B_3 \; 0...0)Z$$

We can define the vectors:

$$U = \begin{bmatrix} u(0) \\ \vdots \\ u(k-N) \end{bmatrix}, \; \Delta = \begin{bmatrix} \delta(0) \\ \vdots \\ \delta(k-N) \end{bmatrix}, \; Z = \begin{bmatrix} z(0) \\ \vdots \\ z(k-N) \end{bmatrix}$$

we can obtain the following equivalent formulation

$$\min_V \frac{1}{2} V^T H V + f^T V \qquad (7)$$

$$A_{inq} V \le B_{inq}$$

where matrix H, A_{inq} et B_{inq} are suitable defined the

the vector $V = \begin{bmatrix} U \\ \Delta \\ Z \end{bmatrix}$ contains both real-valued and integer-valued components.

used during optimal procedure. Consequently, we can get the optimal control sequence by solving the MIQP problem [11].

B. Mixed logical dynamical fault proposed

In this paper, we proposed to use the MLD formalism to model actuator failure. The key idea is to introduce the auxiliary variables z_f, δ_f to represent the fault on the system as follow:

$$u = u - z_f$$

$$z_f = \delta_f \cdot u_{fc} \qquad \delta_f \in \{0,1\} \qquad (8)$$

$$u_{fc} = \alpha \, u$$

With α : Coefficients of failure.

If the fault is detected, the passive fault tolerant control technique consists to reconfigure the model predictive controller for the faulty system by changing the constraints to reflect the identified fault. Therefore, the MPC problem should be modified accordingly by replacing the MLD by MLD fauts.

We can define the MLD fault as follow:

$$x(k+1) = Ax(k) + B_1 u(k) + B_2 \tilde{\delta}(k) + B_3 \tilde{z}(k)$$

$$y(k) = Cx(k) + D_1 u(k) + D_2 \tilde{\delta}(k) + D_3 \tilde{z}(k)$$

$$\tilde{E}_2 \tilde{\delta}(k) + \tilde{E}_3 \tilde{z}(k) \le \tilde{E}_1 u(\tilde{k}) + \tilde{E}_4 x(k) + \tilde{E}_5 \qquad (9)$$

With $\tilde{\delta} = \begin{bmatrix} \delta \\ \delta_f \end{bmatrix}$, $\tilde{z} = \begin{bmatrix} z \\ z_f \end{bmatrix}$

$\tilde{E}_{i,i=1,...,5}$ are the extended matrices involved the faulty mode

In faulty mode, the criteria $J(k)$ is modified and we note in this case another optimization problem MIQP under constraints (7). The optimization problem is similar to a minimum time optimal control problem.

Given a normal model and faulty models of the system subject to the faults another equilibrium state must be found for the faulty system. In fact, if the fault is detected, the passive fault tolerant control technique consists to reconfigure the model predictive controller for the faulty system by changing the constraints to reflect the identified fault. We carry on solving the MPC optimization (4), with the corresponding faults information (7).
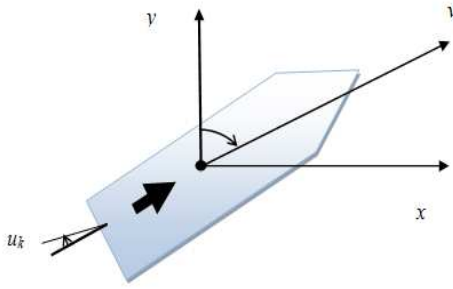
IV. ILLUSTRATIVE SYSTEM

Fig. 2. Motorboat scheme.

In this section, the proposed method is tested using motorboat figure 2, presented in [12]. The model is a simplified version of the submarine model developed in [13]. We adopted MLD formalism for modeling and MPC controller for reconfiguration strategy

The model of the motorboat is nonlinear and is presented as:

$$\begin{bmatrix} \dot{v} \\ \dot{\vartheta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -v + u_m \\ 0.15 * v * u_k \\ v * \cos\vartheta \\ -v * \sin\vartheta \end{bmatrix} \tag{10}$$

where v is the sailing speed, ϑ is the yaw angle, x and y represent the position of the motorboat, u_m is the force produced by the motor and u_k is the rudder position.

The system can be separated into two parts, the first representing the dynamics and the second representing

The position of the motorboat. In this work interested by the first part. Considering the fixed input

$$u_{k1} = -10, \ u_{k2} = 0, u_{k3} = -10$$

The control system can drive the motorboat using discrete inputs. $u_m \in \{0,1\}$ and $u_k \in \{-10,0,10\}$.

if $u_m = 0$, u_k can be -10 or 0 or 10 of same if $u_m = 1$, u_k can be -10 or 0 or 10

TABLE 1 Different possibility of discrete inputs

Inputs	1	2	3	4	5	6
u_m	0	0	0	1	1	1
u_k	-10	0	10	-10	0	10

From Table 1 and using the discretization Eulier (Ts = 0.2s), the system can evolve in several modes of operation. Indeed, each mode depends on the discrete inputs which are represented by the following equations:

$$\begin{bmatrix} v(k+1) \\ \vartheta(k+1) \end{bmatrix} = \begin{bmatrix} 0.8187 & 0 \\ -0.3 & 1 \end{bmatrix}\begin{bmatrix} v(k) \\ \vartheta(k) \end{bmatrix} + \begin{bmatrix} 0.1813 \\ 0 \end{bmatrix} u_m(k) : u_k = -10$$

$$\begin{bmatrix} v(k+1) \\ \vartheta(k+1) \end{bmatrix} = \begin{bmatrix} 0.8187 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v(k) \\ \vartheta(k) \end{bmatrix} + \begin{bmatrix} 0.1813 \\ 0 \end{bmatrix} u_m(k) : u_k = 0$$

$$\begin{bmatrix} v(k+1) \\ \vartheta(k+1) \end{bmatrix} = \begin{bmatrix} 0.8187 & 0 \\ 0.3 & 1 \end{bmatrix}\begin{bmatrix} v(k) \\ \vartheta(k) \end{bmatrix} + \begin{bmatrix} 0.1813 \\ 0 \end{bmatrix} u_m(k) : u_k = 10$$

$$\tag{11}$$

Can be presented as following:

$$\begin{bmatrix} v(k+1) \\ \vartheta(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} z(k) \tag{12}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} z(k) \le \begin{bmatrix} -1.8187 & 0 & 0 & 0 \\ -1.4561 & 0 & 0 & 0 \\ 1.4561 & 0 & 0 & 0 \\ 1.8187 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & -10.6 & 0 & 0 \\ 0 & 10.6 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -10.6 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 10.6 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix} u(k) + \begin{bmatrix} -0.8187 & 0 \\ 0.8187 & 0 \\ -0.8187 & 0 \\ 0.8187 & 0 \\ 0.3000 & -1.0000 \\ -0.3000 & 1.0000 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1.0000 \\ 0 & 1.0000 \\ 0 & 0 \\ 0 & 0 \\ -0.3 & -1 \\ 0.3 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1.6374 \\ 1.6374 \\ 0 \\ 0 \\ 10. \\ 10.6 \\ 0 \\ 0 \\ 10 \\ 10 \\ 0 \\ 0 \\ 10.6 \\ 10 \\ 0 \\ 0 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

With

$$z_1(k) = 0.8187v(k) \text{ or } 0.8187v(k) + 0.1813$$
$$z_2(k) = \vartheta(k) - 0.3v(k)$$
$$z_3(k) = \vartheta(k)$$
$$z_4(k) = \vartheta(k) + 0.3v(k) \tag{13}$$

$$\begin{bmatrix} v(k+1) \\ \vartheta(k+1) \end{bmatrix} = \begin{bmatrix} z_1(k) \\ z_2(k) + z_3(k) + z_4(k) \end{bmatrix}$$

For modeling motorboat system with MLD model we use auxiliary variable where z(k) is an auxiliary continuous variable representing v and ϑ in different operating modes.

Moreover, the transformation of the hybrid system equations into the MLD form requires the application of a set of given rules. A higher level language and associated compiler HYSDEL [14] (see the Appendix) are used here to avoid the tedious procedure of deriving the MLD form by hand. Given the MLD model, the scenarios are simulated using the Hybrid Toolbox for Matlab [15] Matrices $Ei_{1,\ldots,5}$ are defined by the MLD transformation procedure.

In order to show the capability of handling a hybrid control problem, we have simulated the system motorboat. The results of the predictive control described by equation (4) are shown in Figure 3.

With N (horizon of prediction) =3 and Ts was equal to the sampling period 0.2.

The discrete inputs of the motorboat example can be represented as shown in Figure 4. The aim of the control is to drive the motorboat as fast as possible according to the path defined.

We Remarque the both states of motorboat succeed to reach reference imposed by the user.

The switching inputs required to keep track of the reference, the states of the system remain close to the reference, and this objective is reached by a switching u_{k1}, u_{k2}, u_{k3} Figure 4
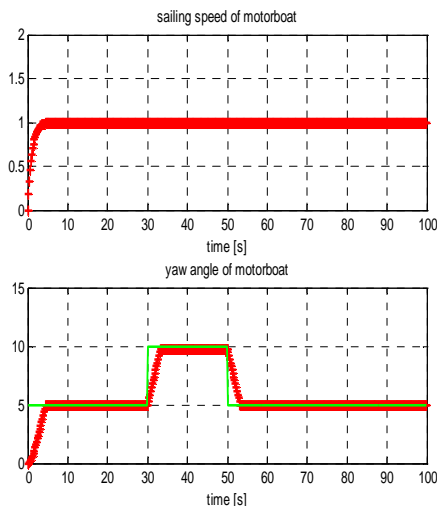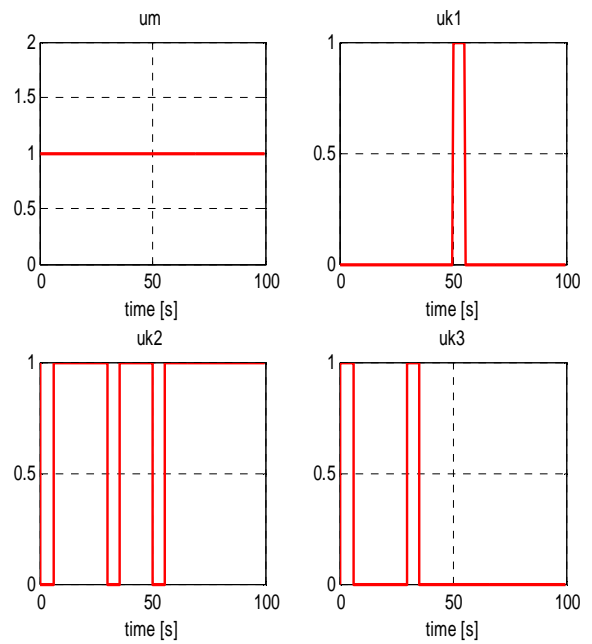

Fig 4. The inputs of motorboat system

We consider a actuator fault affecting the speed of the motorboat (bias). This is maintained throughout the operation of the system
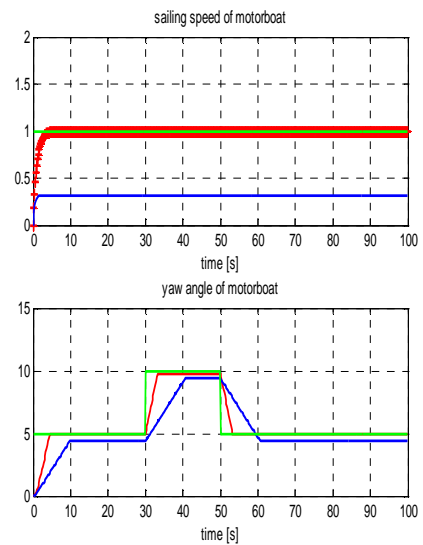
.

Fig 5. The states of normal and faulty mode

We present in the figure 5 p the states of the normal and faulty mode fault tolerant MPC control. Despite the failure, we note that state succeed to reach the trajectory ϑ (the yaw angle) imposed by the user.

Also we note the system tolerant ensures the continuation of desired trajectory after some delay compared to normal mode but with different speed.

Fig 3. The states of motorboat system

In fact, if the fault is affected, the passive fault tolerant control technique consists to reconfigure the model predictive controller for the faulty system by changing the constraints to reflect the identified fault. Therefore, the MPC controller updates this constraint.

V. CONCLUSION

In this paper, we have developed and discussed a methodology of passive fault tolerant control based on Mixed Logical and Dynamical (MLD) approach as model of the hybrid systems and Model Predictive Control (MPC) for control strategy. The approach is applied to motorboat system. The obtained results are very interesting and prove the efficiently of the fault tolerant MPC control.

Appendix

Hysdel code

```
[um=1]  ↔  [um=1]
[uk1=1] ↔ [uk=-10]
[uk2=1] ↔ [uk=0]
[uk3=1] ↔ [uk=10]

SYSTEM BATEAU{
INTERFACE {
  STATE {
  REAL v [0,2];
  REAL th [-10,10];
}
INPUT {
   BOOL um,uk1,uk2,uk3;
}
PARAMETER {
   REAL v1= 0.1813;
   REAL v2=0.8187;
}
}
IMPLEMENTATION {
  AUX {
   REAL vp;
   REAL thp1,thp2,thp3;}
  DA {
    Z1={IF um THEN v2*v+v1 ELSE v2*v};
    Z2={IF uk1 THEN th-0.3*v};
    Z3={IF uk2 THEN th+0*v};
    Z4={IF uk3 THEN th+0.3*v};}

CONTINUOUS {
   v=Z1;
   th=Z2+Z3+Z4;}
MUST {

(uk1&~uk2&~uk3)|(~uk1&uk2&~uk3)|
(~uk1&~uk2&uk3);}
}}
```

REFERENCES

[1] Y. Hao, B. Jiang and V. Cocquempot, Fault Tolerant Control Design for Hybrid Systems Series: Lecture Notes in Control and Information Sciences, (LNCIS). Editio Springer Verlag Vol. 397, (2010).

[2] D. Choe, S. L. Campbell, and R. Nikoukhah, "Optimal piecewise-constant signal design for active fault detection," International Journal of Control, vol. 82, no. 1, pp. 130–146, 2009.

[3] S. Narasimhan and G. Biswas, "Model-based diagnosis of hybrid systems," IEEE transactions on man and cybernetics, vol. 37, no. 3, pp. 347–361, 2007.

[4] J.Lunze, "Diagnosis of quantized systems by means of timed discrete-event representations," in Hybrid systems: Computation and Control, ser. Lecture Notes in Computer Science, N. Lynch and B. Krogh, Eds., vol. 1790. new york: Springer, pp. 258–27, 2000.

[5] R. Mohammadi, S. Hashtrudi-Zad, and K. Khorasani, "A hybrid architecture for diagnosis in hybrid systems with applications to spacecraft propulsion system," in IEEE International Conference on Systems, Man and Cybernetics, 2007, pp. 3184–3190.7

[6] F. Zhao, X. Koutsoukos, H. Haussecker, J. Reich, and P. Cheung, "Monitoring and fault diagnosis of hybrid systems," IEEE Transactions on Systems, Man, and Cybernetics vol. 6, pp. 1225–1240, 2005.

[7] X. Koutsoukos, J. Kurien, and F. Zhao, "Estimation of distributed hybrid systems using particle filtering methods," in Hybrid systems: Computation and Control, ser. Lecture Notes in Computer Science, vol. 2623. Springer, pp. 298–313, 2003.

[8] M. Hofbaur and B. Williams, "Mode estimation of probabilistic hybrid systems," in Hybrid systems: Computation and Control, ser. Lecture Notes in Computer Science, vol. 3927. Springer, pp.253–266, 2002.

[9] G. Ferrari-Trecate.,D. Mignone, and, M. Morari. Moving horizon estimation for hybrid systems. IEEE Transactions on Automatic Control 47 (10), pp 1663-1676. 2002.

[10] D. Mignone, A. Bemporad, and M. Morari, A Framework for Control, Fault Detection, State Estimation, and Verification of Hybrid Systems Proceedings of the American Control Conference, San Diego, pp. 13 Transactions on Automatic Control 47 (10), 1663-1676 2002.

[11] A. Bemporad and, M. Morari, Control of System Integrating Logic, Dynamics, and Constraints, Automatica 35(3), p. 407-427, (1999).

[12] A. Stiver, P. J .Antsaklis and M. D Lemmon, Hybrid control system design based on neural invariants. In Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, LA, USA, pp. 1455–1460(1995)

[13] B.Potocnik, G. Music and B. Zupancic, Model predictive control of discrete-time hybrid systems with discrete inputs ISA Transactions vol. 44 , pp.199–211, (2005).

[14] F.Torrisi and, A. Bemporad, Hysdel - a tool for generating computational hybrid models for analysis and synthesis problems. IEEE Transaction Control. Systems. Technology. 12(2), 235–249, (2004).

[15] A. Bemporad. Hybrid Toolbox for real time applications, User Guide, July (2012).