Heat and Mass Transfer for a Non-Newtonian Fluid Flow along a Surface Embedded in a Porous Medium with Uniform Wall Heat and Mass Fluxes and Heat Generation or Absorption

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***Abstract*—**The problem of steady, laminar, double-diffusive mixed convective flow of a non-Newtonian power-law fluid past a vertical semi-infinite permeable surface embedded in a porous medium with uniform heat and mass fluxes in the presence of heat generation or absorption effects. A mixed convection parameter for the entire range of free-forced-mixed convection is employed and a set of non-similar equations are obtained. These equations are solved numerically by an efficient implicit, iterative, finite-difference method. The obtained results are checked against previously published work for special cases of the problem and are found to be in good agreement. A parametric study illustrating the influence of the concentration to thermal buoyancy ratio, power-law fluid viscosity index, mixed convection parameter, suction or injection parameter, dimensionless heat generation or absorption parameter and the Lewis number on the local Nusselt and the Sherwood numbers is conducted. The obtained results are shown graphically and the physical aspects of the problem are discussed.

***Keywords*—**Mixed convection, porous media, suction or injection, heat generation or absorption, numerical solution, non-Newtonian fluid.

I. INTRODUCTION

CONVECTION heat transfer from vertical surfaces embedded in porous media has been the subject of many investigations. This is due fact that these flows have many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, groundwater pollution, and underground energy transport. Cheng and Minkowycz [1] have presented similarity solutions for free thermal convection from a vertical plate in a fluid-saturated porous medium. Ranganathan and Viskanta [2] have considered mixed convection boundary layer flow along a vertical surface in a porous medium. Nakayama and Koyama [3] have suggested similarity transformations for pure, combined and forced convection in Darcian and non-Darcian porous media. Lai [4] has investigated coupled heat and mass transfer by mixed convection from an isothermal vertical plate in a porous medium. Hsieh et al. [5] have presented non-similar solutions for combined convection in porous media. All of the above references considered Newtonian fluids.

A number of industrially important fluids such as molten plastics, polymers, pulps, foods and slurries and fossil fuels which may saturate underground beds display non-Newtonian fluid behavior. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Chen and Chen [6] have presented similarity solutions for free convection of non-Newtonian fluids over vertical surfaces in porous media. Mehta and Rao [7] have investigated buoyancy-induced flow of non-Newtonian fluids over a non-isothermal horizontal plate embedded in a porous medium. Also, Mehta and Rao [8] have analyzed buoyancy-induced flow of non-Newtonian fluids in a porous medium past a vertical plate with non-uniform surface heat flux. In a series of papers, Gorla and co-workers [9-14] have studied mixed convection in non-Newtonian fluids along horizontal and vertical plates in porous media under various thermal boundary conditions. Jumah and Mujumdar [15] have considered free convection heat and mass transfer of non-Newtonian power-law fluids with yield stress from a vertical flat plate in saturated porous media. Recently, Chamkha and Al-Humoud [16] studied mixed convection heat and mass transfer of non-Newtonian fluids from a permeable surface embedded in a porous medium under uniform surface temperature and concentration species.

The effects of fluid wall suction or injection the flow and heat transfer characteristics along vertical semi-infinite plates have been investigated by several authors (Cheng [17], Lai and Kulacki [18,19], Minkowycz et al. [20] and Hooper et al. [21]). Some of these studies have reported similarity solutions (Cheng [17] and Lai and Kulacki [18,19]) while others have obtained non-similar solutions (Minkowycz, et al. [20] and Hooper et al. [21]). Lai and Kulacki [18,19] have reported similarity solutions for mixed convection flow over

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horizontal and inclined plates embedded in fluid-saturated porous media in the presence of surface mass flux. On the other hand, Minkowycz et al. [20] have discussed the effect of surface mass transfer on buoyancy-induced Darcian flow adjacent to a horizontal surface using non-similarity solutions. Also, Hooper et al. [21] have considered the problem of non-similar mixed convection flow along an isothermal vertical plate in porous media with uniform surface suction or injection and introduced a single parameter for the entire regime of free-forced-mixed convection. Their non-similar variable represented the effect of suction or injection at the wall.

The objective of this paper is to consider double-diffusive mixed convection for a non-Newtonian power-law fluid flow past a permeable vertical surface embedded in a fluid-saturated porous medium in the presence of suction or injection and heat generation or absorption effects under uniform heat and mass fluxes.

II. Problem Formulation

Consider steady mixed convective flow of a non-Newtonian power-law fluid past a permeable semi-infinite vertical surface embedded in a porous medium with constant heat and mass fluxes. The power-law model of Ostwald-de-Waele which is adequate for many non-Newtonian fluids is considered in the present work. Uniform suction or injection with speed v_o is imposed at the surface boundary. The porous medium is assumed to be uniform, isotropic and in local thermal equilibrium with the fluid. All fluid properties are assumed to be constant. Under the Boussinesq and boundary-layer approximations, the governing equations for this problem can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u^n=U_\infty^n+\frac{K}{\mu}\rho g\left[\beta_T(T-T_\infty)+\beta_c(c-c_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_e\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \tag{4}$$

where x and y denote the vertical and horizontal directions, respectively. u, v, T and c are the x- and y-components of velocity, temperature and concentration, respectively. ρ, μ, n, c_p, Q_0 and D are the fluid density, consistency index for viscosity, power-law fluid viscosity index, specific heat at constant pressure, heat generation or absorption coefficient and mass diffusion coefficient, respectively. K and α_e are the porous medium modified permeability and effective thermal diffusivity, respectively. β_T, β_c, U_∞, T_∞ and c_∞ are the thermal expansion coefficient, concentration expansion coefficient and the free stream velocity, temperature and concentration, respectively.

The modified permeability of the porous medium K for flows of non-Newtonian power-law fluids is given by:

$$K=\frac{1}{2C_t}\left(\frac{n\varepsilon}{3n+1}\right)^n\left(\frac{50k^*}{3\varepsilon}\right)^{(n+1)/2} \tag{5}$$

where

$$k^*=\frac{\varepsilon^3 d^2}{150(1-\varepsilon)^2} \tag{6}$$

$$C_t=\begin{cases}\dfrac{25}{12},\ \text{Christopher and Middleman [22]}\\[2ex] \dfrac{2}{3}\left(\dfrac{8n}{9n+3}\right)\left(\dfrac{10n-3}{6n+1}\right)\left(\dfrac{75}{16}\right)^{3(10n-3)/(10n+11)}\\ \text{Dharmadhikari and Kale [23]}\end{cases}, \tag{7}$$

where ε and d is the porosity and the particle diameter of the packed-bed porous medium.

The boundary conditions suggested by the physics of the problem are given by

$$\begin{aligned}&v(x,0)=v_o,\ q_w=\frac{\partial T(x,0)}{\partial x},\ m_w=\frac{\partial c(x,0)}{\partial x}\\ &u(x,\infty)=U_\infty,\ T(x,\infty)=T_\infty,\ c(x,\infty)=c_\infty\end{aligned} \tag{8}$$

where v_o, q_w and m_w are the constant wall normal velocity, uniform wall heat flux and the uniform wall mass flux, respectively.

It is convenient to transform the governing equations into a non-similar dimensionless form which can be suitable for solution as an initial-value problem. This can be done by introducing the stream function such that

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{9}$$

and using

$$\eta=\frac{y}{x}\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)}+Ra_x^{\left(\frac{n}{2n+1}\right)}\right] \tag{10a}$$

$$\xi=\frac{v_0 x}{\alpha_e}\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)}+Ra_x^{\left(\frac{n}{2n+1}\right)}\right]^{-1} \tag{10b}$$

$$\psi=\alpha_e\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)}+Ra_x^{\left(\frac{n}{2n+1}\right)}\right]\cdot f(\xi,\eta) \tag{10c}$$

$$\theta=\frac{(T-T_\infty)\left(Pe_x^{\left(\frac{n+1}{2n+1}\right)}+Ra_x^{\left(\frac{n}{2n+1}\right)}\right)}{\dfrac{q_w x}{\kappa}} \tag{10d}$$

$$C=\frac{(c-c_\infty)\left(Pe_x^{\left(\frac{n+1}{2n+1}\right)}+Ra_x^{\left(\frac{n}{2n+1}\right)}\right)}{\dfrac{m_w x}{D}} \tag{10e}$$

where $Pe_x=U_\infty x/\alpha_e$ and $Ra_x=(x/\alpha_e)[\rho g\beta_T q_w K/(\kappa\mu)]^{1/n}$ are the local Peclet and modified Rayleigh numbers, respectively and κ is the porous medium effective thermal conductivity.

Substituting Eqs. (9) and (10) into Eqs. (1) through (5) produces the following non-similar equations:

$$nf'^{n-1}f''=(1-\chi)^{2n+1}(\theta'+NC') \tag{11}$$

$$\theta''+\frac{n+1}{2n+1}f\theta'+\xi^2\phi=\frac{n}{2n+1}\xi(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}) \tag{12}$$

$$\frac{1}{Le}C''+\frac{n+1}{2n+1}fC'=\frac{n}{2n+1}\xi(f'\frac{\partial C}{\partial\xi}-C'\frac{\partial f}{\partial\xi}) \tag{13}$$

$$f(\xi,0)+\xi\frac{\partial f}{\partial\xi}(\xi,0)=-2\xi,\theta'(\xi,\infty)=-1,C'(\xi,0)=-1$$

$$f(\xi,\infty)=Pe_x^{-\frac{1}{2n+1}}\chi^2,\ \theta(\xi,\infty)=0,\ C(\xi,\infty)=0 \tag{14}$$

where

$$Le=\frac{\alpha_e}{D},\ N=\frac{\beta_c m_w/D}{\beta_T q_w/\kappa},$$

$$\phi=\frac{Q_0\alpha_e}{\rho c_p v_0^2},\ \chi=\left[1+\frac{Ra_x^{\left(\frac{n}{2n+1}\right)}}{Pe_x^{\left(\frac{n+1}{2n+1}\right)}}\right]^{-1} \tag{15}$$

are the Lewis number, concentration to thermal buoyancy ratio, dimensionless heat generation or absorption coefficient ($\phi<0$ corresponds to heat absorption and $\phi>0$ corresponds to heat generation) and the mixed convection parameter, respectively. It should be noted that $\chi = 0$ ($Pe_x = 0$) corresponds to pure free convection while $\chi = 1$ ($Ra_x = 0$) corresponds to pure forced convection. The entire regime of mixed convection corresponds to values of χ between 0 and 1.

Of special significance for this problem are the local Nusselt and Sherwood numbers. These physical quantities can be defined as

$$Nu_x=\frac{hx}{\kappa}=[Pe_x^{(n+1)/(2n+1)}+Ra_x^{n/(2n+1)}]/\theta(\xi,0) \tag{16a}$$

$$Sh_x=\frac{h_m x}{D}=[Pe_x^{(n+1)/(2n+1)}+Ra_x^{n/(2n+1)}]/C(\xi,0) \tag{16b}$$

$$h=\frac{q_w}{T_w-T_\infty},h_m=\frac{m_w}{c_w-c_\infty} \tag{17}$$

III. NUMERICAL METHOD AND VALIDATION

Equations (11) through (14) represent an initial-value problem with ξ playing the role of time. This non-linear problem can not be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. The implicit, tri-diagonal finite-difference method similar to that discussed by Blottner [24] has proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present work.

All first-order derivatives with respect to ξ are replaced by two-point backward-difference formulae when marching in the positive ξ direction and by two-point forward-difference formulae when marching in the negative ξ direction. Then, all second-order differential equations in η are discretized using three-point central difference quotients. This discretization process produces a tri-diagonal set of algebraic equations at each line of constant ξ which is readily solved by the well known Thomas algorithm (see Blottner [24]). During the solution, iteration is employed to deal with the non-linearities of the governing differential equations. The problem is solved line by line starting with line ξ=0 where similarity equations are solved to obtain the initial profiles of velocity, temperature and concentration and marching forward (or backward) in ξ until the desired line of constant ξ is reached. Variable step sizes in the η direction with $\Delta\eta_1 = 0.001$ and a growth factor G = 1.04 such that $\Delta\eta_n = G\Delta\eta_{n-1}$ and constant step sizes in the ξ direction with $\Delta\xi = 0.01$ are employed. These step sizes are arrived at after many numerical experimentations performed to assess grid independence. The convergence criterion employed in the present work is based on the difference between the current and the previous iterations. When this difference reached 10^{-5} for all points in the η directions, the solution was assumed converged and the iteration process was terminated.

Tables 1 and 2 present a comparison of $-\theta'(\xi,0)$ at selected values of ξ and χ between the results of the present work and those reported earlier by Hooper et al. [21] for n=1 and N=0 for the case of uniform wall temperature since no appropriate results were found for the case of uniform heat flux. It is clear from this comparison that a good agreement between the results exists. This lends confidence in the correctness of the numerical results to be reported subsequently. It should be noted that in Table 2, the value of $-\theta'(\xi,0)$ at ξ = -2 and χ =1 seems to be in error or a typo as this value cannot be 1.0502.

Table 1. Values of $-\theta'(\xi,0)$ at selected values of ξ and χ for n=1 and N=0. (Present work)

χ	ξ=-2.0	ξ=0.0	ξ=1.0	ξ=2.0
0.0	1.99894	0.44401	0.14240	0.03408
0.2	1.99762	0.37339	0.09140	0.01431
0.4	1.99757	0.35071	0.06997	0.00754
0.5	1.99824	0.36045	0.07097	0.00725
0.6	2.00066	0.38338	0.08029	0.00884
0.8	2.01485	0.46044	0.12471	0.02055
1.0	2.04971	0.56433	0.19979	0.05036

Table 2. Values of $-\theta'(\xi,0)$ at selected values of ξ and χ for n=1 and N=0. (Hooper et al. [21])

χ	ξ=-2.0	ξ=0.0	ξ=1.0	ξ=2.0
0.0	2.0015	0.4437	0.1417	0.0335
0.2	2.0003	0.3732	0.0907	0.0139
0.4	2.0005	0.3504	0.0693	0.0072
0.5	2.0016	0.3603	0.0704	0.0069
0.6	2.0042	0.3832	0.0797	0.0085
0.8	2.0185	0.4602	0.1242	0.0201
1.0	1.0502	0.5642	0.1996	0.0502

IV. RESULTS AND DISCUSSION

Figures 1 through 3 display representative velocity, temperature and concentration (f', θ and C) profiles for two values of the transformed suction or injection parameter ξ and three distinct values of the buoyancy ratio N and power-law fluid index n=0.5 (shear thinning or pseudo-plastic fluid), respectively. Unlike the case of constant wall temperature and concentration (see Chamkha and Al-Humoud [16]) , increases in the value of N cause less induced flow close to the plate surface and more flow away from the wall. This behavior in the flow velocity takes place with increases in the fluid temperature and species concentration as seen from Figs. 2 and 3. Also, as ξ increases, all of the velocity, temperature and concentration along with their boundary layers are predicted to increase.

Figures 4 through 9 illustrate the influence of the buoyancy ratio N and the transformed suction or injection parameter ξ in the full range of the mixed convection parameter $0 \le \chi \le 1$ on the local Nusselt number $[1/\theta(\xi,0)]$ and the local Sherwood number $[1/C(\xi,0)]$ for power-law fluid viscosity indices n=0.5 (shear-thinning or pseudo-plastic fluid), n=1.0 (Newtonian fluid) and n=1.5 (shear-thickening or dilatant fluid), respectively. As mentioned before, in general, for uniform surface heat and mass fluxes, increases in the value of N have the tendency to cause less induced flow close to the surface and more flow far down stream. This behavior in the flow velocity is accompanied by increases in the fluid temperature and concentration species as well as increases in the thermal and concentration boundary layers as N increases from -1 to 0.5. This causes the inverse of the wall temperature and concentration values to decrease yielding reductions in both the local Nusselt and Sherwood numbers. This is true for all values of the power-law fluid index n. Also, it is noted that as the transformed suction or injection parameter ξ increases for fixed values of N and $\chi \neq 1$, all of the velocity, temperature and concentration species increase. As a result of increasing the value of ξ, the local Nusselt and Sherwood numbers decrease. From the definition of χ, it is seen that increases in the value of the parameter $Ra_x^{n/(2n+1)}/Pe_x^{(n+1)/(2n+1)}$ causes the mixed convection parameter χ to decrease. Thus, small values of $Ra_x^{n/(2n+1)}/Pe_x^{(n+1)/(2n+1)}$ correspond to values of χ close to unity which indicate almost pure forced convection regime. On the other hand, high values of $Ra_x^{n/(2n+1)}/Pe_x^{(n+1)/(2n+1)}$ correspond to values of χ close to zero which indicate almost pure free convection regime. Furthermore, moderate values of $Ra_x^{n/(2n+1)}/Pe_x^{(n+1)/(2n+1)}$ represent values of χ between 0 and 1 which correspond to the mixed convection regime. For the forced convection limit (χ = 1) it is clear from Eq. (11) that the velocity in the boundary layer f' is uniform irregardless of the value of n provided that Pe_x=1 as set in the figures. However, for smaller values of χ at a fixed value of N and n=1.0, the fluid velocity close to the wall increases for values of $\chi \le 0.5$ due to the buoyancy effect which becomes larger for χ = 0 (free convection limit). This decrease and increase in the fluid velocity f' as χ is decreased from unity to zero is accompanied by a respective increase and a decrease in both the wall fluid temperature and concentration. As a result, the local Nusselt and Sherwood numbers tend to decrease and then increase as χ is increased from 0 to 1.

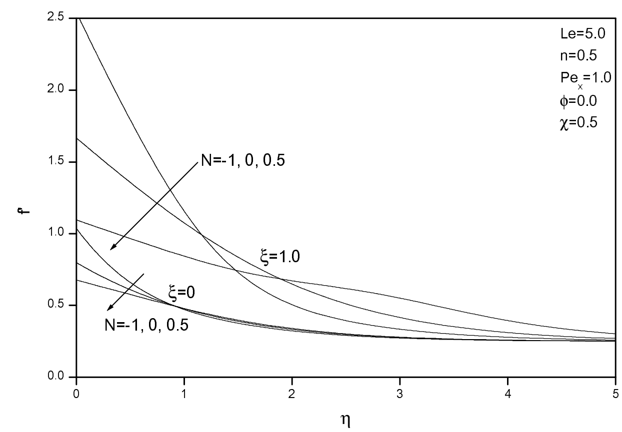

Fig. 1. Velocity profiles for different values of ξ (n=0.5)

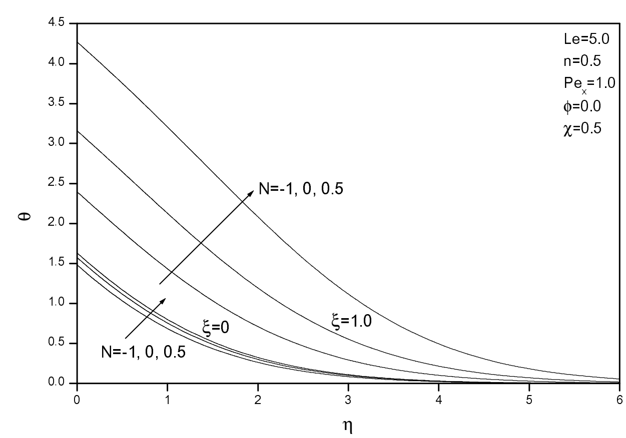

Fig. 2. temperature profiles for different values of ξ (n=0.5)

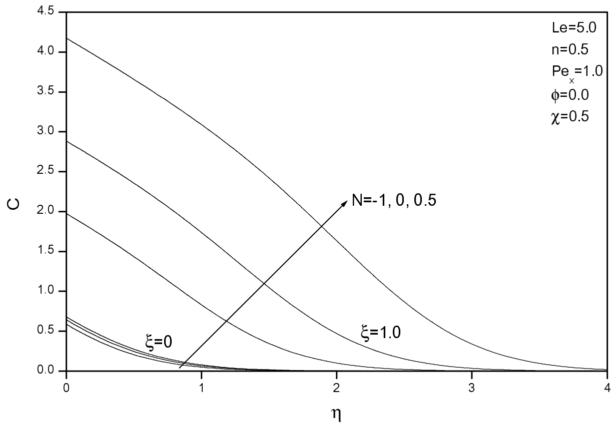

Fig. 3. Concentration profiles for different values of ξ (n=0.5)

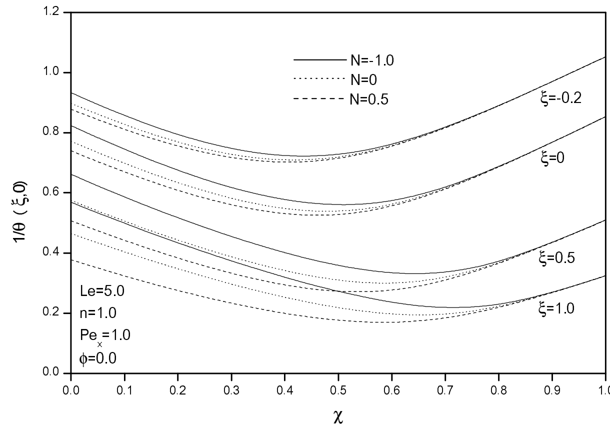

Fig. 6. Effects of N and χ on local Nusselt number for n=1.0 and different ξ values

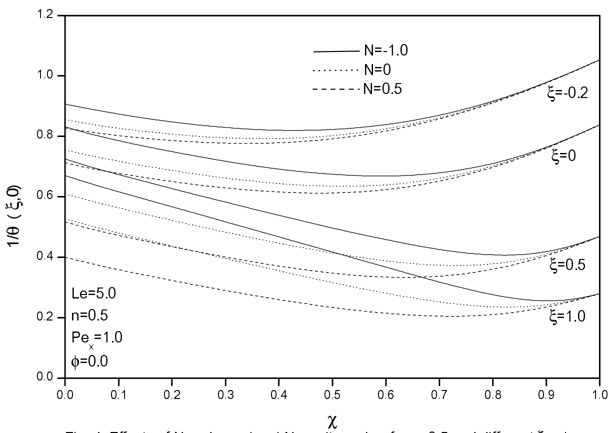

Fig. 4. Effects of N and χ on local Nusselt number for n=0.5 and different ξ values

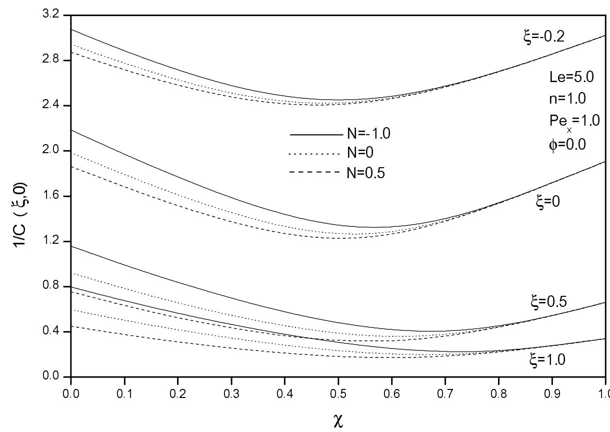

Fig. 7. Effects of N and χ on local Sherwood number for n=1.0 and different ξ values

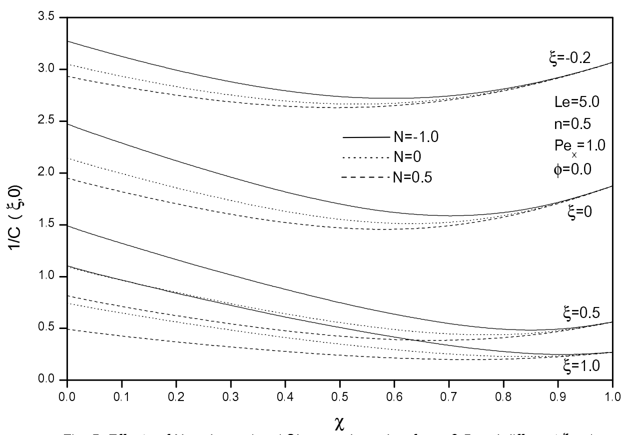

Fig. 5. Effects of N and χ on local Sherwood number for n=0.5 and different ξ values

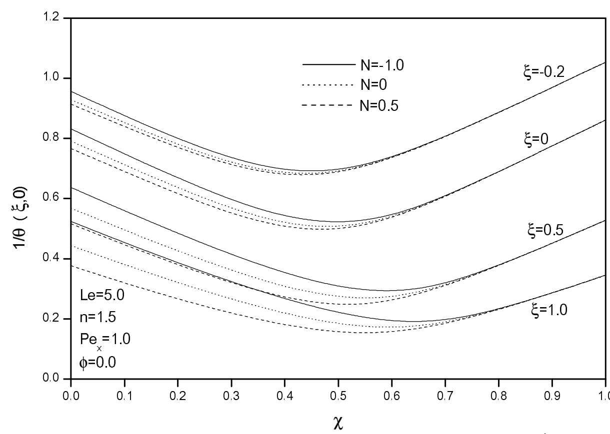

Fig. 8. Effects of N and χ on local Nusselt number for n=1.5 and different ξ values

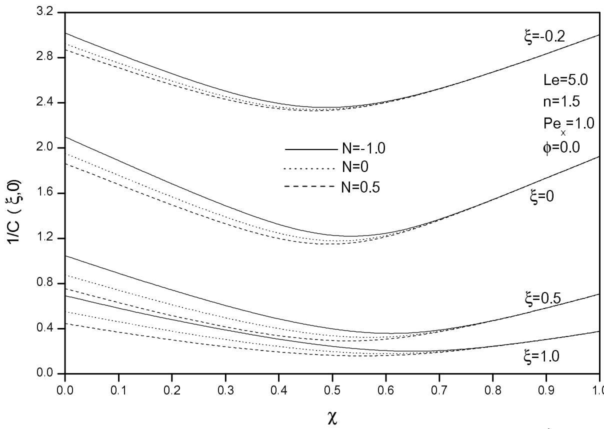

Fig. 9. Effects of N and χ on local Sherwood number for n=1.5 and different ξ values

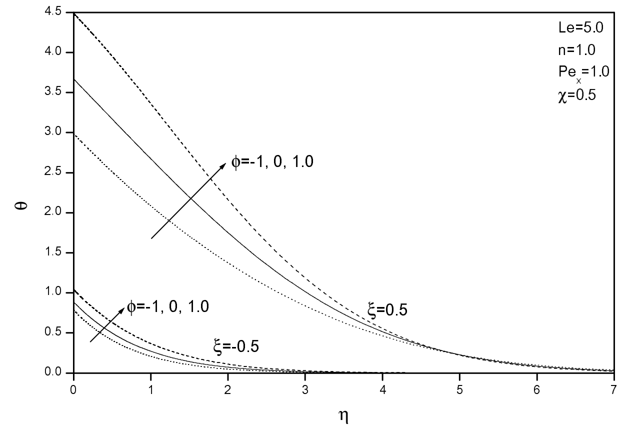

Fig. 11. temperature profiles for different values of ϕ and ξ (n=1.0)

Furthermore, by comparison of Figs. 4-9, one can conclude that the local Nusselt and Sherwood numbers decrease as the power-law fluid index n increases. It is also observed that while the local Nusselt and Sherwood numbers change in the whole range of free and mixed convection regime, they remain constant for the forced-convection regime. This is obvious since for χ=1 and fixed values of Le, the equations are the same and do not depend on n and N. All of the above trends are clearly displayed in Figs. 4-9.

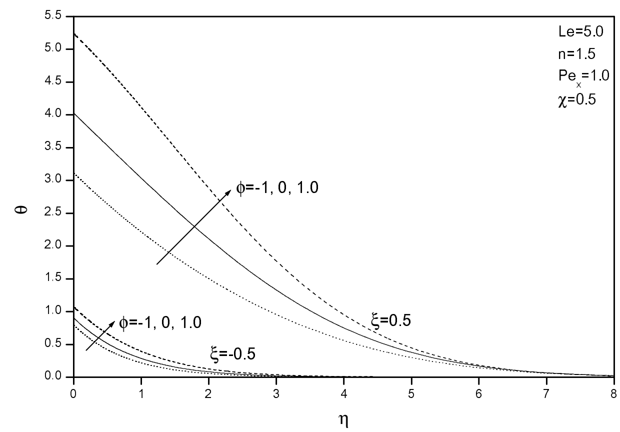

Fig. 12. temperature profiles for different values of ϕ and ξ (n=1.5)

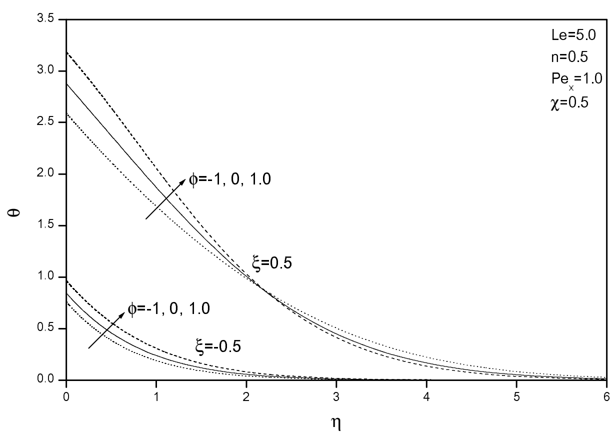

Fig. 10. temperature profiles for different values of ϕ and ξ (n=0.5)

Figures 10 through 12 present the effects of the heat generation or absorption coefficient ϕ on the temperature profiles at different values of ξ for n=0.5, n=1.0 and n=1.5, respectively. The presence of a heat generation source in the flow represented by positive values of ϕ enhances the thermal state of the fluid causing its temperature to increase. On the contrary, the presence of a heat absorption sink in the flow represented by negative values of ϕ reduces the fluid temperature. This is true irregardless of the value of n as clearly seen from Figs. 10-12. Also, it should be noted that for the case of n=0.5 and ξ=0.5 in Fig. 10, while increasing ϕ increases the temperature close to the surface, it causes it to decrease far downstream as it meets the free stream conditions.

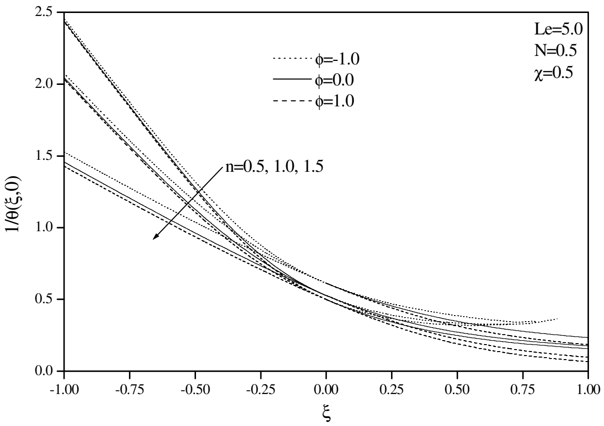

Fig. 13. Variation of local Nusselt number with ξ for different values of n and ϕ

The effect of the heat generation or absorption coefficient ϕ on the local Nusselt number for different values of n (0.5, 1.0, 1.5) in the range $-1 \leq \xi \leq 1$ is displayed in Fig. 13. As mentioned above, in general, the presence of a heat generation effects in the flow causes the fluid temperature to increase. This, in turn, increases the thermal buoyancy effect which produces higher induced flow. On the contrary, the presence of a heat absorption effects in the flow reduces the fluid temperature which, in turn, decreases the induced flow due to thermal buoyancy effects. Thus, the wall temperature increases as ϕ increases causing the local Nusselt number which is inversely proportional to $\theta(\xi,0)$ to decrease for all values of ξ except $\xi=0$ since ϕ does not appear in Eq. (12) at $\xi=0$.

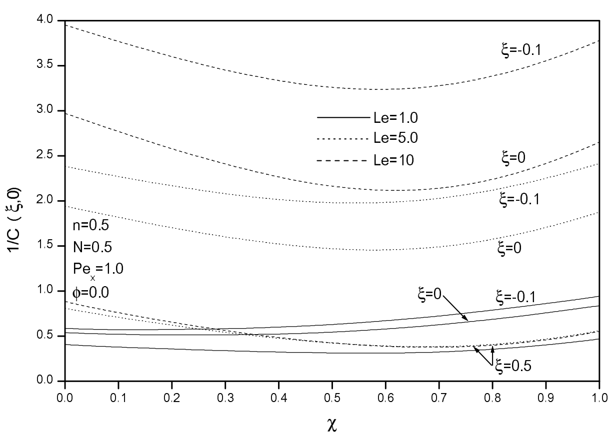

Fig. 14. Effects of Le and χ on local Sherwood number for n=0.5 and different ξ values

Figure 14 depicts the influence of the Lewis number Le on the local Sherwood number in the whole mixed convection range $0 \leq \chi \leq 1$ for different values of ξ. Increasing the value of the Lewis number results in decreasing the concentration species at the wall. This causes the local Sherwood number to increase as seen in Fig. 14.

V. Conclusions

This work considered double-diffusive mixed convective flow of a non-Newtonian power-law fluid along a vertical permeable surface embedded in a porous medium under uniform heat and mass fluxes. A single parameter for the entire range of free-forced-mixed convection regime was employed. The obtained non-similar differential equations were solved numerically by an efficient implicit finite-difference method. The results focused on the effects of the buoyancy ratio, power-law fluid index, mixed convection parameter, suction or injection parameters, heat generation or absorption coefficient and the Lewis number on the local Nusselt and Sherwood numbers. It was found that as the buoyancy ratio was increased, both the local Nusselt and Sherwood numbers decreased in the whole range of free and mixed convection regime while they remained constant for the forced-convection regime for all power-law fluid index values. However, they decreased and then increased forming dips as the mixed-convection parameter was increased from the free-convection limit to the forced-convection limit. Also, the local Nusselt and Sherwood numbers decreased with increasing values of the suction or injection parameter. Furthermore, it was concluded that in general, the local Nusselt and Sherwood numbers decreased as the power-law fluid index was increased. Furthermore, it was found that increasing the heat generation or absorption parameter decreased the local Nusselt number and increasing the Lewis number produced increases in the local Sherwood number.

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