## Voltage and Current Harmonic Variations in Three-phase Induction Motors with Different Stator Coil Pitches

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Abstract: - In this study, firstly a sinusoidal pulsewidth modulation (SPWM) inverter feeding five different chorded three-phase induction motors were tested for low-order odd harmonic voltage component and efficiency at different loads. Secondly the motors fed by sine voltage again were tested for low - order odd harmonic voltage and current component and efficiency at different In both two situations; total harmonic distortion (THD) due to 3rd, 5th, 7th, 9th, 11th and 13th harmonics were less in a motor with (1-7) 120° coil pitch at over load. Particular harmonic order for each coil pitch was suppressed and the increasing quantity of efficiency in a motor with  $(1-6)100^0$  coil pitch was increased by 4, 92 % fed by PWM and 8, 86 % fed by sine voltage. The full pitch motor with (1-10) 180° coil pitch has more harmonics than other motors.

*Keywords:* - Coil pitch; Chording; Harmonics; Stator winding

### 1. Introduction

Due to the increasing requirement of precise control and equipment performance of a modern facility, the appearance of voltage harmonics in the power system has drawn great attention recently. In a power system, induction motors constitute the largest component of the load and are widely used in industrial, commercial and residential applications. Once the power system gets polluted harmonics, the operation characteristics of induction motors will be affected first. Therefore, studying the impacts of induction motors under harmonic voltages has drawn the attention of many researchers. Variable speed drives employing sinusoidal pulse-width modulation (SPWM) inverter fed induction motors are now widespread throughout industry. Unfortunately, losses in an inverter fed machine are always greater than those for the same machine operating on a sinusoidal

supply and in some cases this requires derating of the motor [1]. Rotating machines are considered a source of harmonics [2, 3] because the windings are embedded in slots which can never be exactly sinusoidally distributed so that the mmf is distorted. Low-order harmonics have a larger impact on the three-phase induction motor than that of high-order harmonics. One method to reduce the low-order harmonics is to adopt chording (fractional pitch) of the stator winding. This paper explains the effect of chording on three-phase squirrel-cage induction motors fed from a three-phase inverter and sine voltage [4].

## 2. Chorded (fractional-pitch) windings

A chorded winding is a winding whose coil sides are less than a pole pitch apart, thus saving copper. Pitch factor is the ratio of coil voltages for a fractional-pitch winding to those for a full-pitch one.

$$k_{p1} = \sin\left(\frac{\beta}{2}\right) < 1\tag{1}$$

Where  $\beta$  is the coil span in 'electrical,  $\beta = s\alpha$ ; s is the coil span in slots. Accounting for harmonics, the pitch factor would be:

$$k_{ph} = \sin\left(\frac{\beta h}{2}\right) \tag{2}$$

$$\frac{k_{ph}}{k_{p1}} = \frac{\sin(\beta h/2)}{\sin(\beta/2)} \tag{3}$$

where h is the harmonic order [1].

## 3. The winding factor

In the presence of harmonics, the winding factor becomes:

$$k_{wh} = k_{dh}k_{ph}$$
 so that:

$$\frac{k_{wh}}{k_{wl}} = \frac{k_{dh}k_{ph}}{k_{d1}k_{p1}} = \frac{2\sin(\alpha/2)\sin(\pi h/6)\sin(\beta h/2)}{\sin(\alpha h/2)\sin(\beta/2)}$$
(5)

where  $\beta = s\alpha$ . [1]

## 4. Related definitions and classifications of harmonics

It is well-known that voltage and current harmonics in the power system can come from a number of sources in the network. Theoretically, any nonsinusoidal periodical waveform can be transformed into a different order harmonic waveform through Fourier analysis. Therefore, the nonsinusoidal voltage and current waveform can be expressed as:

$$v(t) = \sqrt{2} \left[ V_1 \sin \omega_o t + \sum_{k=2}^{\infty} V_k \sin(k\omega_o t + \phi_k) \right]$$
 (6)

$$i(t) = \sqrt{2} \left[ I_1 \sin \omega_o t + \sum_{k=2}^{\infty} I_k \sin(k\omega_o t + \theta_k) \right]$$
 (7)

where

 $V_I$ ,  $I_I$  are the fundamental voltage and current,  $V_k$ ,  $I_k$  are the  $k^{th}$  order harmonic voltage and current,  $\phi_k$ ,  $\theta_k$  are the phase angles of the  $k^{th}$  order harmonic voltage and current, and

 $\omega_0$ , is the radian frequency of the fundamental wave.

When a nonsinusoidal voltage source is supplied to a three-phase induction motor, the corresponding slip  $S_k$  to the various harmonics can be expressed as:

$$S_{k} = \frac{kN_{s} + (1-s)N_{s}}{kN_{s}} = \frac{k + (1-s)}{k}$$
 (8)

According to the rotational direction of magneto motive force (MMF), the  $(3n+1)^{th}$  order harmonics (positive sequence harmonics) contribute MMF and torque in the positive (forward) direction; the  $(3n+2)^{th}$  order harmonics (negative sequence harmonics) provide counter MMF and torque; and the  $(3n)^{th}$  order harmonics (zero sequence harmonics) do not contribute any rotating MMF or torque. Although the positive sequence harmonics would add a boost to the

positive sequence (forward) torque and thus be beneficial, the heating effects of the harmonics offset the benefit of the positive sequence torque [1, 7].

According to the definition of IEEE-519 [2], the total voltage harmonics distortion factor (THDv) is defined as:

$$THD_{v}(\%) = \sqrt{\frac{\sum_{k=2}^{\infty} V_{k}^{2}}{V_{1}^{2}}} \times 100\%$$
 (9)

and the amount of voltage distortion due to the kth order harmonic is measured by the voltage distortion

factor (VDF) as: [1]

$$VDF(\%) = \frac{V_k}{V_1} \times 100\% \tag{10}$$

# 5. Configuration of the experimental system

The configuration of the experimental system is shown in Fig. 1. It consists of a three-phase PWM inverter which gives output by comparing the modulating signal with carrier signal technique at 6kHz switching frequency and supplies 50Hz, 380V (rms) voltage to a three-phase squirrel cage induction motor under test. A digital power analyzer with 3,2 kHz sampling frequency is used to measure the stator voltage harmonics, stator voltage, stator current and input power to the motor. The operating data of the induction motor are transmitted to the PC through RS-485 for later analysis. Each motor was mounted in turn on a drive bed and loaded by an electromagnetic brake which is controlled by the dc voltage applied to the brake provided with two arms, one of which with balances weight for measuring the out put torque of the motor. The brake includes a cooling fan that is supplied by the main voltage. Force applied to the induction motor is measured with a dynamometer which is mounted on the electromagnetic brake's one arm to obtain the applied torque. The stator winding of five commercial, 1100W, 36-slot, three-phase, four-pole squirrel cage induction motors were rewounded with different coil pitches. The coil pitch for each motor was re-wound to pitch 180° (Full pitch, 1-10 slots pitch),  $160^{\circ}$  (1-9),  $140^{\circ}$  (1-8),  $120^{\circ}$  (1-7) and  $100^{0} (1-6)$ 

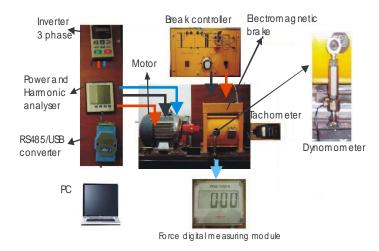


Fig. 1 Experimental setup

for M1, M2, M3, M4 and M5 motors, respectively. All the windings were a simple lap configuration. Fig. 2 shows the three-phase double layer windings embedded in slots.

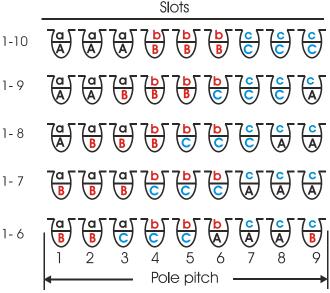


Fig. 2 Three – Phase winding with two layer configuration in the stator slots

The letters (a, b, and c) indicate the conductors corresponding with phases  $L_1$ ,  $L_2$ ,  $L_3$  and their vertical position designate conductors in the same slot. The direction of current is indicated by a, A etc. The pole pitch is 9 slots with 3 conductor slots per pole per phase. The slot pitch is  $20^0$  so for full pitch winding the coil pitch is  $180^0$  and the coil pitch is reduced by  $20^0$  each time for other motors resulting in coil pitch of  $160^0$ ,  $140^0$ ,  $120^0$  and  $100^0$  respectively. To measure the winding temperature, K-type thermocouples were attached to the stator winding of all five motors. Motors were loaded with applied torque of from 1 to 9, 74 Nm (full load was 8,18 Nm). The power and harmonic analyzer employs the fast Fourier

transformation to obtain the harmonic voltage components with PWM supply was used.

### 6. Results at the first situation

At the first situation the motors have been fed by PWM inverter. Figs. 3–5 are the stator low-order voltage harmonics for the different motors fed from PWM inverter with (1-10), (1-9), (1-8), (1-7) and (1-6) coil pitch at half load, full load and overload, respectively. If the coil pitch is shortened by 1/n of the pole pitch then the nth harmonic will be suppressed or the harmonics near to n will be with low voltage, because of the harmonic cancellation at that coil pitch [3].

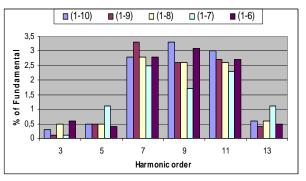


Fig.3 Low – order voltage harmonics at half load for motors fed by PWM.

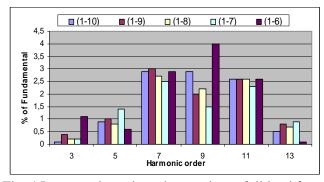


Fig. 4 Low – order voltage harmonics at full load for motors fed by PWM.

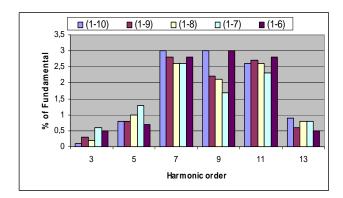


Fig. 5 Low – order voltage harmonics at over load for motors fed by PWM.

As the motor full pole pitch is (1-10) and for M5 (1-6) motor the coil pitch is reduced by 44,44 % of the full pole pitch, the 5th harmonics voltage is reduced dramatically compared to other motor with a different coil pitch. The same effect occurs at all loads as seen from Figs. 3–5. If we consider motor M4 (1-7), the coil pitch is reduced by 33.33% of pole pitch, so the 3rd, harmonics voltage is reduced at half load and at full load. Also the 7th, 9th and 11th order harmonics voltages are reduced as seen in Figs. 3–5.

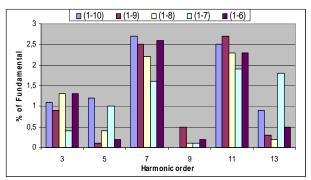


Fig. 6 Low – order current harmonics at half load for motors fed by PWM

In motor M3 (1-8) the coil pitch is reduced by 22.22%, the 5th harmonics is suppressed at all loads in fig. 6-8. The 7th harmonics is less than in the (1-9) and (1-10) motors (Fig. 6). If we consider motor M2 (1-9) the coil pitch is reduced by 11, 11%, the 13th harmonics voltage and current are reduced compared almost all motors with different coil pitch. If we consider M1 with coil pitch (1-10), upper and lower slots have the same phase and direction of currents in each slot. But for motor M2 (1-9) the slot numbers 3, 6 and 9 have two different phase conductors with different current directions, thus overlapping between adjacent phase-bands benefits on their air gap flux pattern [6]. This overlapping is bigger in motor M3 and even higher in motor M4 and also higher in motor M5. So there are more possibilities of harmonics cancellation in motor M5. But if we see the THD due to 3rd, 5th, 7th, 9th, 11th and 13th for five motors at three different loads, motor M2 with (1-9) coil pitch and M3 with (1-8) coil pitch have fewer harmonic compared with the other motors both for current and voltage in fig. 9-10 has lowest harmonics and other motors have more. This is due to the aiding of a loworder odd harmonics (3rd, 5th and 9th) due to overlapping of MMF waveform caused by chording of coil. This aiding of harmonics is less in motor M2 than in other motors. The effect of chording can also be seen in THD in current due to 3rd, 5th, 7th, 9th, 11th and 13th harmonics (Fig. 10) [7,8]

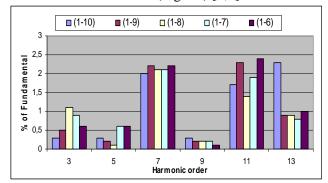


Fig. 7 Low – order current harmonics at full load for motors fed by PWM

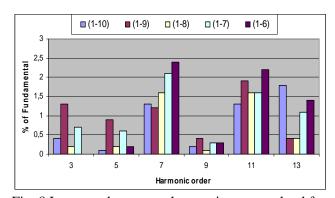


Fig. 8 Low – order current harmonics at over load for motors fed by PWM

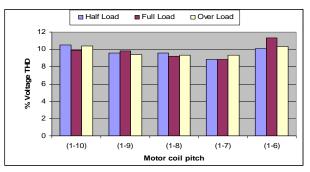


Fig. 9 Value of Voltage THD at different loads for motors fed by PWM.

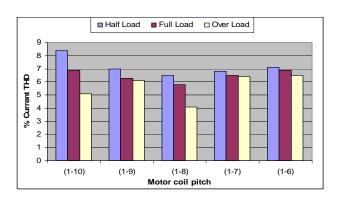


Fig. 10 Value of Current THD at different loads for motors fed by PWM.

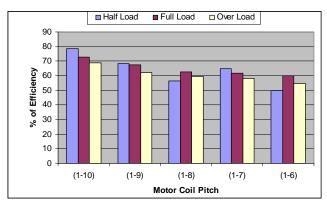


Fig. 11 Efficiency at different motors fed by PWM

Fig. 11 shows that the efficiency of motor M5 (1-6) at all load is less than the others. This is due to the increase in flux caused by the decrease of pitch factor. This increase in flux causes more core losses. However the increasing quantity of efficiency was increased 4,92 % so there is a significant effect of the winding design at over load.

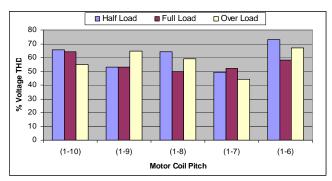


Fig. 12 Value of voltage THD due to the first thirty harmonics at different loads for motors fed by PWM

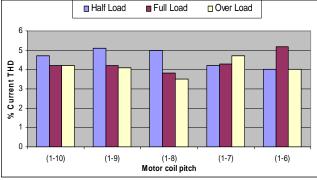


Fig. 13 Value of current THD due to the first thirty harmonics at different loads for motors fed by PWM

### 7. Results at the second situation

At the second situation the motors have been fed by sine voltage. In this case, voltage total harmonic distortion due to low order odd harmonics 3rd, 5th, 7th, 9th, 11th and 13th has not been seen. But voltage THD due to first thirty harmonics has been seen as shown in fig. 18. Figs. 15-17 are the stator low-order current harmonics for the different motors fed from sine voltage with (1-10), (1-9), (1-8), (1-7) and (1-6) coil pitch at half load, full load and overload, respectively. As seen in the first situation, as the motor full pole pitch is (1-10) and for M5 (1-6) motor the coil pitch is reduced by 44,44 % of the full pole 5th harmonics current is reduced pitch, dramatically compared to other motor with a different coil pitch. The same effect occurs at half and full loads as seen from Figs. 15-16. If we consider motor M4 (1-7), the coil pitch is reduced by 33.33% of pole pitch, so the 3rd, 13th and 9th harmonics order currents is reduced at over load, full load and half load respectively as seen in Figs. 15–17. In motor M3 (1-8) the coil pitch is reduced by 22.22%, the 5th 7th and 9th harmonics is suppressed at all loads in fig. 15-17. If we consider motor M2 (1-9) the coil pitch is reduced by 11, 11%, the 5th, 7th, 9th and 11th harmonics currents are reduced compared almost all motors with different coil pitch. If we see the THD due to 3rd, 5th, 7th, 9th, 11th and 13th for five motors at three different loads, motor M2 with (1-9) coil pitch and has fewer harmonic compared with the other motors for current in fig. 20 has lowest harmonics and other motors have more. This is due to the aiding of a low-order odd harmonics (3rd, 5th and 9th) due to overlapping of MMF waveform caused by chording of coil. This aiding of harmonics is less in motor M2 than in other motors. The effect of chording can also be seen in THD in current due to first thirty harmonics order (Fig. 19) [7].

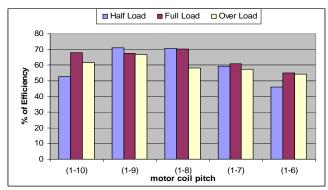


Fig.14 Efficiency at different motors fed by sine voltage

Fig. 14 shows that the efficiency of motor M5 (1-6) at all load is less than the others. This is due to the increase in flux caused by the decrease of pitch factor.

This increase in flux causes more core losses. However the increasing quantity of efficiency was increased 8, 86 % so there is a significant effect of the winding design at over load.

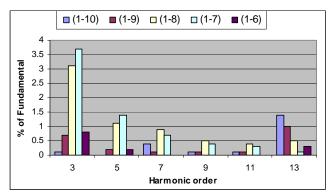


Fig. 15 Low – order current harmonics at half load for motors fed by sine voltage

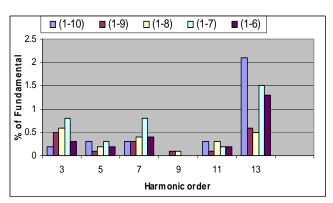


Fig. 16 Low – order current harmonics at full load for motors fed by sine voltage

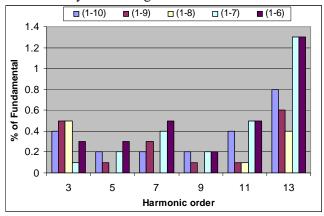


Fig. 17 Low – order current harmonics at over load for motors fed by sine voltage

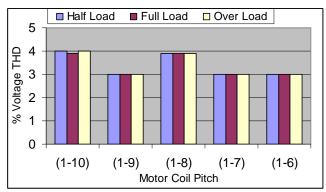


Fig. 18 Value of voltage THD due to the first thirty harmonics at different loads for motors fed by sine voltage

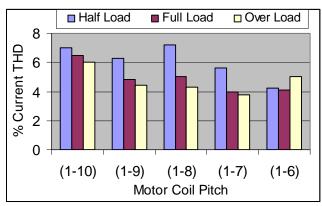


Fig. 19 Value of current THD due to the first thirty harmonics at different loads for motors fed by sine Voltage

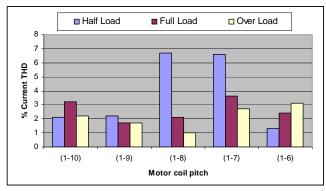


Fig. 20 Value of current THD due to the low order harmonics at different loads for motors fed by sine voltage

Table 1. % Value of THD at half loads

	Sine		PWM	
Coil	THD V	THD I	THD V	THD I
Pitch				
(1 - 10)	0	2.1	10.5	8,4
(1 – 9)	0	2.2	9.6	7
(1 - 8)	0	6.7	9,6	6.5

(1-7)	0	6.6	8.8	6.8
(1-6)	0	1.3	10.1	7.1

Table 2. % Value of THD at full loads

	Sine		PWM	
Coil	THD V	THD I	THD V	THD I
Pitch				
(1–10)	0	3.2	9.9	6.9
(1 - 9)	0	1.7	9.8	6.3
(1 - 8)	0	2.1	9.2	5.8
(1-7)	0	3.6	8.8	6.5
(1-6)	0	2.4	11.3	6.9

Table 3. % Value of THD at over loads

	Sine		PWM	
Coil	THD V	THD I	THD V	THD I
Pitch				
(1-10)	0	2.2	10.4	5.1
(1 - 9)	0	1.7	9.4	6.1
(1 - 8)	0	1	9.3	4.1
(1-7)	0	2.7	9.3	6.4
(1 – 6)	0	3.1	10.3	6.5

### 8. Conclusions

Low-order harmonics in stator voltage of three phase induction motor fed by PWM voltage could be reduced by chording the stator winding. This suppresses particular harmonic components with different type of coil pitch but also aids the other low order harmonics. Motor M1 is considered efficient however increasing quantity of efficiency decreased. The increasing quantity of efficiency was increased when the low order harmonics up to 13th were dealt with and motor M5 efficiency was increased 4, 92 % fed by PWM and 8, 86% fed by sine voltage so there is a significant effect of the winding design at over load. The full pitch motor has more harmonics than other motors.

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#### References

[1] J. Wakileh, Harmonic In Rotating Machines, *Electric Power System Research* vol. 66, 2003, pp. 31-37.

- [2] C. Y. Lee, W. J. Lee, Y.N. Wang, J. C. Gu, Effect of Voltage Harmonics on the Electrical and Mechanical Performance of a Three-Phase Induction Motor, *Industrial and Commercial Power Systems Technical Conference*, Atlanta, Canada, 1998, IEEE 88-94.
- [3] R. Deshmukh, A. J. Moses, F. Anayi, Improvement in Performance of Short Chorded Three Phase Induction Motors With Variable PWM Switching Frequency, *Transection on Magnetics*, *IEEE*, Vol. 42, No: 10, 2006, pp. 3452-3454.
- [4] C. Y. Lee and W. J. Lee, Effects of nonsinusoidal voltage on the operation of a three-phase induction motor, *IEEE Trans. Energy Convers.*,vol. 14, no. 2, pp. 193–201, Jun. 1999.
- [5] G.Chang, Modeling devices with nonlinear voltage-current characteristics for harmonic studies, *IEEE Trans. Power Del.* vol.19,no.4, pp.1802–1811, Oct. 2004.
- [6] J. Nazarko, M. Poplawski, W. Zalewski, G. Cross, A. E. Walbridge, The Determination of Supply Harmonic Currents Produced by Multiple PWM Voltage Source Inverter Induction Motor Drives: An Expert System Approach, *Electric Power Engineering, PowerTech Budapest 99. International conference 29 Aug.-2 Sept.* 1999 Page(s):93
- [7] R. Deshmukh, A. J. Moses, F. Anayi, Voltage harmonic variation in three-phase induction motors with different coil pitches, *Journal of Magnetism and Magnetic Materials* 304 (2006) e810–e812
- [8] B. K. Bose, Modern Power Electronics and AC Drives, *Prentice Hall PTR*, *USA*, 2002, pp. 30-70.
- [9] C. A. Hernandez-Aramburo, T. C. Green, S. Smith, Assessment of Power Losses of an Inverter-Driven Induction Machine With Its Experimental Validation, *IEEE Transaction on Industry Applications*, Vol. 39, NO. 4, July/August. 2003
- [10] A. R. Muhoz and T. A. Lipo, Dual Stator Winding Induction Machine Drive, *IEEE Transactions on Industry Applications*, Vol. 36, No. 5, September/October 2000.
- [11] D. H. Hwang, K. C. Lee, Y. J. Kim, Y. J. Lee, M. H. Kim, D. H. Kim, Analysis of Voltage Distribution in Stator Winding of IGBT PWM Inverter-Fed Induction Motors, *IEEE ISIE* 2005, June 20-23, Dubrovnik, Croatia, 2005.
- [12] K. Yamazaki, Induction Motor Analysis Considering both Harmonics and End Effects using Combination of 2D and 3D Finite Element Method *IEEE Transactions on Energy Conversion*, Vol. 14, No. 3, September 1999
- [13] T.M. Wolbank, R. Woehrnschimmel, B. Haidvogl, Comparison of different winding schemes reference to their application on speed sensorless control of induction machines, *Industrial Electronics*

Society, 2000. IECON 2000. 26th Annual Confierence of the IEEE Volume 4, 22-28 Oct. 2000 Page(s):2340-2345 vol.4