A new algorithm for smart grid protection based on synchronized sampling

Francesco Muzi, Antonio De Sanctis, and Pasquale Palumbo

***Abstract*—** Conventional distribution systems are usually radially operated, supplied at one end through a main source. The presence of a massive Distributed Generation (DG) from renewable sources radically changes the radial operation of these systems. Indeed, from passive the network becomes active, and bidirectional power flows can appear in the distribution system, which causes a number of problems related both to normal and fault conditions. In this paper, a new protection procedure is proposed, based on distance protection concepts and synchronization of the voltage and current samples coming from the two terminals of the faulted MV line segment. The fault identification algorithm suitably exploits the model equations of the lumped parameter circuit by applying the Recursive Least Square approach. A thorough set of simulations are carried out in order to validate the proposed algorithm, which is able to estimate the fault distance with good performances and great speed, even in case of high distortion of the acquired voltage and current waveforms. Moreover, the algorithm provides a very good estimate of the fault resistance, which means it works correctly also whenever high fault resistances appear in the distribution systems herein examined.

***Keywords*—** Distance protection, Parameter estimate, Recursive least square method, Renewable sources, Smart grid protection, Synchronized sampling.

I. Introduction

THE great increase in the diffusion of Distributed Generation (DG) requires a substantial evolution of electrical distribution systems so as to ensure high levels of automation, protection and stability. On the other hand, a massive penetration of DG in existing distribution networks brings out many problems related to the operation and protection of the distribution system, such as:

- the coordination failure of protective devices and relay de-sensitization; in some cases over-current protection may even not respond or may take a long time to respond [1];
- the degradation of the current and voltage waveforms [2];
- the difficulty in controlling the voltage profiles due to changes in the power injected by distributed generators [2-3];
- the presence of electromechanical transients and dynamic instability phenomena [3-4].

The above scenario emphasises the need to solve a number of problems in upgrading present distribution networks with the aim to obtain more and more efficient and reliable smart grids in the future. In this paper particular attention is paid to improving protection systems by developing a new algorithm for fault-distance estimation.

The main techniques adopted in distance protection, which are now mainly used in high voltage transmission lines, are based on the symmetrical component theory (phasor method). Even though these procedures use digital techniques, they actually depend on concepts derived from analogical formulations and frequency domain approaches [5-12]. As a matter of fact, two particularly important features (usually offered by digital techniques) are ignored:

1) the overcoming of the signal processing, which means a drastic reduction in processing time, since it is possible to operate directly on the acquired samples of voltage and current instead of on their associated signals;
2) the use of more complex, but more precise mathematical schemes, compared with those drawn from simplified circuits based on phasor representation.

Unlike most common distance protection systems, which usually refer to an identification scheme based on stand-alone relays, in this paper a new identification algorithm is proposed, based on the idea that couples of relays co-operate with each other. In this way, thanks to a double stream of data the algorithm is able to deal also with a non-negligible fault resistance, in spite of the common practice to assume only pure metallic faults. Indeed, according to the proposed algorithm, also the fault resistance is estimated as a by-product, with a very good degree of precision. More specifically, the fault distance identification algorithm is based on the Recursive Least Squares (RLS) approach, and it suitably exploits the circuit equations in the time domain. This approach has been preferred to others (e.g. the ones based on neurofuzzy models [13-14], black-box models [15] or other techniques based on statistical local approaches [16]) because the RLS only requires a mathematical relationship between parameters and data to be successfully exploited in the absence of any statistical characterization of the noises involving the acquired measurements, as in the case we are

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dealing with.

From a technical point of view, the idea of co-operating relays requires the hypothesis to assume a synchronization mechanism, ensuring simultaneous measurements coming from the different relays installed on the network.

The wide range of simulations (performed in Matlab) shows that the algorithm is able to estimate the fault distance with a very good precision and great speed (below 10*ms*).

II. Power Systems and Sample Synchronization

In the following, a single MV feeder divided into a number of segments is taken into consideration. Each line segment is equipped with two distance relays, each placed at one of the two line ends. It is assumed that each relay is connected to a circuit breaker that exhibits reclosing capabilities. When a fault occurs, the protection system must detect and isolate only the faulted line segment. The assumed MV radial system can feed any kind of load (linear and nonlinear). Nowadays, in residential areas particularly important nonlinear loads must be taken into account, due to the development of the building automation [17]. In addition, in the considered power system along the line there may be a number of distributed generators (besides the main equivalent source), normally connected to nodes materialized by MV/LV transforming stations. It is also assumed that the distribution system can be operated in islanded mode if the local DG generation is higher than the load demand. In this case, problems concerning power/frequency control can often appear [18].

In order to take into account the islanded mode operation of the system, the circuit breakers adopted must have open and close capabilities as well as receive remote open and close command. In addition, the adopted breakers must be equipped with check-synchronization devices in order to maintain zones synchronization in case two islanded zones should be re-connected to each other.

As far as the samples synchronization is concerned, different technologies can be used. As an example, Fig. 1 shows a measurement system acquiring in real-time at both line terminations voltage and current signals that are properly sampled at a pre-established frequency. In this case, the sample synchronization is obtained by means of a synchronization source that refers to a typical Global Positioning System (GPS).

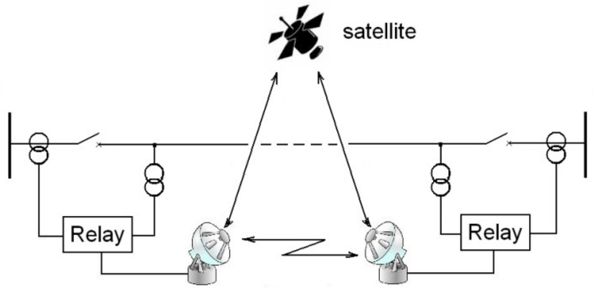

Fig. 1 Single line segment equipped with a synchronization system of GPS type.

Commercial synchronization devices usually acquire signals coming from a 50*Hz* network with a sampling frequency that varies from 600 to 2500 samples per second.

III. The Digital Identification Algorithm

Consider a simplified lumped parameter model of a faulted line segment of an MV power line connecting bus 2 to bus 3 as in Fig. 2. This figure is taken from a more complete network which will be used as a test system to evaluate the performance of the proposed algorithm in the following Section (see Fig. 3).

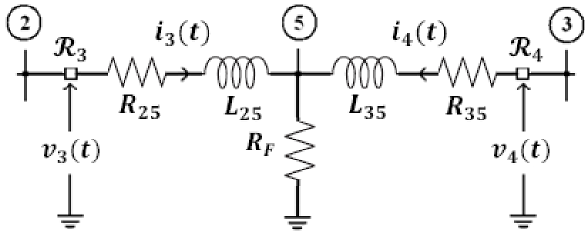

Fig. 2 Electrical scheme for a pair of synchronized relays.

Here and in the following $\mathcal{R}_i$ stands for the i-th relay, while R_{jk} and L_{jk} stand for the lumped resistance and inductance between nodes j and k. The fault is assumed to occur in node 5, with R_F being the the fault resistance. The total resistance-inductances of the power line, R_{23} and L_{23} respectively, are known so that R_{35}, L_{35} can be expressed in terms of R_{25} and L_{25}:

$$R_{35} = R_{23} - R_{25} \quad L_{35} = L_{23} - L_{25} \tag{1}$$

The distance protection problem is solved by identifying the lumped parameters R_{25}, L_{25}, and therefore the fault distance is readily obtained, assuming that the line resistance-inductance values are frequency-independent and uniformly distributed along the whole line. Indeed, by denoting with R'_{23}, L'_{23} the line unit resistance-inductance, and with $\hat{R}_{25}$, $\hat{L}_{25}$ the estimated lumped parameters, the fault distance from relay $\mathcal{R}_3$, computed according to the resistance and to the inductance estimates, will be:

$$\hat{S}_{R,\mathcal{R}_3} = \hat{R}_{25}/R'_{23}, \qquad \hat{S}_{L,\mathcal{R}_3} = \hat{L}_{25}/L'_{23} \tag{2}$$

respectively.

Remark 1. It is important to emphasize that, according to (2), we have a pair of estimates of the fault distance. On some occasions there are technical reasons which suggest to prefer the resistance-based estimate $\hat{S}_{R,\mathcal{R}_3}$ instead of the inductance-based $\hat{S}_{L,\mathcal{R}_3}$, or *vice versa*; at other times, when there is no *a priori* knowledge, the mean value of the two estimates provided by (2) is suggested.

The differential equations describing the scheme of Fig. 2 are:

$$\begin{cases} v_3(t) = R_{25} i_3(t) + L_{25} \frac{di_3}{dt} + R_F(i_3(t) + i_4(t)) \\ v_4(t) = (R_{23} - R_{25}) i_4(t) + (L_{23} - L_{25}) \frac{di_4}{dt} \\ \qquad\qquad + R_F(i_3(t) + i_4(t)) \end{cases} \tag{3}$$

Voltage and current measurements (v_3, i_3), (v_4, i_4) are acquired by the relays $\mathcal{R}_3$, $\mathcal{R}_4$ respectively, according to the sample interval T_S, starting from time 0, which is the instant when the fault occurs. The idea of synchronized relays is that at each sample time kT_S, four new measurements are available, $v_3(kT_S)$, $v_4(kT_S)$, $i_3(kT_S)$, $i_4(kT_S)$.

According to the digital feature of acquired data measurements, a discrete-time version of system (3) is required. To this aim, the central difference approximation is adopted for the time derivative:

$$\left.\frac{di}{dt}\right|_{t=kT_S} \approx \frac{i((k+1)T_S) - i((k-1)T_S)}{2T_S} \tag{4}$$

Such a choice is preferred with respect to others such as the forward or the backward difference approximation because the central difference yields a more accurate approximation, since it involves smaller truncation errors with respect to forward or backward (one order of magnitude), see e.g. [28], [29]. Then, by using the following notation for the samples:

$$v_{i,k} = v_i(kT_S), \quad i_{j,k} = i_j(kT_S), \quad i,j = 3,4, \tag{5}$$

for $k = 0,1,2, \ldots$, system (3) is discretized as:

$$\begin{cases} v_{3,k} = R_{25} i_{3,k} + L_{25} \frac{i_{3,k+1} - i_{3,k-1}}{2T_S} + R_F(i_{3,k} + i_{4,k}) \\ v_{4,k} = (R_{23} - R_{25}) i_{4,k} + (L_{23} - L_{25}) \frac{i_{4,k+1} - i_{4,k-1}}{2T_S} \\ \qquad\qquad + R_F(i_{3,k} + i_{4,k}) \end{cases} \tag{6}$$

Denote with $\theta = [R_{25} \ L_{25} \ R_F]^T$ the parameter vector to be estimated according to a finite set of m pairs of equations as given by (6), that is $k = 1, \ldots, m$. According to (6), the whole set of measurement equations can be put in the more compact form:

$$\Phi_m \cdot \theta = \Gamma_m \tag{7}$$

where matrix $\Phi_m \in \mathbb{R}^{2m \times 3}$ and vector $\Gamma_m \in \mathbb{R}^{2m}$ appropriately collect the voltage/current measurements:

$$\Phi_m = \begin{bmatrix} i_{3,1} & \frac{i_{3,2} - i_{3,0}}{2T_S} & i_{3,1} + i_{4,1} \\ -i_{4,1} & -\frac{i_{4,2} - i_{4,0}}{2T_S} & i_{3,1} + i_{4,1} \\ \vdots & \vdots & \vdots \\ i_{3,m} & \frac{i_{3,m+1} - i_{3,m-1}}{2T_S} & i_{3,m} + i_{4,m} \\ -i_{4,m} & -\frac{i_{4,m+1} - i_{4,m-1}}{2T_S} & i_{3,m} + i_{4,m} \end{bmatrix}, \tag{8}$$

$$\Gamma_m = \begin{bmatrix} v_{3,1} \\ v_{4,1} - R_{23} i_{4,1} - L_{23} \frac{i_{4,2} - i_{4,0}}{2T_S} \\ \vdots \\ v_{3,m} \\ v_{4,m} - R_{23} i_{4,m} - L_{23} \frac{i_{4,m+1} - i_{4,m-1}}{2T_S} \end{bmatrix}. \tag{9}$$

Due to the measurement/discretization noises involved in the equations, there is no exact solution to the linear problem presented in (7). A way to cope with the absence of any *a priori* knowledge about the noise statistics is the Weighted Least Square (WLS) method, which is a pure deterministic approach strictly based on the mathematical relationship between parameters and data (see e.g. [19] and references therein). The WLS approach provides the estimate $\hat{\theta}_m$ which minimizes the following index:

$$\begin{aligned} \hat{\theta}_m &= \min_\theta J(\theta) \\ J(\theta) &= (\Gamma_m - \Phi_m \theta)^T W_m (\Gamma_m - \Phi_m \theta), \end{aligned} \tag{10}$$

where W_m is a symmetric, positive-definite weight matrix. In other words, the J index is the square of the norm of the measurement error $\Gamma_m - \Phi_m \theta$, weighted by the matrix W_m:

$$J(\theta) = \|\Gamma_m - \Phi_m \theta\|_{W_m}^2. \tag{11}$$

In case of W_m given by a diagonal matrix, i.e. $W_m = diag\{w_1, \ldots, w_{2m}\}$, (which means all independent measurements), the J index is reduced to the following sum:

$$\begin{aligned} J(\theta) = &\sum_{k=1}^{m} w_{2k-1} \Bigg(v_{3,k} - R_{25} i_{3,k} - L_{25} \frac{i_{3,k+1} - i_{3,k-1}}{2T_s} \\ &\qquad - R_F(i_{3,k} + i_{4,k}) \Bigg)^2 \\ &+ \sum_{k=1}^{m} w_{2k} \Bigg(v_{4,k} - (R_{23} - R_{25}) i_{4,k} \\ &\qquad - (L_{23} - L_{25}) \frac{i_{4,k+1} - i_{4,k-1}}{2T_s} - R_F(i_{3,k} + i_{4,k}) \Bigg)^2 \end{aligned} \tag{12}$$

The problem stated in (10) is solved by means of the following equation (see e.g. [19]):

$$\hat{\theta}_m = \Phi_{W_m}^{\dagger} \cdot \Gamma_m \tag{13}$$

where

$$\Phi_{W_m}^{\dagger} = (\Phi_m^T W_m \Phi_m)^{-1} \Phi_m^T W_m \tag{14}$$

is the Moore-Penrose pseudo-inverse of Φ_m.

Remark 2. In case of partial knowledge of the noises affecting the system, the weighting matrix W_m may well assume a precise statistical meaning. For instance, suppose to write equation (7) as:

$$\Phi_m \cdot \theta + \eta_m = \Gamma_m \tag{15}$$

where $\eta_m \in \mathbb{R}^{2m}$ is a Gaussian zero-mean random vector, with known covariance matrix Ψ_m. Then, it can be shown that the Maximum Likelihood estimate of θ is provided by the optimal solution (13-14), with $W_m = \Psi_m^{-1}$ (see [19] for more details). In other contexts, when no *a priori* information is available about the noises and all measurements are equivalent, then matrix W_m may well be set equal to the identity matrix.

Remark 3. It must be noted that, according to the non-causal discretization adopted in (4), the $\hat{\theta}_m$ estimate requires voltage measurements up to the sample time mT_S and current measurements up to the sample time $(m+1)T_S$. It means that $\hat{\theta}_m$ is computed at time $(m+1)T_S$. Such a delay in the estimate availability may be exploited for the synchronization of cooperating relays.

According to (13-14), whenever a new measurement is available, a matrix inversion is required, and the matrix to be inverted is computed by using an increasing number of sums and products; a further drawback is that a great amount of memory is necessary to take into account all the previous measurements. The Recursive Least Squares (RLS) approach allows to overcome all these drawbacks, providing a recursive implementable solution to problem (10), by suitably exploiting equations (13-14). Indeed, suppose that the parameter vector $\hat{\theta}_m$ is estimated from data matrices Φ_m, Γ_m (i.e. from voltage and current measurements $v_{3,k}$, $v_{4,k}$, $i_{3,j}$, $i_{4,j}$, $k = 1, \dots, m$, $j = 0, \dots, m+1$); then, by exploiting the measurements at the next sample time (i.e. $v_{3,m+1}$, $v_{4,m+1}$, $i_{3,m+2}$, $i_{4,m+2}$), assuming matrix W_m in a diagonal form, it is:

$$\Phi_{m+1} = \begin{bmatrix} \Phi_m \\ \phi_{m+1} \end{bmatrix}, \qquad W_{m+1} = \begin{bmatrix} W_m & 0 \\ 0 & \omega_{m+1} \end{bmatrix},$$

$$\Gamma_{m+1} = \begin{bmatrix} \Gamma_m \\ \gamma_{m+1} \end{bmatrix}, \tag{16}$$

with (recall that, due to the cooperative relays, each sample time two pairs of voltage/current measurements are available):

$$\phi_{m+1} = \begin{bmatrix} i_{3,m+1} & \frac{i_{3,m+2} - i_{3,m}}{2T_s} & i_{3,m+1} + i_{4,m+1} \\ -i_{4,m+1} & -\frac{i_{4,m+2} - i_{4,m}}{2T_s} & i_{3,m+1} + i_{4,m+1} \end{bmatrix} \in \mathbb{R}^{2\times 3} \tag{17}$$

$$\omega_{m+1} = \begin{bmatrix} w_{2m+1} & 0 \\ 0 & w_{2m+2} \end{bmatrix} \in \mathbb{R}^{2\times 2} \tag{18}$$

$$\gamma_{m+1} = \begin{bmatrix} v_{3,m+1} \\ v_{4,m+1} - R_{23} i_{4,m+1} - L_{23} \frac{i_{4,m+2} - i_{4,m}}{2T_s} \end{bmatrix} \in \mathbb{R}^2 \tag{19}$$

Now let matrix S_m be defined as:

$$S_m = (\Phi_m^T W_m \Phi_m)^{-1}, \tag{20}$$

according to which, solution (13-14) can be written as

$$\hat{\theta}_m = S_m \Phi_m^T W_m \Gamma_m. \tag{21}$$

Then, the following recursive formula is readily obtained:

$$\begin{aligned} S_{m+1} &= \left(\begin{bmatrix} \Phi_m^T & \phi_{m+1}^T \end{bmatrix} \begin{bmatrix} W_m & 0 \\ 0 & \omega_{m+1} \end{bmatrix} \begin{bmatrix} \Phi_m \\ \phi_{m+1} \end{bmatrix} \right)^{-1} \\ &= (\Phi_m^T W_m \Phi_m + \phi_{m+1}^T \omega_{m+1} \phi_{m+1})^{-1} \\ &= \left(\Phi_m^T W_m \Phi_m (I_3 + (\Phi_m^T W_m \Phi_m)^{-1} \phi_{m+1}^T \omega_{m+1} \phi_{m+1}) \right)^{-1} \\ &= (I_3 + S_m \phi_{m+1}^T \omega_{m+1} \phi_{m+1})^{-1} S_m \end{aligned} \tag{22}$$

with I_3 denoting the identity matrix in $\mathbb{R}^{3\times 3}$. Thus, the $\hat{\theta}_{m+1}$ estimate is computed as:

$$\begin{aligned} \hat{\theta}_{m+1} &= S_{m+1} \Phi_{m+1}^T W_{m+1} \Gamma_{m+1} \\ &= S_{m+1} (\Phi_m^T W_m \Gamma_m + \phi_{m+1}^T \omega_{m+1} \gamma_{m+1}) \\ &= \hat{\theta}_m + S_{m+1} \left(\Phi_m^T W_m \Gamma_m - S_{m+1}^{-1} \hat{\theta}_m + \phi_{m+1}^T \omega_{m+1} \gamma_{m+1} \right) \\ &= \hat{\theta}_m + S_{m+1} \phi_{m+1}^T \omega_{m+1} \left(\gamma_{m+1} - \phi_{m+1} \hat{\theta}_m \right). \end{aligned} \tag{23}$$

As far as the algorithm initialization is concerned, one method is to consider a small initial segment of data (enough to ensure the invertibility of matrix S), according to which the *non-recursive* WLS algorithm summarized by eq.(20-21) provides the initial values for both the estimate $\hat{\theta}$ and the S matrix. In this way there is a perfect match between the general (20-21) and the recursive (22-23) solutions.

The steps of the algorithm are summarized below:

0. Initialization of the algorithm:
   - define a criterion (if any) to set the weights w_k
   - starting from the instant when a fault is known to have occurred, compute $\hat{\theta}_{\bar{m}}$ and $S_{\bar{m}}$ from the first set of samples acquired (that is from $v_{3,1}$, ..., $v_{3,\bar{m}}$, $v_{4,1}$, ..., $v_{4,\bar{m}}$; $i_{3,0}$, ..., $i_{3,\bar{m}+1}$, $i_{4,0}$, ..., $i_{4,\bar{m}+1}$), according to (20) and (21)
1. set $m = \bar{m}$
2. acquire the next set of samples, $v_{3,m+1}$, $v_{4,m+1}$, $i_{3,m+2}$, $i_{4,m+2}$, from which define ϕ_{m+1}, γ_{m+1}, equations (17) and (19)
3. compute S_{m+1} according to (22)

4. compute $\hat{\theta}_{m+1}$ according to (23); consequently, the fault distance is obtained according to the resistance-inductance estimates given by eq. (2)
5. increment the counter $m = m + 1$
6. GO TO step 2.

Remark 4. A common drawback in the RLS approach is that matrix S_m converges to zero as $m \to \infty$, and, therefore, the correction term in (23) becomes useless as time goes on. This is clearly a problem in case of (slowly) time-varying parameters, since the algorithm is not able to update the estimate in case of parameter variations. A possible way to cope with this problem is to use a *forgetting factor* which gives less weight to older error samples (see e.g. [19]). However, the above mentioned drawback does not affect the algorithm in the present framework. Indeed, the algorithm starts to work as soon as the fault is detected and runs for a short time period (milliseconds) before providing the fault distance estimate: during this period it is reasonable to assume time-invariant model parameters.

Remark 5. It must be emphasized that the RLS equations may be seen as the Kalman Filter equations [20-21] applied to the following linear system:

$$\begin{cases} \theta_{m+1} = \theta_m \\ \gamma_m = \phi_{m+1}\theta_m + \omega_m^{-\frac{1}{2}} N_m \end{cases} \tag{24}$$

where N_m is a sequence of zero-mean independent random vectors, with an identity covariance matrix (white noise sequence). Indeed, the Kalman recursive state estimate equation is given by (23), where the Kalman one-step prediction error covariance matrix is equal to the Kalman error covariance matrix which, in its turn, according to a common initialization, coincides with S_m.

IV. Test System and Simulation Results

In the following, the most significant simulation results are reported in order to evaluate the validity limits of the proposed method subordinately to the fault distance estimates and to the time needed to obtain such estimates.

A. The Test System

The proposed method has been tested in a simulation on a $20kV$ system as shown in Fig. 3. Though the rated voltage of the system is $20kV$, simulations consider a voltage supply of $12kV$, which corresponds to the single phase voltage +4%. According to Fig. 3, the fault is assumed to be somewhere on the line segment connecting busses 2 and 3.

The distribution feeder is supplied at one end by the main source $e_{s1}(t)$, which injects only the base frequency. A short circuit power of 300 MVA, with $\cos\varphi_{cc} = 0.3$, is assumed, which corresponds to an internal impedance $Z_{s1} = 1.30\ \Omega$.

The DG is connected at the other end of the line. It is assumed to be a voltage generator $e_{s2}(t)$, with an internal impedance of 38.4 Ω, with $\cos\varphi_{cc} = 0.3$, which includes the step-up transformer of DG. The DG capacity is limited to 2.5 MVA. In order to take into account random disturbances caused by nonlinear devices [10], [22-24] higher, sub- and inter- harmonics are added to the fundamental ($50Hz$, $12kV$) according to the following model:

$$e_{S2}(t) = 0.1e_{S2,28Hz}(t) + e_{S2,50Hz}(t) + 0.1e_{S2,100Hz}(t) + 0.2e_{S2,150Hz}(t) + 0.1e_{S2,117Hz}(t) \tag{25}$$

where, for instance, $e_{S2,28Hz}(t)$ stands for a sub-harmonic at $28Hz$ added to the fundamental.

A second source of nonlinear disturbances is modeled by the harmonic current generator $i_s(t)$ (connected on bus 2), which injects the base frequencies ($50Hz$, $30A$) and other harmonics, whose amplitudes and frequencies are defined by the following relation [10]:

$$i_S(t) = i_{S,50Hz}(t) + 0.20i_{S,250Hz}(t) + 0.14i_{S,350Hz}(t) + 0.091i_{S,550Hz}(t) + 0.077i_{S,650Hz}(t) + 0.059i_{S,850Hz}(t) + 0.053i_{S,950Hz}(t) \tag{26}$$

Current harmonics are often injected by power converters.

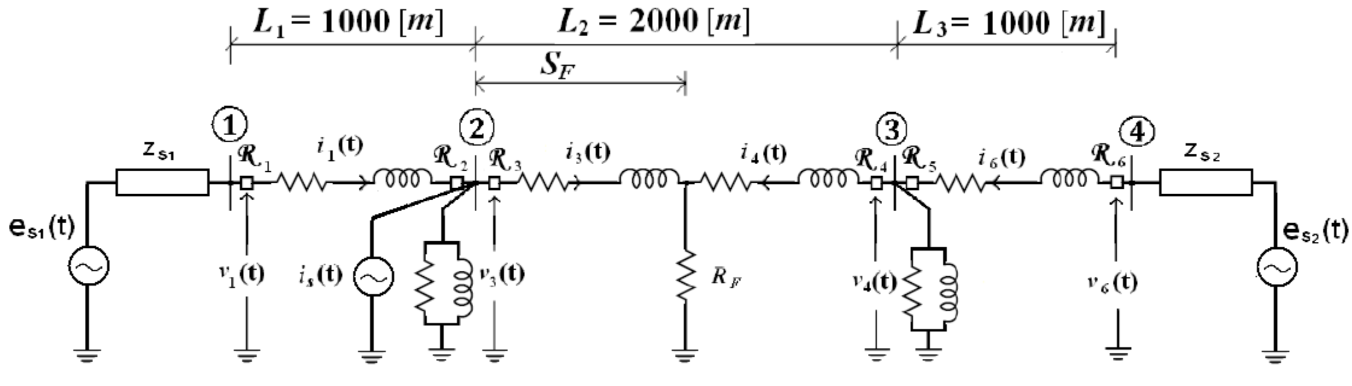

Fig. 3 The examined MV-distribution feeder with the locations of distance relays ($\mathcal{R}_i$, i = 1, … ,6) and fault resistance (R_F); e_{s1} and e_{s2} are the main equivalent source and the equivalent of the distributed generator, respectively.

Commercially, different kind of converters are available; among them there are apparatuses characterized by a very low level of harmonic pollution [25-27].

The MV-line consists of a copper conductor of $140mm^2$-section which exhibits the following unit parameters:

$$R' = R'_{12} = R'_{23} = R'_{34} = 0.145\ \Omega/km$$
$$L' = L'_{12} = L'_{23} = L'_{34} = 1.1\ mH/km \tag{27}$$

The adopted sampling frequency is $f_s = 2.5kHz$, corresponding to a sampling time $T_s = 400\mu s$, i.e. 2500 samples per second.

The two linear loads, which are connected on busses 2 and 3, draw 3MVA, $\cos\varphi = 0.9$, and 2MVA, $\cos\varphi = 0.9$, respectively.

B. The Simulation Results

A number of simulations were carried out by means of the Matlab code with different fault distances and fault resistances. In all the proposed test cases, the fault distances were computed as the average value of both estimates coming from the resistance-inductance estimates, according to (2). With reference to relay $\mathcal{R}_3$, three cases were considered for the fault distance:

$$S_F = 100m, \qquad S_F = 900m, \qquad S_F = 1900m,$$

in order to take into account close, medium and far distances, while five cases were considered for the fault resistance:

$$R_F = 0\Omega, \quad R_F = 1\Omega, \quad R_F = 10\Omega, \quad R_F = 50\Omega, \quad R_F = 100\Omega$$

Table 1 shows the results of 15 tests for different values of S_F and R_F (columns 2 and 3); columns 4 and 5 report the percentage error (absolute value) of the lumped resistance-inductance estimates, obtained after a given time (in $[ms]$):

$$\Delta R(m) = 100\frac{\hat{R}_{25}(m)-R_{25}}{R_{25}}, \qquad \Delta L(m) = 100\frac{\hat{L}_{25}(m)-L_{25}}{L_{25}} \tag{28}$$

Column 6 reports the percentage error (absolute value) of the fault distance estimate. For instance, Test 1 says that for a fault occurring at a distance of $1900m$ from relay $\mathcal{R}_3$, with a fault resistance R_F equal to zero:

- resistance R_{25} is estimated with an error percentage (absolute value) definitely lower than 1% after $4.8ms$;
- inductance L_{25} is estimated with an error percentage (absolute value) definitely lower than 1% after $2.0ms$;
- the fault distance S_F is estimated with an error percentage (absolute value) definitely lower than 1% after $3.6ms$.

It has to be stressed that the algorithm is very fast, since it allows to estimate the fault distance with a high degree of precision within the first $10ms$ from the fault occurrence. And this happens whatever the value of the fault resistance. For instance, in case of high distances from relay $\mathcal{R}_3$ (Tests 1-5), the algorithm is able to estimate a fault at $1900m$ with an error lower than $20m$ within the first $8ms$; in case of medium distances (Tests 6-10), the algorithm is able to estimate a fault at $900m$ with an error lower than $10m$ within the first $8ms$; in case of small distances (Tests 11-15), the algorithm is able to estimate a fault at $100m$ with an error lower than $5m$ within the first $8ms$.

Table 1 Percentage errors (absolute values) for R_{25}, L_{25} and the fault distance, obtained after a time period expressed in $[ms]$.

	S_F	R_F	ΔR (% / ms)	ΔL (% / ms)	ΔS_F (% / ms)
Test 1	1900	0	< 1.0 / 4.8	< 1.0 / 2.0	< 1.0 / 3.6
Test 2	1900	1	< 1.0 / 6.0	< 1.0 / 3.6	< 1.0 / 3.6
Test 3	1900	10	< 1.0 / 7.6	< 1.0 / 8.8	< 1.0 / 5.6
Test 4	1900	50	< 1.0 / 8.0	< 1.0 / 9.6	< 1.0 / 5.6
Test 5	1900	100	< 1.0 / 9.2	< 1.0 / 9.6	< 1.0 / 8.0
Test 6	900	0	< 1.0 / 4.8	< 1.0 / 2.0	< 1.0 / 3.6
Test 7	900	1	< 1.0 / 6.0	< 1.0 / 3.6	< 1.0 / 4.8
Test 8	900	10	< 1.0 / 7.6	< 1.0 / 9.6	< 1.0 / 5.6
Test 9	900	50	< 2.0 / 8.0	< 1.0 / 5.6	< 1.0 / 8.0
Test 10	900	100	< 2.0 / 8.4	< 1.0 / 8.0	< 1.0 / 8.0
Test 11	100	0	< 1.0 / 5.2	< 1.0 / 2.0	< 1.0 / 4.8
Test 12	100	1	< 1.0 / 5.6	< 1.0 / 2.0	< 1.0 / 4.8
Test 13	100	10	< 2.0 / 9.2	< 2.0 / 4.8	< 2.0 / 5.2
Test 14	100	50	< 6.0 / 9.2	< 2.5 / 11.6	< 2.5 / 8.4
Test 15	100	100	< 6.0 / 19.2	< 6.0 / 12.0	< 5.0 / 8.0

It must be noted that many of these results are obtained with high fault resistances (10Ω, 50Ω, 100Ω), which usually would not allow acceptable estimates with non co-operating relays.

Figures 4-6 refer to Test 8 of Table 1, where a fault at $900m$ is considered, with a fault resistance of 10Ω. More in detail, Fig. 4 plots the time evolution of the percentage errors concerning the estimates of resistance R_{25} and inductance L_{25}, according to which the fault distance is computed by means of (2). Note that both error percentages are reduced below 2% within $10ms$. Fig. 5 plots the time evolution of the fault distance percentage error as computed from resistance estimates (squares), from inductance estimates (dots) and from the mean average of both (circles). Finally, Fig. 6 displays the percentage error of the fault resistance estimate versus time. Indeed, in this case, R_F is estimated with a very good precision and high speed, since it reduces below 0.1% within the first $3ms$.

Table 2 reports the time instants (in $[ms]$) when the percentage error (absolute value) of the fault resistance estimate is definitely below 0.01%.

Table 2 Time instants $[ms]$ when the error percentage (absolute value) of the fault resistance estimate is definitely below 0.01%.

	$S_F = 100m$	$S_F = 900m$	$S_F = 1900m$
$R_F = 1\Omega$	5.6	5.6	6.0
$R_F = 10\Omega$	4.4	4.0	4.0
$R_F = 50\Omega$	3.6	4.0	4.0
$R_F = 100\Omega$	3.6	2.4	2.4

Note that in this case the fault resistance estimates are even faster (times below 6ms) and provide an excellent degree of precision.

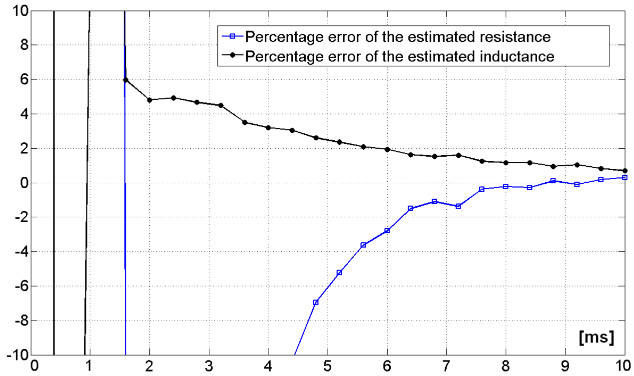

Fig. 4 Time evolution of the percentage error of $\hat{R}_{25}$ (dots) and $\hat{L}_{25}$ (squares). $S_F = 900m$, $R_F = 10\Omega$.

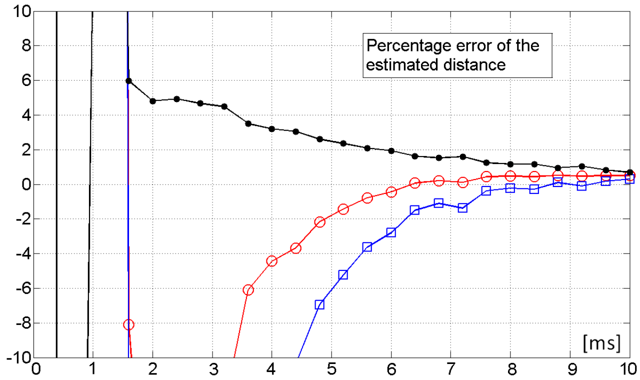

Fig. 5. Time evolution of the fault distance percentage error as computed from resistance (squares), inductance (dots), and both (circles) estimates. $S_F = 900m$, $R_F = 10\Omega$.

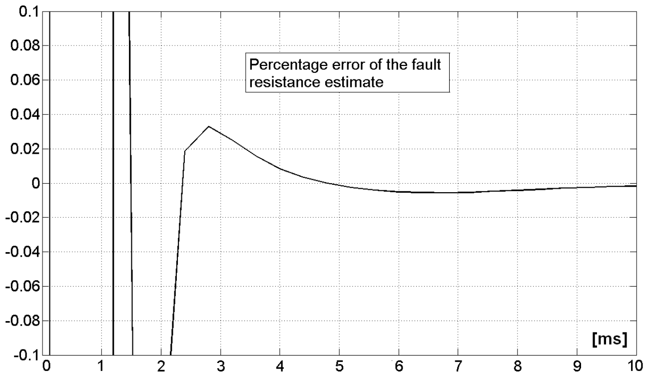

Fig. 6. Time evolution of the fault resistance percentage error. $S_F = 900m$, $R_F = 10\Omega$.

V. Conclusion

In order to adequately protect smart grids with massive presence of DG from renewable sources, GPS synchronized relays are proposed that use a new type of algorithm developed on the basis of distance protection concepts. The estimate of the line lumped parameters is obtained through a particular application of the Recursive Least Square method, which works directly on the synchronized samples of voltage and currents, acquired at both terminations of each MV line segment. As demonstrated by the performed simulations, the proposed technique is robust, very fast and exhibits good performances also in case of high distortion of the voltage and current waveforms. Moreover, a parametric analysis is carried out to evaluate the relay time response, assuming both the fault resistance and fault distance as variables. The results obtained show that the proposed protection system works correctly in any condition, allowing an accurate and timely selectivity between relays as well. Although the kind of fault herein examined is a three phase symmetrical fault, the validity of the method can be easily extended to unsymmetrical faults.

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