Use of data assimilation in an integrated model of the economics of marine ecosystem

Shoichi Kiyama

***Abstract*—** To realize sustainable real-world fishery management, this paper attempts to increase the predictability the fishery needs under uncertainties of model and measurements. Practically, a data assimilation method of the extended Kalman filter is employed to estimate parameters of the bioeconomic model integrated with a computable general equilibrium model. Different from previous studies of a single bioeconomic model, the integrated model extends the state variable of fish catches to describe the trading of fish products as well as the production process of fishing. This enables the observation model to deal with not only fish catches but also the value of production. Assimilating time series data on Japanese clam fishery into the integrated model, the selection problem of observation variables is investigated to increase predictability. As a result, this study demonstrates that the identification of model parameters by the extended Kalman filter can be stably performed. An analysis of predictability shows that the most suitable observation model consists of both the fish catches and value of production.

***Keywords*—** Bioeconomic model, Data assimilation, Extended Kalman filter, Parameter estimation, Sustainable fishery management.

I. INTRODUCTION

THE difficulty of performing sustainable fishery management is a major concern in world resource management. One of the main obstacles is believed to be the depletion of aquatic resources, which is caused by deterioration in ocean water quality, the existence of predators and pathogenic organisms, and the improper management of fishing activity for economic incentives (e.g., overexploitation).

However, the estimation of fish stock and catches is the basis of fishery management. To precisely estimate the stock and catches is considerably difficult because of the uncertainty of measurements, modeling, and policy implications. Clark [1] suggested that many fisheries have failed to prevent overfishing because of a lack of economic incentives, even when fish stock management was considered to apply sound science. Worm et al. [2] indicated that human-dominated marine ecosystems have accelerated the loss of species populations with largely unknown consequences. The Food and Agriculture Organization of the UN (FAO) [3] has found that overexploitation and resource depletion in world fisheries are increasing; in 2008, the share of fisheries considered overexploited was 30 %, and 80 % of world fisheries lacked abundant aquatic resources.

In such a situation, the deterministic bioeconomic model has been well developed toward sustainable fishery management (Gordon [4], Schaefer [5]). For example, an early empirical study predicted the population of Pacific Halibut and California sardine by a single species model (Schaefer [6]). Flaaten [7] developed a deterministic bioeconomic model of multi-species to analyze maximum sustainable yields of capelin and cod in the Barents Sea.

However, with the number of fishery collapses in the world expected to increase, fishery management requires a more robust model that accounts for uncertainties. Roughgarden and Smith [8] illustrated that the ecologically recommended stock size was much larger than the economic optimal stock size under the uncertainty of a marine ecosystem. Therefore, ignoring the uncertainty could lead to excessive exploitation. Given difficult situation for obtaining sufficient ecological measurements due to the uncertainty of ecosystems (e.g., multi-species predator–prey relationships), we must rationally determine poorly known parameters of a bioeconomic model in some way.

In addition, a long-term fishery management model should consider the uncertainty created by economic fluctuations in consumer preferences, life quality, and the global market as well as ecological fluctuations responsible for fish growth rates.

To address the issues mentioned above, more suitable is the employment of a stochastic model, which is an approach to deal with the uncertainty of bioeconomics. For example, there is a data assimilation method in which measurements (observation values) with randomness are incorporated into the equation of a dynamic model with certain state variables. Then, the state variables are estimated through a filtering process. Fishery economics increasingly applies the following data assimilations approaches; a variational assimilation (Lowson et al. [9] and Ussif et al. [10]) and a sequential assimilation including the extended Kalman filter (Berck and Grace [11], Meyer and Millar [12], Peterman et al. [13], Dorner et al. [14], and Kvamsdal and Sandal. [15]). In addition, there is a circuit analysis that uses the block-diagram on a state-space system (Keller [16]).

However, all of these studies are limited to a single industry problem with the observed variable of fish catches or fish stock. In these cases, because of a lack of price variables, it cannot be understood how the fisheries are influenced by other industry and consumer demands under global economic fluctuations that

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then contribute to their collapse. Therefore, a consideration of both multi-industries and multi-regions becomes increasingly important for a more accurate prediction of real-world fisheries management.

In the context of a society subjected to fishery collapse, this study attempted to estimate a bioeconomic model parameter with the aid of the extended Kalman filter (Kalman [17] and Jazwinski [18]), a sequential assimilation method. This has the potential to predict real-world fisheries catches and production value under the uncertain environment of an aquatic ecosystem and economic market and helps to explain the mechanism of fishery collapse. For this reason, this study set out to introduce the value of production into observation equations. A bioeconomic model endogenously deals with two factors likely to cause fishery collapse: feeding damage by predators and the abandonment of fishery management. The bioeconomic model is incorporated into a dynamic computable general equilibrium (CGE) model (i.e., a two-country model [19]).

This paper provides details of the model development and interpretation of results based on a previous work (Kiyama [20]). This paper is organized as follows. Section 2 outlines the integration of a two-country model and a bioeconomic model. Then, to estimate model parameters by the iterated extended Kalman filter, necessary procedures are described. Noting a case of fishery collapse, section 3 demonstrates the parameter estimation and examines a proper definition of the observation equation. The predictability of models estimated by the extended Kalman filter is discussed.

II. Materials and Methods

A. Two-country Model

A two-country model was assumed with two regions ($k = 1, 2$), two industries of a certain fishery ($j = 1$), and the rest of the industry ($j = 2$). Following is the formulation of the fishery economics used in the estimation of the model parameters. Fig. 1 represents a CGE model structure relevant to the regional fishery industry. We considered two processes in the fishery industry; the inputs of necessary factors for fish catches (the harvest process) and the distribution of caught fishes (the trading process).

The input process for the fish catches is described as follows. In a region k, a fishery inputs a resource stock XM_k, labor $E_{k,1}$, and capital $K_{k,1}$, and then produces the goods of a resource–labor–capital bundle $Y_{k,1}$. In addition, inputs of $Y_{k,1}$ and intermediate goods $XX_{k,j}$ provide the domestic output (i.e., the fish catches $Z_{k,1}{}^{(1)}$).

$$Z_{k,1}{}^{(1)} = \frac{Y_{k,1}}{ay_{k,1}} = \frac{q_{k,1}}{ay_{k,1}} E_{k,1}\Big|_t^{\sigma_{k,1}} K_{k,1}\Big|_t^{(1-\sigma_{k,1})} XM_k\Big|_t \tag{1}$$

where $ay_{k,1}$ is the input coefficient of $Y_{k,1}$, $q_{k,1}$ is the catchability coefficient representing a technology level of the production efficiency in fishing. The parameter $\sigma_{k,1}$ is the Cobb–Douglas power of labor (Schaefer [5]). The technology level was assumed to be time-dependent.

$$q_{k,1}(t) = q_{0k,1} t^{-kq_{k,1}} \qquad t \geq 1 \tag{2}$$

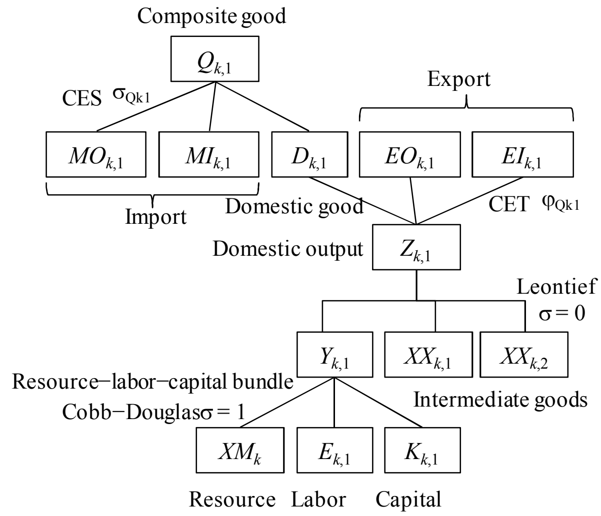


Fig. 1 Model structure of fishery production.

where $q_{0k,1}$ is the initial technology level and $kq_{k,1}$ is a parameter of the rate of change in the technology level. Equation (2) describes the cumulative effect of shifts in the production function (1) over time (Clark [21], Anderson [22] and Hannesson [23]). For example, a greater $q_{k,1}$ describes that the industry has a technology to increase production with a certain amount of inputs. The industry improves or degenerates the technology for production when the sign of the parameter $kq_{k,1}$ is a negative or positive in (2).

Next, the following trading in fish catches was considered. The fish caught in a region are transformed to domestic goods $D_{k,1}$ and exported goods $EI_{k,1}$ and $EO_{k,1}$ to maximize the profit from trading. As a result, the following relation between the domestic goods and catches $Z_{k,1}{}^{(2)}$ is given (see Appendix A).

$$Z_{k,1}{}^{(2)} = D_{k,1} \left(\frac{p_{d\,k,1}}{\delta d_{k,1}\, \vartheta_{k,1}{}^{\phi_{k,1}}\, p_{z\,k,1}} \right)^{\frac{1}{1-\phi_{k,1}}} \tag{3}$$

where $\phi_{k,1}$ is the coefficient of the elasticity of transformation, $\delta d_{k,1}$ denotes the share parameter of domestic goods, $\theta_{k,1}$ represents a scale parameter, p_d is the domestic price, and p_z is the producer's fish price. The scale parameter is defined as a time function.

$$\vartheta_{k,1}(t) = \vartheta_{0k,1} t^{-kf_{k,1}} \quad t \geq 1 \tag{4}$$

where $\theta_{0k,1}$ is the initial value of $\theta_{k,1}$ and $kf_{k,1}$ is a parameter. Equation (4) denotes the influence of trading on regional fish catches over time. However, the CGE model requires that the catches variables, $Z_{k,1}{}^{(1)}$ and $Z_{k,1}{}^{(2)}$, must be equal.

$$Z_{k,1}{}^{(1)} = Z_{k,1}{}^{(2)} \equiv Z_{k,1} \tag{5}$$

B. Fisheries Bioeconomics

The conventional bioeconomic model described by the following dynamics of resource stock was used.

$$\begin{aligned} dXM_k/dt = {} & F_k(XM_k) - Z_{k,1}{}^{(1)}(XM_k) \\ & + GX_k(XM_k) - H_k(XM_k) \end{aligned} \tag{6}$$

where F_k is the natural growth rate of a fish, GX_k is the incremental rate of fish stocks by a seedling release, and H_k denotes the reduction rate of fish stocks by feeding damage.

The multi-species bioeconomic model has been developed by Flaaten [7], Clark [21], Hannesson [24], Hsu et al. [25], and Suwandechochai et al. [26]. On the basis of the previous model, this study utilizes a predator–prey model with the Michaelis–Menten-type function. In addition, this study considered a situation of no data about the predator's biology because of a lack of predator data in many cases. To simplify the predator stock dynamics, the accumulated predator weight was assumed to be equivalent to the weight of the predator mixed with the released seedling. The corresponding formulation is written as follows.

$$F_k = \left\{ -\left(\bar{b} - \bar{c}\,Ke\right)Ke + \bar{b}\frac{XM_k}{A_k} - \bar{c}\left(\frac{XM_k}{A_k}\right)^2 \right\} A_k \qquad (7)$$

$$GX_k = \left(1 - p_{mk}\right) SE_k \qquad (8)$$

$$H_k = V_{xs\,k}\frac{XM_k}{K_{xs\,k} + XM_k} SS_k \qquad (9)$$

$$SS_k\big|_t = \sum_{T=1}^{t} p_{mk}\, SE\big|_T \qquad (10)$$

where $\bar{b}$ and $\bar{c}$ are parameters to describe the natural growth rates of a species, Ke is the carrying capacity (Gordon [4]), A_k is a fishing area, p_{mk} is the percentage of predators mixed in with the seedlings, SE_k is the amount of seedlings released, V_{xs} is the capture rate, K_{xs} is the half saturation constant, and SS_x is the weight of the predators.

C. State-space System

For a nonlinear stochastic system, the following state-space system was considered.

$$d\boldsymbol{x}_t / dt = \boldsymbol{f}(\boldsymbol{x}_t, t) + \boldsymbol{G}(t)\,\boldsymbol{\omega}_t \qquad (11)$$

$$\boldsymbol{y}_t = \boldsymbol{h}(\boldsymbol{x}_t, t) + \boldsymbol{v}_t \qquad (12)$$

where $\boldsymbol{x}_t$ is a system state vector, t is a time, $\boldsymbol{f}$ denotes a vector of system's dynamics without error sources, $\boldsymbol{G}$: $\partial \boldsymbol{f} / \partial \boldsymbol{x}$, $\boldsymbol{\omega} \sim \boldsymbol{N}(0, \boldsymbol{R}_t)$ is a white Gaussian vector with zero mean and covariance matrix $\boldsymbol{R}$, $\boldsymbol{y}$ is an observation vector, $\boldsymbol{h}$ is a vector of measurement functions without error sources, and $\boldsymbol{v} \sim \boldsymbol{N}(0, \boldsymbol{Q}_t)$ is a white Gaussian vector with zero mean and covariance matrix $\boldsymbol{Q}$.

In the bioeconomic model, (11) was associated with the dynamics of fish resource stock in (6). A state vector consisting of regional fish stocks, a predator stock, and seven model parameters was defined as follows.

$$\boldsymbol{x} = \left(XM_1, XM_2, SS_1, K_e, kq_{11}, kq_{21}, V_{xs}, p_{m1}, kf_{11}, kf_{21}\right)^T \qquad (13)$$

This study assumed that a predator-prey relation was established in a region (k = 1). In this case, components of the system's dynamics vector $\boldsymbol{f}$ are written as follows.

$$f_i = FF_i - Z_i^* - H_i + GX_i\,, i = 1,2\text{, i.e.,}$$
$$f_1 = f_1\left(XM_1, SS_1, K_e, kq_{11}, V_{xs}, p_{m1}, kf_{11}\right),$$
$$f_2 = f_2\left(XM_2, K_e, kq_{21}, kf_{21}\right),$$
$$f_3 = SS_1 = f_3\left(p_{m1}\right),$$
$$f_i = 0\,,\;\; i = 4, 10 \qquad (14)$$

Different from the conventional model, it was considered that the variable $Z_{k,1}^{(1)}$ in (6) was replaced by the following extended catches $Z^*_{k,1}$ with a weighting factor α.

$$Z^*_{k,1} = \alpha Z^{(1)}_{k,1} + (1-\alpha) Z^{(2)}_{k,1} = Z_{k,1}\,,\;\; 0 \le \alpha \le 1 \qquad (15)$$

Thus, the parameter kf is introduced in the system on contact with $Z_{k,1}^{(2)}$, and kf can be estimated.

In addition, this study considered that a vector of observation equation consisted of regional values of production, $PV^{(obs)}$ and regional catches $Z^{(obs)}$; $\boldsymbol{y} = \left(PV^{(obs)}_{1,1}, PV^{(obs)}_{2,1}, Z^{(obs)}_{1,1}, Z^{(obs)}_{2,1}\right)^T$. The regional catches were assumed to be the extended catch variables $Z^*_{k,1}$. Therefore, the measurement function vector $\boldsymbol{h}$ can be written as follows.

$$\boldsymbol{h} = \left(p_{z1,1}\left(Z^{(2)}_{1,1}\right)Z^{(1)}_{1,1},\, p_{z2,1}\left(Z^{(2)}_{2,1}\right)Z^{(1)}_{2,1},\, Z^*_{1,1},\, Z^*_{2,1}\right)^T \qquad (16)$$

where the fish price can be calculated from (3).

D. Iterative Extended Kalman Filter

This study employed the iterated extended Kalman filter (IEKF) to identify bioeconomic parameters. The following calculation algorithm was used for the estimation of bioeconomic parameters (Jazwinski [18]).

1. Consider the last filtered state estimate in a time step k, $\hat{\boldsymbol{x}}(k|k)$.
2. Predict the system's state at the next time step k+1, $\hat{\boldsymbol{x}}(k+1|k) = \hat{\boldsymbol{x}}(k|k) + \int_k^{k+1} \boldsymbol{f}\left(\hat{\boldsymbol{x}}(t|k), t\right) dt$.
3. Compute the predicted error covariance,
$$\boldsymbol{P}(k+1|k) = \boldsymbol{\Phi}\left(k+1, k; \hat{\boldsymbol{x}}(k|k)\right) \boldsymbol{P}(k|k) \times \boldsymbol{\Phi}^T\left(k+1, k; \hat{\boldsymbol{x}}(k|k)\right) + \boldsymbol{Q}(k+1),$$
$$\boldsymbol{\Phi}\left(k+1, k; \hat{\boldsymbol{x}}(k|k)\right) = \boldsymbol{I} + \Delta\, \boldsymbol{G}\left(\hat{\boldsymbol{x}}(k|k)\right),$$
Δ : Time interval.
4. Store $\hat{\boldsymbol{x}}(k+1|k)$ as an iterator $\boldsymbol{\eta}_1$ and begin the iteration.
5. Update the iterator $\boldsymbol{\eta}_i$ by the gain filter $\boldsymbol{K}$.
$$\boldsymbol{\eta}_{i+1} = \hat{\boldsymbol{x}}(k+1|k) + \boldsymbol{K}(k+1; \boldsymbol{\eta}_i) \times \left[\boldsymbol{y}(k+1) - \boldsymbol{h}(\boldsymbol{\eta}_i, k+1) - \boldsymbol{M}(k+1; \boldsymbol{\eta}_i)\right],$$
$$\boldsymbol{K}(k+1; \boldsymbol{\eta}_i) = \boldsymbol{P}(k+1|k)\, \boldsymbol{M}^T(k+1; \boldsymbol{\eta}_i) \times \left[\boldsymbol{M}(k+1; \boldsymbol{\eta}_i)\, \boldsymbol{P}(k+1|k) \boldsymbol{M}^T(k+1; \boldsymbol{\eta}_i) + \boldsymbol{R}(k+1)\right]^{-1},$$
$$\boldsymbol{M}(k+1; \boldsymbol{\eta}_i) = \partial \boldsymbol{h}(\boldsymbol{\eta}_i, k+1) / \partial \boldsymbol{\eta}$$
6. The iteration terminates with no significant difference between consecutive iterates. Otherwise, return to step 5. The last iterator becomes the system estimate at the next time step k+1, $\hat{\boldsymbol{x}}(k+1|k+1) = \boldsymbol{\eta}_{i+1}$.
7. Compute the new error covariance.
$$\boldsymbol{P}(k+1|k+1) = \left[\boldsymbol{I} - \boldsymbol{K}(k+1; \boldsymbol{\eta}_l)\, \boldsymbol{M}(k+1; \boldsymbol{\eta}_l)\right] \times \boldsymbol{P}(k+1|k) \left[\boldsymbol{I} - \boldsymbol{K}(k+1; \boldsymbol{\eta}_l)\, \boldsymbol{M}(k+1; \boldsymbol{\eta}_l)\right]^T + \boldsymbol{K}(k+1; \boldsymbol{\eta}_l)\, \boldsymbol{R}(k+1)\, \boldsymbol{K}(k+1; \boldsymbol{\eta}_l)^T$$

The details of the matrix used in this study are described in Appendix B. The IEKF was used for the bioeconomic model part after solving the general equilibrium equations of the two-country model. Then, the parameters and stocks updated by the IKEF scheme were used to solve the aquatic resource stocks and general equilibrium equations in the subsequent step. This calculation procedure continued to the final time step. The resultant parameters were used to investigate the model predictability.

E. Data

Two regions were defined as Maizuru City in Kyoto Prefecture, Japan ($k = 1$) and the rest of Kyoto Prefecture ($k = 2$). Annual time series data of the 1980–2006 regional clam catches and their values of production were assimilated into the stock dynamics of the aquatic species model containing trading parameters of the two-country model. The data is provided by the Kyoto Prefectural Agriculture, Forestry and Fisheries Technology Center [27], [28].

According to the results, the recent clam catches in Maizuru City have been very small and there is no prospect to recover the clam catches. This situation was found after the clam fishery released clam seedlings in Maizuru Bay. The seedlings may be a cause of the clam fishery depression. Therefore, the model considered the increase of clam stocks by seedlings in (8) and the decrease of clam stocks by feeding damage in (9). In the rest of the city in Kyoto Prefecture, however, the model excluded these effects on stock change because the clam catches stayed at a low level with a certain fluctuation band but showed signs of recovery.

F. Initial Condition

The data assimilation requires the initial values of model parameters. The initial values of parameters and initial clam stocks are determined by minimizing the square of the sum of the errors between the estimated and observed catches. Other parameters of the two-country model are determined to satisfy the initial equilibrium condition shown by the 1980 regional social accounting matrix. All values of the parameters are shown in reference [19].

To build a suitable system for the parameter estimation, three combinations of measurements were considered: (Case 1) regional catches and regional values of production; (Case 2) regional values of production; and (Case 3) regional catches. The corresponding measurement function $\boldsymbol{h}$ is as follows.

Case 1: $\boldsymbol{h} = \left(p_{z1,1}\left(Z_{1,1}^{(2)}\right)Z_{1,1}^{(1)},\, p_{z2,1}\left(Z_{2,1}^{(2)}\right)Z_{2,1}^{(1)},\, Z_{1,1}^{*},\, Z_{2,1}^{*}\right)^T$, $\alpha = 0.5$.

Case 2: $\boldsymbol{h} = \left(p_{z1,1}\left(Z_{1,1}^{(2)}\right)Z_{1,1}^{(1)},\, p_{z2,1}\left(Z_{2,1}^{(2)}\right)Z_{2,1}^{(1)}\right)^T$, $\alpha = 1$.

Case 3: $\boldsymbol{h} = \left(Z_{1,1}^{(1)},\, Z_{2,1}^{(1)}\right)^T$, $\alpha = 1$.

Case 1 estimates the parameter kf. Case 3 corresponds to the data assimilation of the traditional bioeconomic model. On the basis of the error in the statistics, a regional measurement error was given as the average error for the analytical period (Table 1). Table 2 shows the initial measurements of $\boldsymbol{y}$ and their covariance matrix $\boldsymbol{Q}$, that are calculated on the basis of the measurement errors listed in Table 1.

The initial state estimate error was assumed to be 10% at a maximum. The initial value of the state estimate and the covariance are listed in Table 3. Case 1 adds to consider two state estimates kf_{11} and kf_{21}. The covariance matrix of system noise $\boldsymbol{R}$ was assumed to be one hundredth of the matrix $\boldsymbol{P}$.

Table 1. Definition of observation variables.

	Case 1	Case 2	Case 3
Observation variables (uncertainity %)	$Z_{11}(3)$, $Z_{21}(5)$, $PV_{11}(3)$, $PV_{21}(5)$	$PV_{11}(3)$, $PV_{21}(5)$	$Z_{11}(3)$, $Z_{21}(5)$

Table 2. Initial values of observation and covariance.

Observation	Unit	$\boldsymbol{y}$	$\boldsymbol{Q}$
PV_{11}	Million JPN	43.83	1.728
PV_{21}	Million JPN	30.17	2.276
Z_{11}	t	199	35.6
Z_{21}	t	137	46.9

Table 3. Initial value of state estimate and covariance.

State variable	Unit	$\boldsymbol{x}(1\|1)$	$\boldsymbol{P}(1\|1)$
XM_1	t	771.6	5953.7
XM_2	t	15432	23815
SS_1	t	0	0.4402
Ke	kt/km^2	19.67	0.24
Vxs	y^{-1}	40.46	16.37
p_{m1}	-	0.107	0.000115
kq_{11}	-	0.3	0.0009
kq_{21}	-	0.3	0.0009
kf_{11}	-	-0.02	0.000004
kf_{21}	-	-0.02	0.000004

III. Results

A. Parameter estimation

Fig. 2 shows estimated values of the parameters for the whole time period. Case 1 adds 95% confidence intervals (CI). The estimation of the two parameters, Vxs and p_{m1}, begins in the year after the clam seedlings are released; that is, in 1996.

It is observed that the parameter estimation is influenced by the definition of the observation equation. For example, the estimated carrying capacity of the clam Ke varies from 19.3 kt/km^2 to 21.7 kt/km^2. The previous field survey [27] shows that a proper habitat density is 4.0–24.0 kt/km^2. From the comparison showing that the estimated carrying capacity is close to the upper habitat density, the model estimation is reasonable. For all state estimates, the average ratio of the final covariance value to the initial covariance value is 0.24 in Case 1, 0.38 in Case 2, and 0.55 in Case 3. A smaller ratio represents the estimation with less uncertainty. As a result, Case 1 may be the most probable estimation. However, it is found that all case parameters converge at certain values from 2000. Therefore, it

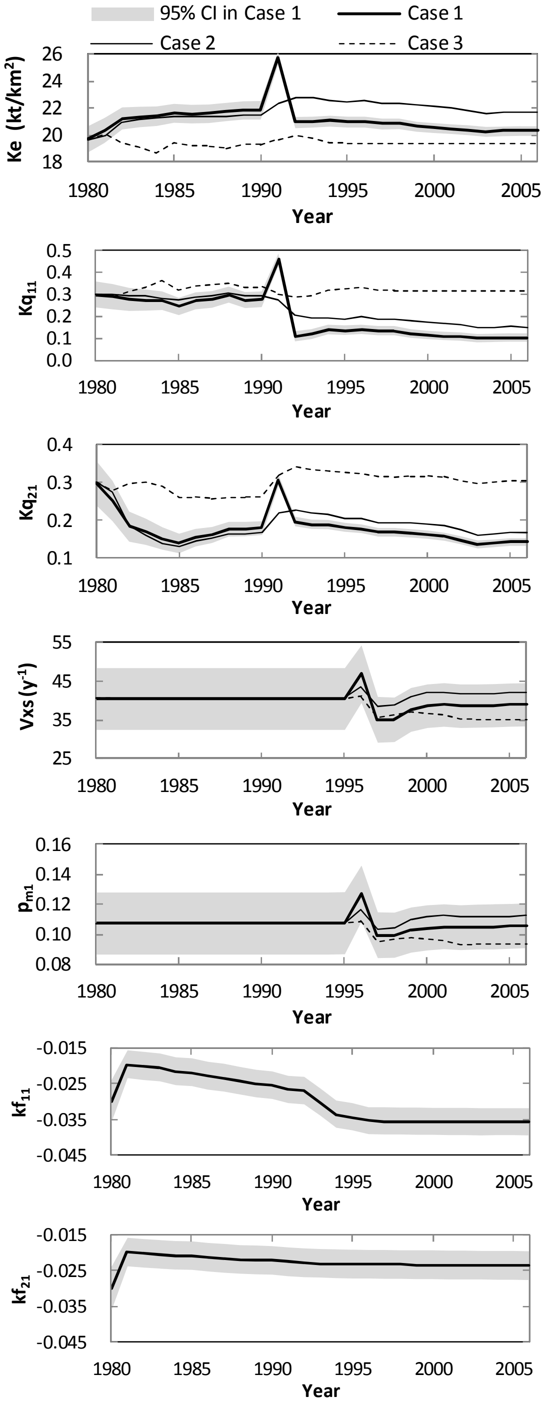
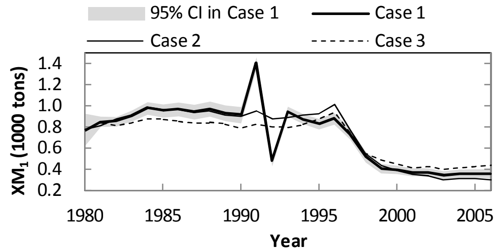


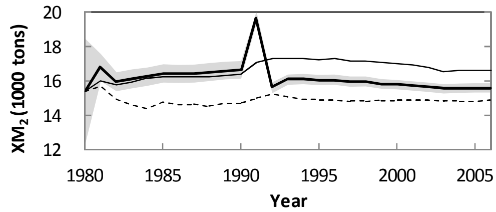
Fig. 2 Estimated values of parameters.

can be said that all cases can estimate the parameters over the whole time period and predictions of all cases are available.

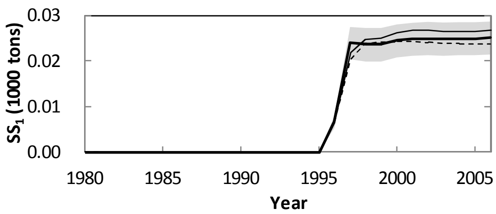
Figs. 3(a) and (b) illustrate the regional clam stock trend during the parameter identification. Case 1 shows that the estimated resource stock changes with a peak magnitude in 1991–1993. However, in the subsequent period, the estimated value stabilizes with a considerably small variance. Other cases show a smaller peak magnitude of the clam stock and a stable transition. In Fig. 3(c), Case 1 shows that the value of the



(a) Clam stock in Maizuru (k = 1)



(b) Clam stock in rest of the city (k = 2)



(c) Predator stock in Maizuru (k = 1)

Fig. 3 Aquatic species stock during parameter identification

variance of the predator stock is approximately constant because the observation model excludes the predator stock. In fact, this study has no observation data on the predator's stock. Therefore, improving the error of this is not covered in this study. However, in all cases, the final fish stocks of the clam and the predator approaches a certain value.

B. Predictability

To investigate predictability of the IEKF, the prediction of all cases was performed with the parameter values estimated over the whole time period. Fig. 4 compares the actual data (observations) with the predicted regional clam catches Z_{11} and Z_{21} and the value of production $PV_{11}$and PV_{21}

However, the base line was defined to investigate the effect of the data assimilation. The base line denotes the prediction with the model parameter without the data assimilation. Thus, the values of parameters were equal to the initial values of parameters before the data assimilation, $\boldsymbol{x}(1|1)$ in Table 1.

For the predicted clam catches Z_{11} and Z_{21}, Case 3 (the observation equation with the catches) performs the best prediction to minimize the prediction error. The base line considerably overestimates the catches. In addition, Cases 1 and 2 overestimate the catches. However, both the cases draw the catches trend close to the actual catches. In particular, Case 1 significantly reduces the prediction error, considering the clam catches as an observation variable. Therefore, the use of parameter values estimated by the extended Kalman filter has an advantage in model predictability.

For the predicted value of production PV_{11} and PV_{21}, Cases 1 and 2 show comparatively good predictions. In particular, Case 2 minimizes the prediction error of the value of production by considering the value of production as an observation variable. On the other hand, Case 3 significantly underestimates the value of production due to the exclusion of the value of production from the observation equation.

From the discussion, it can be said that when an observation variable takes into account either the clam catches or the value of production, the predictability of another item, i.e., the value of production or the catches, worsens. However, the management of fishery resources should seek to precisely grasp the ecological and economic quantities. Therefore, it can be said that Case 1 provides good prediction.

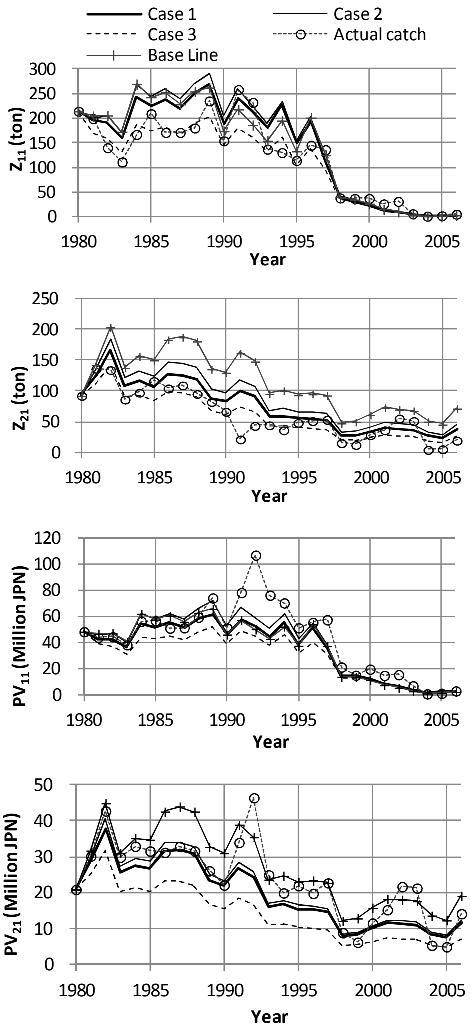


Fig. 4 Predicted catches and value of production.

Finally, a suitable setting of the observation equation for increasing the predictability is investigated by a comparison of the prediction errors of the three cases. Thus, we considered a total prediction error of the three cases, and then divided it by a total prediction error of the base line (the case without the data assimilation). The resultant ratio represents a change in the prediction error associated with the data assimilation. It is clear that Case "x" increases predictability when the ratio of that case is smaller than 1.

Figs. 5 and 6 are comparison charts of the change ratio of the prediction error for the clam catches and value of production, respectively.

In Case 1, all ratios have a smaller value than 1 and have an intermediate value between the ratio in Case 2 and the ratio in Case 3. On the other hand, in Case 2, the ratio for the catches Z_{11} exceeds 1. In Case 3, the ratios for the value of production PV_{11} and PV_{21} are greater than 1. Therefore, it is confirmed that Case 1 provides the best way to reduce any prediction error.

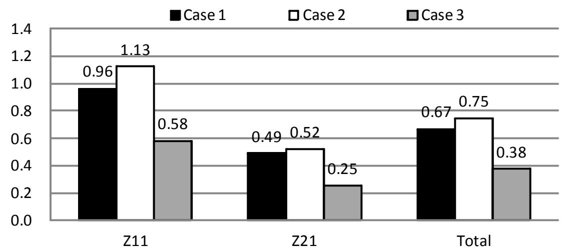


Fig. 5 Change ratio of prediction errors of clam catches.

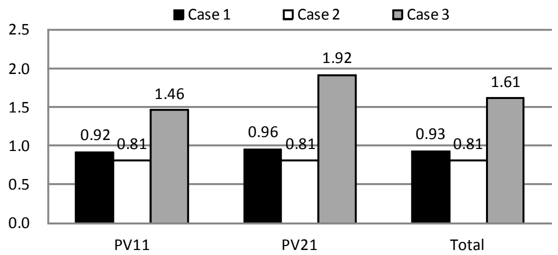


Fig. 6 Change ratio of prediction errors of the value of production.

IV. Discussion

This study analyzes two relationships affecting the implementation of real-world sustainable fishery management: the estimation of bioeconomic parameters under uncertainty and the association between marine ecology and the global market.

Thus, this study demonstrates that data assimilation provides more reliable predictions when bioeconomic parameters are identified with observation variables of the fish catch and its price. This is realized in a procedure wherein a bioeconomic model of multi-species is incorporated into a general equilibrium model with multi-regions and multi-industries. Increasing predictability is a crucial for a sustainable fishery management; however, the deterministic bioeconomic model has failed to provide sufficiently accurate forecasting. Few studies are available on the data assimilation processes, which consider a fish price for the parameter identification.

For studies of sequential data assimilation, Berck and Grace

[11] applied the extended Kalman filter to identify the parameters of the Pacific Halibut fishery model. Expanding on these results, Kvamsdal and Sandal denoted that the extended Kalman filter may be inconvenient when a state-space system has strong nonlinearity and discovered the applicability of the ensemble Kalman filter [15]. In addition, the Kalman filter random-walk model is useful to estimate the parameters of the Ricker stock-recruitment model (Peterman et al. [13], Dorner et al. [14]). However, this study proves that parameter identification by the extended Kalman filter increases predictability of the fish catches and price when a bioeconomic model is incorporated into a general equilibrium model and the corresponding state-space model adds a fish catch variable associated with a fish price.

At least one factor to watch out for is the price of the fishery product because the fish price highly influences the incentives of fishermen to make a living by fishing activity. A decline in price may result in withdrawal from fishing. A resultant abandonment of appropriate resource management may accelerate the depletion of aquatic resources. Thus, the development and collapse in fisheries is greatly influenced by pricing in the global market. In such a situation, Flaaten [29] developed a multi-species bioeconomic model and theoretically demonstrated that fishing in lower valued fish leads to loss. Radulescu et al. [30] noted the importance of minimizing fisheries' financial risk and developed the Portfolio theory for compatible fishery management. Their study adopted the Portfolio theory to consider the uncertainty of multi-species ecosystems. However, those studies treated sustainable fishery management as a single industry problem and ignored the uncertainty of the global market when determining the model parameters.

Other studies do consider a broader economy market with multi-industries and multi-regions for fishery management. Heen [31] investigated the employment and income impact on the regional economy of multi-species harvesting. Kiyama [19] investigated causes of the fishery depression using a two-country model. These studies help to understand the influence of multi-industries on fisheries development. However, these studies use a deterministic model without considering the uncertainty of ecosystems and economy over time. Therefore, we do not gain a sufficient understanding of how the uncertainty of bioeconomics acts on the failure of real-world fishery management.

In conclusion, it is increasingly important to consider fishery management in the framework of a dynamic general equilibrium condition with a stochastic process. Thus, this study covers important market aspects that previous studies exclude; the interaction between fisheries activity and the global market under uncertainty, and resultant fish harvests, prices, and labor inputs. A methodology of bioeconomic parameter identification provided by this study makes it possible to perform a real causal analysis of fishery collapse and policy design to rebuild the fishery.

APPENDIX

A. Trading and fish catches

Consider a profit maximization in trading situation in which industry firms obtain production goods from home region *Z* and supply to home region *D* and other regions *EI* and *EO*. This problem can be written as follows.

$$\max.\left(p_{k,i}^{eo}\, EO_{k,i} + p_{k,i}^{ei}\, EI_{k,i} + p_{k,i}^{d}\, D_{k,i} - p_{k,i}^{z}\, Z_{k,i}\right) \tag{A1}$$

$$\text{s.t.}\ \ Z_{k,i} = \vartheta_{k,i}\Big(\delta eo_{k,i}\, EO_{k,i}{}^{\phi_{k,i}} + \delta ei_{k,i}\, EI_{k,i}{}^{\phi_{k,i}} + \delta d_{k,i}\, D_{k,i}{}^{\phi_{k,i}}\Big)^{\frac{1}{\phi_{k,i}}} \tag{A2}$$

where *EI*: exports into the region defined by the two-country model, *EO*: exports into rest of the world, p^{ei} and p^{eo}: export prices, and p^{d}: home price. Applying the Armington assumption, the output transformation is defined as the constant elasticity of the transformation function with the scale parameter ϑ and the export share parameters δei, δeo, and δd.

The corresponding Lagrange function has a Lagrange multiplier λ, i.e., $L(EO_{k,i}, EI_{k,i}, D_{k,i}, Z_{k,i}, \lambda)$. A stationary point is given when partial derivatives of the Lagrange function are zero ($\partial L/\partial EO_{k,i} = 0$, $\partial L/\partial EI_{k,i} = 0$, $\partial L/\partial D_{k,i} = 0$, $\partial L/\partial Z_{k,i} = 0$, and $\partial L/\partial \lambda = 0$). Equation (3) is given, eliminating the Lagrange multiplier λ from equations of $\partial L/\partial D_{k,i} = 0$ and $\partial L/\partial Z_{k,i} = 0$.

B. Matrix used by the IEKF

Consider numerical calculations by the iterative extended Kalman filter (IEKF). For time step *t*, the vector ***f*** has the following components.

$$\begin{aligned} f_1 = &\left\{-\left(\bar{b} - \bar{c}\,Ke\right)Ke + \bar{b}\frac{XM_1}{A_1} - \bar{c}\left(\frac{XM_1}{A_1}\right)^2\right\}A_1 \\ &- \left\{\alpha\frac{q_{1,1}}{ay_{1,1}}E_{1,1}{}^{\sigma_{1,1}}K_{1,1}{}^{(1-\sigma_{1,1})}XM_1 \right. \\ &\left. + (1-\alpha)D_{1,1}\left(\frac{p_{d1,1}}{\delta d_{1,1}\,\vartheta_{1,1}{}^{\phi_{1,1}}\,p_{z1,1}}\right)^{\frac{1}{1-\phi_{1,1}}}\right\} \\ &- V_{xs1}\frac{XM_1}{K_{xs1} + XM_1}SS_1 + (1 - p_{m1})SE_1\Big|_t \end{aligned} \tag{B1}$$

$$\begin{aligned} f_2 = &\left\{-\left(\bar{b} - \bar{c}\,Ke\right)Ke + \bar{b}\frac{XM_2}{A_2} - \bar{c}\left(\frac{XM_2}{A_2}\right)^2\right\}A_2 \\ &- \left\{\alpha\frac{q_{2,1}}{ay_{2,1}}E_{2,1}{}^{\sigma_{2,1}}K_{2,1}{}^{(1-\sigma_{2,1})}XM_2 \right. \\ &\left. + (1-\alpha)D_{2,1}\left(\frac{p_{d2,1}}{\delta d_{2,1}\,\vartheta_{2,1}{}^{\phi_{1,1}}\,p_{z2,1}}\right)^{\frac{1}{1-\phi_{2,1}}}\right\} \end{aligned} \tag{B2}$$

$$f_3 = \sum_{T=1}^{t} p_{m1} SE\big|_T \tag{B3}$$

The matrix $\boldsymbol{G}$ with a dimension of 10×10 consists of partial derivatives of the function $\boldsymbol{f}$ by state variable $\boldsymbol{x}$ as follows.

$$\partial f_1/\partial x_1 = \bar{b} - 2\bar{c}x_1/A_1 - \alpha\, q_{11}(x_5) E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}}/ay_{11} - x_3\, x_7\, K_{xs}/(K_{xs}+x_1)^2 \tag{B4}$$

$$\partial f_1/\partial x_3 = -x_1\, x_7/(K_{xs}+x_1) \tag{B5}$$

$$\partial f_1/\partial x_4 = -A_1(\bar{b} - 2\bar{c}x_4) \tag{B6}$$

$$\partial f_1/\partial x_5 = \alpha\, q_{11}(x_5) E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} \ln t/ay_{11} \tag{B7}$$

$$\partial f_1/\partial x_7 = -x_1\, x_3/(K_{xs}+x_1) \tag{B8}$$

$$\partial f_1/\partial x_8 = -SE \tag{B9}$$

$$\partial f_1/\partial x_9 = \frac{(1-\alpha) D_{11}\phi_{11}}{(1-\phi_{11})}\left[\frac{p_{d11}}{p_{z11}\delta d_{11}\{\theta_{11}(x_9)\}^{\phi_{11}}}\right]^{\frac{1}{1-\phi_{11}}} \times \ln t \tag{B10}$$

$$\partial f_2/\partial x_2 = \bar{b} - 2\bar{c}x_2/A_2 - \alpha\, q_{21}(x_6) E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}}/ay_{21} \tag{B11}$$

$$\partial f_2/\partial x_6 = \alpha\, q_{21}(x_6) E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} \ln t/ay_{21} \tag{B12}$$

$$\partial f_2/\partial x_{10} = \frac{(1-\alpha) D_{21}\phi_{21}}{(1-\phi_{21})}\left[\frac{p_{d21}}{p_{z21}\delta d_{21}\{\theta_{21}(x_{10})\}^{\phi_{21}}}\right]^{\frac{1}{1-\phi_{21}}} \times \ln t \tag{B13}$$

$$\partial f_3/\partial x_8 = SE \tag{B14}$$

and other components are $\partial f_i/\partial x_j = 0$.

Consider that the equilibrium condition has the relation of $Z_{k1}^{(1)} = Z_{k1}^{(2)}$ from (15). This permits the replacement of the catch $Z_{k1}^{(1)}$ in (5) by $Z_{k1}^{(2)}$. In Case 1, a resultant observation vector consists of the following components.

$$h_1 = p_{z11}\left(Z_{11}^{(1)}\right) Z_{11}^{(1)} = \frac{D_{11}{}^{(1-\phi_{11})} p_{d11}}{\delta d_{11}\{\theta_{11}(x_9)\}^{\phi_{11}}}\left(\frac{q_{11}(x_5)}{ay_{11}} E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} x_1\right)^{\phi_{11}} \tag{B15}$$

$$h_2 = p_{z21}\left(Z_{21}^{(1)}\right) Z_{21}^{(1)} = \frac{D_{21}{}^{(1-\phi_{21})} p_{d21}}{\delta d_{21}\{\theta_{21}(x_{10})\}^{\phi_{21}}}\left(\frac{q_{21}(x_6)}{ay_{21}} E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} x_2\right)^{\phi_{21}} \tag{B16}$$

$$h_3 = \alpha Z_{11}^{(1)} + (1-\alpha) Z_{11}^{(2)} \tag{B17}$$

$$h_4 = \alpha Z_{21}^{(1)} + (1-\alpha) Z_{21}^{(2)} \tag{B18}$$

The matrix $\boldsymbol{M}$ has a dimension of 4×10 and its components are given as a partial derivative of the observation function vector $\boldsymbol{h}$ as follows.

$$\partial h_1/\partial x_1 = \frac{D_{11}{}^{(1-\phi_{11})} p_{d11}\phi_{11}}{\delta d_{11}\{\theta_{11}(x_9)\}^{\phi_{11}} x_1} \times \left(\frac{q_{11}(x_5)}{ay_{11}} E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} x_1\right)^{\phi_{11}} \tag{B19}$$

$$\partial h_1/\partial x_5 = \frac{-D_{11}{}^{(1-\phi_{11})} p_{d11}\phi_{11}}{\delta d_{11}\{\theta_{11}(x_9)\}^{\phi_{11}}}\left(\frac{q_{11}(x_5)}{ay_{11}} E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} x_1\right)^{\phi_{11}} \times q_{11}(x_5)\ln t \tag{B20}$$

$$\partial h_1/\partial x_9 = \frac{D_{11}{}^{(1-\phi_{11})} p_{d11}\phi_{11}}{\delta d_{11}\{\theta_{11}(x_9)\}^{1+\phi_{11}}}\left(\frac{q_{11}(x_5)}{ay_{11}} E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} x_1\right)^{\phi_{11}} \times \theta_{11}(x_9)\ln t \tag{B21}$$

$$\partial h_2/\partial x_2 = \frac{D_{21}{}^{(1-\phi_{21})} p_{d21}\phi_{21}}{\delta d_{21}\{\theta_{21}(x_{10})\}^{\phi_{21}} x_2} \times \left(\frac{q_{21}(x_6)}{ay_{21}} E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} x_2\right)^{\phi_{21}} \tag{B22}$$

$$\partial h_2/\partial x_6 = \frac{-D_{21}{}^{(1-\phi_{21})} p_{d21}\phi_{21}}{\delta d_{21}\{\theta_{21}(x_{10})\}^{\phi_{21}}}\left(\frac{q_{21}(x_6)}{ay_{21}} E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} x_2\right)^{\phi_{21}} \times q_{21}(x_6)\ln t \tag{B23}$$

$$\partial h_2/\partial x_{10} = \frac{D_{21}{}^{(1-\phi_{21})} p_{d21}\phi_{21}}{\delta d_{21}\{\theta_{21}(x_{10})\}^{1+\phi_{21}}}\left(\frac{q_{21}(x_6)}{ay_{21}} E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} x_2\right)^{\phi_{21}} \times \theta_{21}(x_{10})\ln t \tag{B24}$$

$$\partial h_3/\partial x_1 = \alpha\, q_{11}(x_5) E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}}/ay_{11} \tag{B25}$$

$$\partial h_3/\partial x_5 = -\alpha\, q_{11}(x_5) E_{11}{}^{\sigma_{11}} K_{11}{}^{1-\sigma_{11}} \ln t/ay_{11} \tag{B26}$$

$$\partial h_3/\partial x_9 = (1-\alpha)\frac{-D_{11}\phi_{11}}{1-\phi_{11}}\left(\frac{p_{d11}}{\delta d_{11} p_{z11}\{\theta_{11}(x_9)\}^{\phi_{11}}}\right)^{\frac{1}{1-\phi_{11}}} \times \ln t \tag{B27}$$

$$\partial h_4/\partial x_2 = \alpha\, q_{21}(x_6) E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}}/ay_{21} \tag{B28}$$

$$\partial h_4/\partial x_6 = -\alpha\, q_{21}(x_6) E_{21}{}^{\sigma_{21}} K_{21}{}^{1-\sigma_{21}} \ln t/ay_{21} \tag{B29}$$

$$\partial h_4/\partial x_{10} = (1-\alpha)\frac{-D_{21}\phi_{21}}{1-\phi_{21}}\left(\frac{p_{d21}}{\delta d_{21} p_{z21}\{\theta_{21}(x_{10})\}^{\phi_{21}}}\right)^{\frac{1}{1-\phi_{21}}} \times \ln t \tag{B30}$$

and other elements are $\partial h_i/\partial x_j = 0$.

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