Design of an Optimal Control for Carbon Dioxide Enrichment in a Tomato Crop of Greenhouse Based on a Dynamic System

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Abstract– The tomato model-space has been developed and it is called a big leaf-big fruit model because no distinction is made between leaf or fruit number. this model is formed by the mass balances: the non-structural biomass (nutrients) and the structural biomass of fruits and leaves. An optimal control for the carbon dioxide enrichment in a tomato greenhouse gives benefits, because it is possible get a saving for energy consumption and get more tomato production. The optimal control theory is applied to the integrated system crop-greenhouse, which is based in four variables state: the consumption of nutrients, the fruits and leaves growth and the carbon dioxide concentration, also it is necessary select a cost functional. This work contributes with the control law deduced from optimal control theory and it reduces the cost for CO_2 enrichment. The parameters used for the simulations are taken from the Puebla region, in order to bring the system closer to reality for it application. Finally, the simulations was made in order to know the variables state behaviour, and the control law deduced is compared with a step input.

keywords– Greenhouse, carbon dioxide, structural biomass of leaves, structural biomass of fruit, optimal control, functional, state space.

1. INTRODUCTION

In the last two decades researchers have done many efforts to develop advanced climate control systems in greenhouses [1,2,3,9,12]. They have proposed different optimal control methods although it has not been applied in the practice because this methods are difficult for the application [12].

The growth in a crop is based in different variables, all of them are very important, but in this research the variable that has our interest is the carbon dioxide. The carbon dioxide enrichment is practised in the crops of greenhouse in order to increase the yield and the benefit. There are studies that show the CO_2 enrichment improves the net photosynthesis in the plants, it makes the total weight, height, and the number of leaves and branches increase [9]. Other research has demonstrated the CO_2 enrichment makes physic-chemical changes in the crop, like color and firmness [8].

Optimization problems with two or more objectives are very common in engineering and many other disciplines, the process of optimizing a collection of objectives function is called multi-objective optimization and it is difficult because de large number of conditions and variables involved in the system [4], in this case specific the optimization problem has two objectives, first, decrease the energy consumption for carbon dioxide enrichment and second, increase the tomato production. Process optimization is designated for the best answer of all that are available in the system design. The search process can be accomplished in two ways: deterministic and stochastic search algorithms [6]. Optimal strategies for CO_2 enrichment can be deduced experimentally or analytically. Experimentation is not able to produce a valid result for all condition set. The analysis as a tool gives us a mathematical idea for an optimal strategy, because it considers the all variables set involved in the system, this method is based on ventilation, photosynthesis, dry matter and production rate models. The method used for calculate the optimal enrichment level is valid for different models which describe the production rate, dry matter, photosynthesis and ventilation.

There are develops about optimal strategies for the CO_2 enrichment [12], which are supported in reduction of expenses for energy consumption, reduction of CO_2 consumption and increase in the production, which results in a higher net gain for the crop. Figure (1) shows three different types of control for the crop, the first one is a traditional method of the farmer, the second one and the last one are optimal controllers, note the two last have a better impact for energy save, production and total gain.

One of the main objectives is to contribute with the optimal control problem, and its implementation in real time. Having control over CO_2 we have an extra advantage talking about production. The tomato crop has been chosen because it is one of the most important crop in our country and is the second farm product consumed on the world. To achieve the objective, we part from the tomato and greenhouse mathematical set model considering the variable of plant and fruit dry weight, the availability of nutrients and the quantity of carbon dioxide.

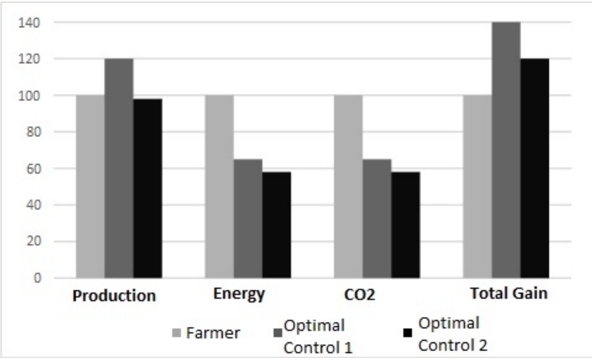

Figure 1. Energy and CO_2 consumption, total production and gain with different control strategies.

2. GENERAL FORMULATION OF THE OPTIMAL CONTROL PROBLEM

Optimal control problems appeared as essential tools in modern control theory, several authors have proposed different basic mathematical formulations of fixed time problems [5]. The optimal control of any system has to be based on three concepts: the dynamic model of the system, a functional and the system restrictions. In matrix notation the equation of state is represented as follow:

$$\dot{x} = f\left(x(t), u(t), t\right). \tag{1}$$

Where $x(t)$ is the states vector, $u(t)$ is the control signal and t is the time. A criterion is required to help to evaluate the performance of the system, normally, the functional is defined by:

$$J = \phi\left(x(t_f), t_f\right) + \int_{t_0}^{t_f} L\left(x(t), u(t), t\right)\, dt, \tag{2}$$

where t_0 and t_f are the initial and final time, ϕ and L are scalar functions, t_f can be fixed or free. Starting at the initial state $x(t_0) = x_0$ and applying the control signal $u(t)$ for $t \in [t_0, t_f]$, it makes that system follows some trajectory of states, then the cost functional assigns a unique real number for each trajectory of the system. The function (2) is called form o *Bolza* [5], it has the final conditions for all state variables and the integral part which has the control input.

The fundamental problem of optimal control is determinate an admissible control u^* which makes that equation (1) follows one admissible trajectory x^* that minimize the value of the functional showed in the equation (2). Then, u^* is named optimal control and x^* is an optimal trajectory.

Necessary conditions for a solution.

Restrictions (1) are added to the functional (2) with a Lagrange multipliers vector time variant $\Psi(t)$, the functional is rewritten as follow:

$$J = \phi(x(t_f)) + \int_{t_0}^{t_f} [L(x(t), u(t), t) - \Psi^T f(x(t), u(t), t) - \dot{x}]dt, \tag{3}$$

Then, the Hamiltonian scalar function is defined, which depends on the variable state vector, the control signal and the new vector $\Psi(t)$

$$H(x(t), u(t), \Psi(t), t) = = L(x(t), u(t), t) + \Psi(t) f(x(t), u(t), t) \tag{4}$$

An infinitesimal variation in $u(t)$ is considerate and it is denominated like $\delta u(t)$, this variation produces a change in the functional. If $x(t_0)$ is specified then $\delta x(t_0)$ also is specified. This variation can be calculated as follow:

$$\delta J = \Psi^T(t_0)\delta x(t_0) + \int_{t_0}^{t_f} [\tfrac{\partial H}{\partial u}\delta u]dt$$

We choose the multiplier $\Psi(t)$, from this, the auxiliary system equations is formed:

$$\dot{\Psi}^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \Psi^T \frac{\partial f}{\partial x} \tag{5}$$

And the final conditions can be obtained as follow:

$$\Psi^T(t_f) = \frac{\partial \phi}{\partial x}(t_f) \tag{6}$$

For a stationary solution it is required that the functional with an arbitrary variation must be equal to zero, $\delta J = 0$. This is true when:

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \Psi^T \frac{\partial f}{\partial u} = 0 \tag{7}$$

Note from the Hamiltonian function (7) it is possible get the control form. The control form depends on the variable Ψ_4 in each time instant. Then to find the vector function of control $u(t)$ that produce a stationary value of the functional we must solve the following differential equation system:

$$\begin{cases} \dot{x}(t) = f\left(x(t), u(t), t\right), \\ \dot{\Psi}(t) = -\dfrac{\partial H^T}{\partial x}, \end{cases} \tag{8}$$

The boundary conditions for this differential equations are separated, it means, some of them are defined in $t = t_0$ and the others in $t = t_f$. This is a *problem with boundary values of two points.* Note the equations that describe the states $x(t)$ and the auxiliary states $\psi(t)$ in the equation (8) are coupled, for this reason $u(t)$ depends on $\psi(t)$ through the stationary condition and the auxiliary states depend on $x(t)$ and $u(t)$. And the first system in (8) has the initial conditions of he system while the last system in (8) has the final condition of the system.

3. MATHEMATICAL MODEL OF THE CROP

The model in space states of the tomato crop has three principles states (Van Straten et al., 2011)[13]:

- Nonstructural Biomass (Nutrients).
- Leaves Structural Biomass.
- Fruits Structural Biomass.

The basic mass balances are as follow.

- Assimilates:

$$\frac{dW_B}{dt} = P - G_V - \theta_V G_V - G_F - \theta_F G_F - R_{B,V} - R_{B,F}, \tag{9}$$

- Leaves:

$$\frac{dW_V}{dt} = G_V - R_{V,V} - H_L, \tag{10}$$

- Fruits:

$$\frac{dW_F}{dt} = G_F - R_{F,F} - H_F, \tag{11}$$

where:

- Production of assimilates by photosynthesis (P).
- Conversion of assimilates to vegetative biomass by growth (G_V).
- Use of assimilates as energy to drive vegetative growth ($\theta_V G_V$).
- Conversion of assimilates to generative (fruit) biomass by growth (G_F).
- Use of assimilates as energy to drive generative (fruit) growth ($\theta_F G_F$).
- Drain of assimilates for maintenance of vegetative parts ($R_{B,V}$).
- Drain of assimilates for maintenance of generative parts ($R_{B,F}$). - Use of biomass for maintenance there is lack of assimilates ($R_{V,V}$).
- Leaf picking rate (H_L).
- Use of biomass for maintenance when there is a lack of assimilates ($R_{F,F}$).
- Fruit harvest rate (H_F).

All biomass and rates are expressed in dry weight per unit greenhouse projected area.

3.1 Biomass balance of nutrients

Nutrients are being produced by photosynthesis. The gross canopy photosynthesis rate in dry matter per unit area is P. Nutrients are converted to leaf and fruits, this is known as growth. Leaf and fruits have a demand for nutrients, which will be honored if there are sufficient nutrients available. We denote W_B like total nutrients in the plant, it is expressed like dry weight per area unit, the biomass balance equation of nutrients is the follow:

$$\begin{aligned}\frac{dW_B}{dt} &= P - h\{\cdot\}\left(\frac{(1+\theta_V)}{z}G_L^{dem} + (1+\theta_F)\,G_F^{dem}\right) - \\ &\quad -h\{\cdot\}\left(\frac{R_L}{z} + R_F\right).\end{aligned} \tag{12}$$

The biomass balance equation of nutrients (12) can take two values depending on the nutrients abundance $h\{\cdot\}$ where the first expression is taken when $h\{\cdot\} = 1$ (abundance of nutrients) and the second one is taken when $h\{\cdot\} = 0$ (lack of nutrients).

$$\frac{dW_B}{dt} = \begin{cases} P - \dfrac{(1+\theta_V)}{z}G_L^{dem} - (1+\theta_F)\,G_F^{dem} - \\ -\dfrac{R_L}{z} - R_F, \\ P, \end{cases} \tag{13}$$

where

R_F.- Respiration needs of fruits
θ_V.- Additional amount of assimilates needs for one unit of structural vegetative parts.
G_L^{dem}.- Unit area growth demand of leaves.
θ_F.- Additional amount of assimilates needs for one unit of structural fruit parts.
G_F^{dem}.- Unit area growth demand of fruit.
z.- Total vegetative parts.
$h\{\cdot\}$.- Nutrients abundance.

3.2 Biomass balance of leaves

The leaf growth is equal to the amount of nutrients converted to structural leaf biomass in the plant and it is given by $h\{\cdot\}G_L^{dem}$. The model does not incorporate an extra state for stem and roots, but the factor z assumes that each increment in leaf will be accompanied by an increment in stem and roots. If there are no sufficient assimilates (nutrients), growth stops, normally the assimilates are used for the maintenance, but in lack of nutrients, maintenance in the model goes at the expense of structural parts (leaves and fruit). The biomass balance of leaves is expressed like:

$$\frac{dW_L}{dt} = h\{\cdot\}G_L^{dem} - (1 - h\{\cdot\})\,R_L - H_L, \tag{14}$$

Depending on the abundance of nutrients $h\{\cdot\}$, the biomass leaf balance equation (14) can take two values:

$$\frac{dW_L}{dt} = \begin{cases} G_L^{dem} - H_L, & \text{si } h\{\cdot\} = 1, \\ -R_L - H_L, & \text{si } h\{\cdot\} = 0. \end{cases} \tag{15}$$

where

H_L is the leaf picking rate.

The term G_L^{dem} depends principally on the pivotal temperature, cultivation temperature level and the reference temperature.

3.3 Biomass balance of fruit

Similarly to the biomass of leaf case, the growth of fruits in the plant from the nutrients is given by

$h\{\cdot\}G_F^{dem}$. The term G_F^{dem} depends principally on the pivotal temperature, cultivation temperature level and the reference temperature.

$$\frac{dW_F}{dt} = h\{\cdot\}G_F^{dem} - (1 - h\{\cdot\})\, R_F - H_F, \qquad (16)$$

Finally, the equation (16) of biomass balance of fruits can take two different values depending on nutrient abundance $h\{\cdot\}$ where H_F is the fruit harvest rate.

$$\frac{dW_F}{dt} = \begin{cases} G_F^{dem} - H_F, & \text{si } h\{\cdot\} = 1, \\ -R_F - H_F, & \text{si } h\{\cdot\} = 0. \end{cases} \qquad (17)$$

4. GREENHOUSE MATHEMATICAL MODEL

4.1 Balance of CO_2 energy in the greenhouse

The balance of carbon dioxide energy within greenhouse is given by the equation [13]:

$$\frac{V_g}{A_g}\frac{dC_{CO_2}}{dt} = -\eta_{CO_2/dw}P + \eta_{CO_2/dw}R - -\varphi_{CO_2,g\text{-}o}^{vent} + u_{CO_2}, \qquad (18)$$

Then each term is described.

* Carbon dioxide taken from the greenhouse air for plant photosynthesis:

$$\eta_{CO_2/dw}P,$$

* Carbon dioxide returned to the greenhouse air for plant respiration:

$$\eta_{CO_2/dw}R,$$

The term $\frac{V_g}{A_g}$ is the reason of the volume of greenhouse per unit of area.
R is the total respiration plant per unit of time.

* Lost of carbon dioxide mass by ventilation:

$$\varphi_{CO_2,g\text{-}o}^{vent} = u_V(C_{CO_2} - C_{CO_2\text{-}o}),$$

where: u_V ventilation flow rate per unit of area.
C_{CO_2} (kgm^{-3}), is the carbon dioxide concentration within greenhouse.
$C_{CO_2,o}$ (kgm^{-3}), is the carbon dioxide concentration on the outside greenhouse.

* Carbon dioxide supply:

$$u_{CO_2} = u_{CO_2}^{Vp}\varphi_{CO_2,in\text{-}g}^{max},$$

where: $u_{CO_2}^{Vp}$, is the opening supply valve.
$\varphi_{CO_2,in\text{-}g}^{max}$ $\left(kg[CO_2]m^{-2}[gh]s^{-1}\right)$, is the maximum flow rate of carbon dioxide.

In this greenhouse model, the position of the carbon dioxide supply valve is the control input. For this reason, the valve relates directly to the actuator that is present on a physic way in the greenhouse.

5. INTEGRATED MODEL CROP-GREENHOUSE

From previous description of greenhouse and crop models is possible get a complete system formed by three crop equations and greenhouse equation. This new equation system describes the complete system behaviour and it is important to note that all of the equations are related principally by the P element and the state variables of the crop. It is important to say that the three equations related to the crop taken with the assumption that there is an abundance of nutrients ($h\{\cdot\} = 1$). The general system is as follows.

$$\dot{W}_L(t) = G_L^{dem} - H_L,$$
$$\dot{W}_F(t) = G_F^{dem} - H_F,$$
$$\dot{W}_B(t) = P - \tfrac{1+\theta_v}{z}G_L^{dem} + (1+\theta_F)G_F^{dem} - \tfrac{R_L}{z} + R_F,$$
$$3\dot{C}_{CO_2}(t) = -\eta CO_{2/dw}P + +\eta CO_{2/dw}R - \varphi_{CO_{g-0}}^{vent} + uCO_2,$$

6. SYNTHESIS OF OPTIMAL CONTROL

We consider the system formed by the state equations (13, 15, 17, 18), the first three of them are relative to the crop and the last one is relative to the greenhouse. We suppose there is nutrients abundance for the three equations relative to the crop. The terms for the equation systems (19) and (22) have been substituted using the equation table of the mathematical model (table 1) and the values have been substituted using the table of physical parameters (table 2).

$$\begin{cases} \dot{W}_L(t) = 2{,}2996 \times 10^{-6}\, W_L(t), \\ \dot{W}_F(t) = 4{,}3925 \times 10^{-6}\, W_F(t), \\ \dot{W}_B(t) = P(t) - 5{,}39 \times 10^{-6}W_L(t) - 5{,}92 \times 10^{-6}W_F(t), \\ 3\dot{C}_{CO_2}(t) = 1{,}0266(R(t) - P(t)) + 0{,}155 \times 10^{-10}u_{CO_2}^{v_p}, \end{cases} \qquad (19)$$

P and R are as show following:

$$P(t) = \frac{3{,}7192 \times 10^{-11}\, W_L^{2{,}511}(t)}{1{,}6353 \times 10^{-9} + 4{,}0439 \times 10^{-5}\, W_L^{2{,}511}(t)},$$
$$R(t) = 1{,}5942{\times}10^{-6}W_F(t) + 0{,}4856{\times}10^{-6}W_L(t) + 1{,}668{\times}10^{-7}.$$

It is important to note that the terms P(t) and R(t) have involved two of the three state variables of the crop and they are time depending functions, so the entire system is connected and it can be solved simultaneously.

We consider the following functional, which has the same form shown above (3).

$$J = \frac{1}{2}[W_L^2(tf) + W_F^2(tf) + W_B^2(tf) + C_{CO_2}^2(tf) + + \int_{t_0}^{t_f}[W_L^2(t) + W_F^2(t) + + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^{v_p})^2(t)]dt] \qquad (20)$$

Table 1. Greenhouse and crop mathematical model equations

Term	Description
$P = P^{max}\left(\frac{I^{PAR}}{I^{PAR}+K_I}\right)\left(\frac{C_{CO_2}}{C_{CO_2}+K_C}\right)f_m\{\cdot\}$	Production of assimilates by photosynthesis.
$R = h\{\cdot\}\left(\frac{\theta_V}{z}G_L^{dem} + \theta_F G_F^{dem}\right) + \frac{R_L}{z} + R_F$	Total amount breathed plant per unit of time.
$I^{PAR} = f_{PAR/I}\tau_r I_o$	The PAR light intensity at the crop level.
$f_m\{\cdot\} = \frac{(W_L/p_m)^m}{1+(W_L/p_m)^m}$	Maturity factor.
$G_L^{dem} = f_{L/F}(T)k_{GF}^{ref}f_{TG}(T)f_D\{\cdot\}W_L$	Growth leaves demand.
$G_F^{dem} = k_{GF}^{ref}f_{TG}(T)f_D\{\cdot\}W_F$	Growth fruits demand.
$f_{L/F}(T) = f_{L/F}^{ref}e^{v_2(T-T_{L/F}^{ref})}$	Temperature-dependent ratio.
$f_{TG}(T) = Q_{10R}^{T-T_G^{ref}/10}$	Temperature dependent with a Q_{10G} relation.
$f_{TR}(T) = Q_{10R}^{T-T_R^{ref}}$	Function of temperature with a Q_{10G} relation.
$f_D\{\cdot\} = \frac{c_{f1}-c_{f2}D}{c_{f1}-c_{f2}}$	Correction factor for the fruit growth rate.
$R_L = k_{RL}^{ref}f_{TR}(T)W_L$	Respiration demand of the leaves.
$R_F = k_{RF}^{ref}f_{TR}(T)W_F$	Respiration demand of the fruits.
$H_L = k_{HL}W_L$	Leaf picking rate.
$H_F = k_{HF}W_F$	Harvest rate.
$K_{HL} = C_{yL}K_H$	Coefficient of harvest.
$K_{HF} = C_{yF}K_H$	Coefficient of harvest.
$K_H = Cd1 + Cd2ln(T/Cd3) - Cd3 - Cd4e_D$	Harvest rate.
$u_V = \left(\frac{p_{V1}u_V^{A_{plsd}}}{1+p_{V2}u_V^{A_{plsd}}} + p_{V3} + p_{V4}u_V^{A_{pwsd}}\right)v + p_{V5}$	Ventilation flow rate.

The first term of performance index involves the three first variables at the end time, they are related to the final production and the nutrients, and the integral contains the control input in order to avoid the risk for big control inputs. The idea is minimize the functional (20), related with the equations system (19).

6.1 Method solution description

The Hamiltonian scalar function is obtained considering the relation (4) with the Lagrange multipliers and the functional (20).

$$
\begin{aligned}
&H(\mathbf{x},\mathbf{u},\mathbf{\Psi},t)) = \\
&= \frac{1}{2}[W_L^2(t) + W_F^2(t) + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^{v_p})^2(t)] + \\
&+ 2{,}2996\times10^{-6}W_L(t)\Psi_1(t) + 4{,}3925\times10^{-6}W_F(t)\Psi_2(t) + \\
&+ \left[P - 5{,}39\times10^{-6}W_L(t) - 5{,}92\times10^{-6}W_F(t)\right]\Psi_3(t) + \\
&+ \frac{1}{3}\left[1{,}0266(R-P) + 0{,}1554\times10^{-10}u_{CO_2}^{v_p}\right]\Psi_4(t). \quad (21)
\end{aligned}
$$

The system of auxiliary variables is formed using the expression (5), it has the following form:

$$
\begin{cases}
\dot{\Psi}_1 = W_L + 2{,}2996\times10^{-6}\,\Psi_1 + \dfrac{\partial P}{\partial W_L}\Psi_3 - \\
\quad -5{,}39\times10^{-6}\Psi_3 + \dfrac{1}{3}\dfrac{\partial(R-P)}{\partial W_L}\Psi_4(1{,}0266) \\
\dot{\Psi}_2 = W_F + 4{,}3925\times10^{-6}\,\Psi_2 - \\
\quad -5{,}92\times10^{-6}\,\Psi_3 + \dfrac{1}{3}\dfrac{\partial R}{\partial W_F}\Psi_4(1{,}0266), \\
\dot{\Psi}_3 = W_B, \\
\dot{\Psi}_4 = C_{CO_2},
\end{cases} \quad (22)
$$

The stationary condition give us the following control form, which was obtained from equation (7) and depends on fourth appended state:

$$u_{CO_2}^{v_p} = -\frac{1}{3}0{,}1554\times10^{-10}\Psi_4(t). \quad (23)$$

It is necessary to solve the equation systems (19) and (22), in this way we can know the Ψ_4 value and finally we will get the control form. The system (19) has initial condition and the system (22) has final conditions. The systems are coupled, because the control form (23) has been substituted. To

Table 2. Physic Parameters.

Variable	Value	Description
z	0,6081	Fraction leaf of total vegetative mass
θ_v	0,23	Surplus assimilate requirement factor per unit fruit increment.
θ_F	0,2	Surplus assimilate requirement factor per unit vegetative increment.
p_h	$2{,}7 \times 10^{-3}$	Parameter of switching function, $[m^2\ kg^{-1}]$
p_m	$1{,}8 \times 10^{-2}$	Parameter in maturity factor, $[kg\ m^{-2}]$
m	2,511	Parameter in maturity factor
p^{max}	$2{,}2 \times 10^{-6}$	Maximum gross canopy photosynthesis rate, $[kg\ m^{-2}\ s^{-1}]$
K_1	577	Monod constant for PAR, $[W\ m^{-2}]$
K_c	0.211	Monod constant for CO_2, $[kg\ m^{-3}]$
$f_{PAR/I}$	0.475	PAR fraction of global radiation
τ_r	0.7	Transmittance of the roof
k_{GF}^{ref}	$3{,}8 \times 10^{-6}$	Reference fruit growth rate coefficient, $[s^{-1}]$
T_{GF}^{ref}	20	Reference temperature, $[^0C]$
Q_{10G}	1.6	Temperature function parameter growth
$f_{L/F}^{ref}$	1.38	Reference leaf-fruit partitioning factor
v_2	-0.168	Parámetro de partición de fruta-hoja, $[K^{-1}]$
$T_{L/F}^{ref}$	19	Fruit-leaf partitioning reference temperature, $[^0C]$
k_{RL}^{ref}	$2{,}9 \times 10^{-7}$	Maintenance respiration coefficient leaf, $[s^{-1}]$
Q_{10R}	2	Temperature function parameter respiration
T_R^{ref}	25	Reference temperature for respiration, $[^0C]$
k_{RF}^{ref}	$1{,}2 \times 10^{-7}$	Maintenance respiration coefficient leaf, $[s^{-1}]$
η	0,7	Absorbed in relation to the total energy of the net radiation heat received.
$Cd1$	$2{,}13x10^{-7}$	Parameter in development rate function, s^{-1}
$Cd2$	$2{,}47x10^{-7}$	Parameter in development rate function, s^{-1}
$Cd4$	$7{,}50x10^{-11}$	Parameter in development rate function, s^{-1}
C_{yL}	1,636	Parameter in harvest function (fruit)
C_{yF}	0,4805	Parameter in harvest function (leaf)
$C_{CO_{2,0}}$	1,6637	
$C_{CO_{2/dw}}$	1,4667	Ratio CO_2 per unit dry weight, $Kg[CO_2]Kg^{-1}[dw]$
$C_{CO_{2,in_g}}$	$2{,}10x^{-6}$	Ratio CO_2 per unit dry weight, $Kg[CO_2]m^{-2}[gh]s^{-1}$
$\frac{V_g}{A_g}$	3	Volume per unit greenhouse area
p_{v1}	$7{,}17x10^{-5}$	Parameter.
p_{v2}	0,0156	Parameter.
p_{v3}	$2{,}71x10^{-5}$	Parameter.
p_{v4}	$6{,}32x10^{-5}$	Parameter.
p_{v5}	$7{,}40x10^{-5}$	Parameter.

solve the complete system like a system with initial conditions, the auxiliary equations are considerate in reverse time, then the behaviour of the auxiliary variables is returned to the direct time. When we solve the appended equation system in reverse time the system becomes in a system with initial conditions. It is important to note that the equation (23) depends on fourth state but this state depends on the other three states. Using MatLab tools we solve the equation systems (19) and (22), then we can get the state Ψ_4 in reverse time and finally return it to the direct time.

7. SIMULATION AND RESULTS

The MatLab tools were used to elaborate the program that solve the differential equations system. The simulation period is for two weeks. The simulations was made using a step function like control input and then the simulation was made using the control law deduced in this paper in order to compare the results of a simple control against the optimal control. Below all the results obtained are described.

7.1 Analysis with a step input.

It is important to say that the above control was performed using synthesized constant parameters, the first simulation is about constant temperature control and solar radiation. Further on the simulations are presented taking into account these variables parameters. A step control signal is introduced to the system (19), the behaviour is described in the figure 2. The graphic shows how nutrients decrease with the time, and fruits and leaves grow with the time. The Figure 3 shows the CO_2 behaviour.

Dry matter of fruits grows more than dry matter of leaves, which is very acceptable, also the behaviour of nutrients is acceptable because the crop uses the nutrients while it is growing. However, it is important to note the carbon dioxide is very high, the CO_2 concentration increase until 9000 ppm, it means so much consumption energy which means the cost for CO_2 enrichment will be expensive for the farmer.

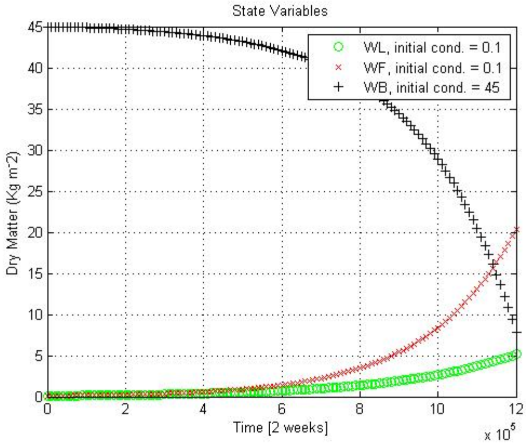

Figure 2. Behaviour of fruits, nutrients and leaves dry matter with an step input.

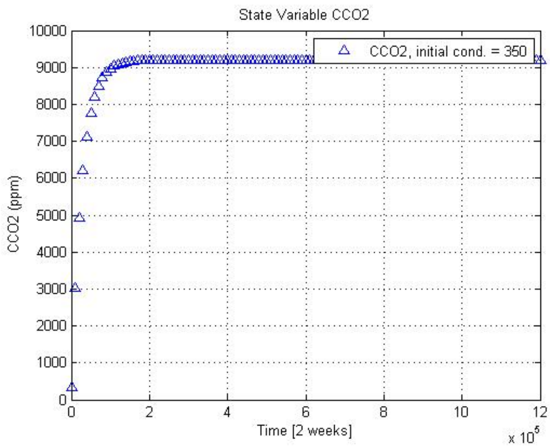

Figure 3. Carbon dioxide behaviour with an step input.

7.2 Analysis of the synthesized control.

Simulation considering the control law determinate in this paper is presented bellow, also, for this results the temperature and solar radiation are constant parameters.
The figure 4 shows like dry matter of fruits, leaves and nutrients is a similar case where the ramp was simulated for the system. It is important to note the fact that in the simulation with the control law obtained in this research, the carbon dioxide was significantly reduced, note that CO_2 concentration decreases to zero and then increases until 400-450 ppm, this is very acceptable because he amount of CO_2 required for the crop is about 400. And it could give benefits to the farmers (Figure 5).

The figure 6 shows the behaviour of the auxiliary variable Ψ_4, note the variable was solve in reverse time and in the figure 6 the variable is represented in real time. The behaviour is important because the control expression depends on it for each time instant.

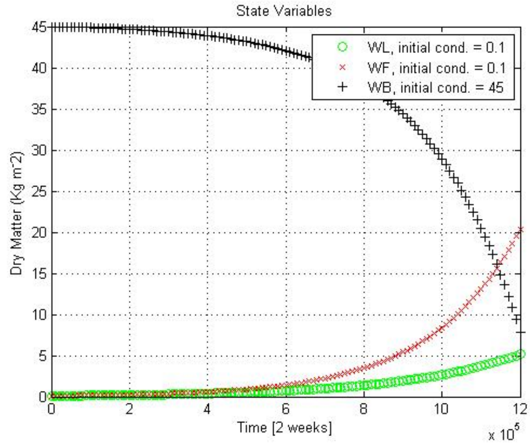

Figure 4. Fruits, leaves and nutrients dry matter behaviour with control law deduced. $u^{v_p}_{CO_2}$.

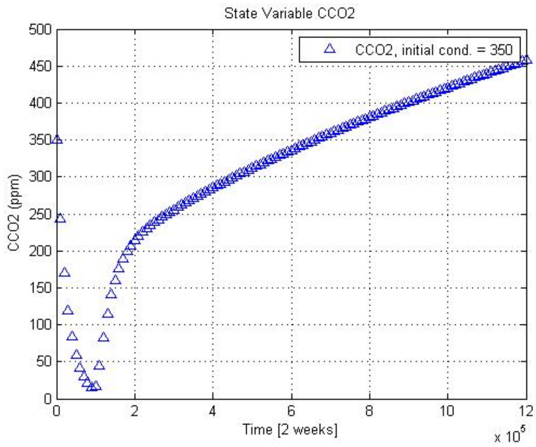

Figure 5. Carbon dioxide behaviour with control law deduced. $u^{v_p}_{CO_2}$.

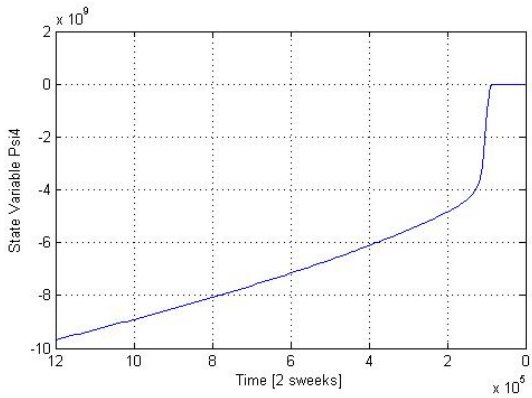

Figure 6. Behaviour of auxiliary variable Ψ_4.

In agreement with expression (23), the behaviour in each instant time of the appended state variable $\Psi_4(t)$ must be multiplied by the value $(-\frac{1}{3}0{,}1554 \times 10^{-10})$, this gives us the control behaviour.

7.3 Analysis with temperature and solar radiation variables.

In previous cases, the system has constant parameters which made it easier to solve the complete system, but in real life some of these parameters are not constant, as they are natural effects such as temperature and solar radiation. For the following simulations we consider these parameters as variables functions over time, the synthesized control was the same, but now we have new simulation results. Figures 7 and 8 shows the temperature and solar radiation inside the greenhouse.

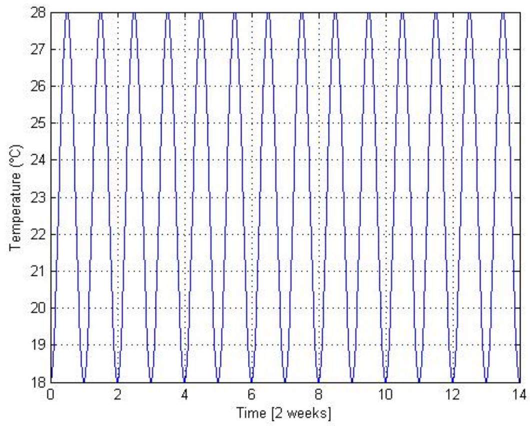

Figure 7. Variable temperature in greenhouse.

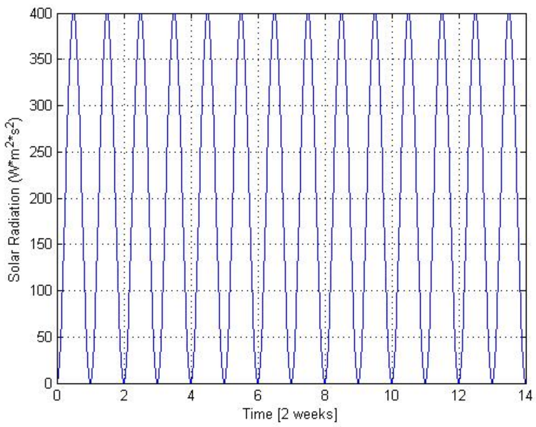

Figure 8. Variable solar radiation in greenhouse.

7.4 Analysis with a step input.

Also for the next simulation we use a different control input, in this case the input was an unit step. Now, we can get a new result for the simulation with variable parameters that allow us to be closest of the reality. The Figure 9 shows the behaviour of three crop state variables. Note the result is similar to the case where there was constant parameters.

The Figure 10 shows the carbon dioxide behaviour, note the CO_2 concentration increase until 9000 ppm, this is very high and it means so much

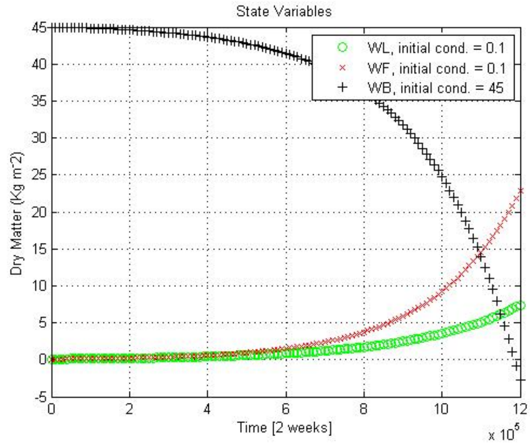

Figure 9. Behaviour of fruits, leaves and nutrients dry matter with an input step and variables temperature and radiation.

consumption energy just like the case where the temperature and radiation were constant.

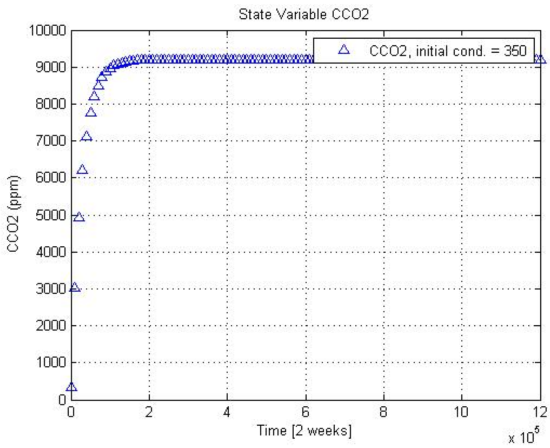

Figure 10. Behaviour of CO_2 concentration with variables temperature and radiation and a step like input control.

7.5 Analysis with the synthesized control.

For the next simulation, we get results using the variable parameters, but now we simulated the control law deduced in this paper. Figure 11 shows the results. Note the behaviour of three crop state variables are similar to the previous cases.

In this case, the carbon dioxide behaviour is different, in Figure 12 we can note that CO_2 concentration decreases just like the case were temperature and radiation were not variables.

8. CONCLUSION

The tomato and greenhouse model was analysed and we obtained the synthesized control law that

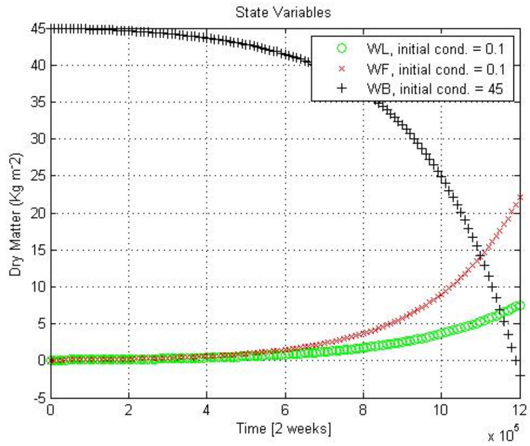

Figure 11. Behaviour of fruits, leaves and nutrients dry matter with the control deduced and variables temperature and radiation.

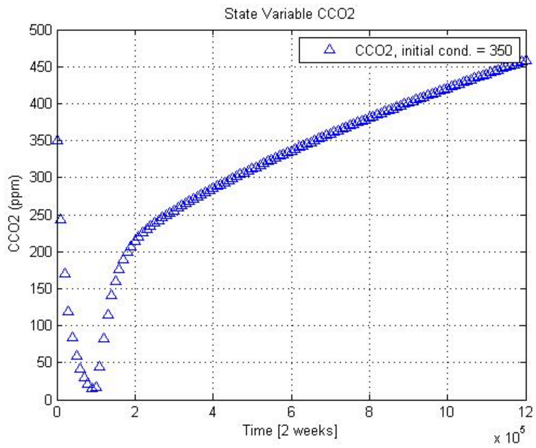

Figure 12. Behaviour of CO_2 concentration with variables temperature and radiation and the control input deduced.

give benefits to the farmers, because it is possible to having the same growth for the dry matter of fruits, but the consumption of carbon dioxide is reduced significantly. Two important results was obtained, the first one was we checked the control law is correct because increase the production and reduce the energy consumption, the second result is we knew the behaviour of the system with real parameters and we noted the control law has the same results. On the other hand, the next short-term work is the design and construction of the electronic system which will control the carbon dioxide and its application in a greenhouse.

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