Aggregated indicator of environmental damage caused by air pollution

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Abstract - Searching for synthetic information in the form of aggregated indicators for decision-making is very topical. In the article attention is paid to selected air quality data which is assigned about monetary terms of damage to the environment.Mathematical expression is based on the scalar representation of comparative indicators. In the literary sources misinterpretations can be found, e.g. the price level is defined by the basic price index. The base has an associated unit level but in principle, indices indicate a change.An indicator of pollution is possible to separate into two different aspects. It is possible to define pollution indices and the indices for quantity and indices for prices. The article presents the process and outcome on real data of air pollution in the Czech Republic.

Keywords – Environmental damage, quality of the air, aggregated indicators, air pollution in the Czech Republic.

I. INTRODUCTION

DUE to the increasingly stringent environmental regulations and growing pressure to apply principles of sustainable development into practice in countries around the world, there is a growing demand for aggregated indicators for monitoring and measuring environmental development [1], [2], [3]. Decision makers on local, regional or national level, general public and all interested parties need detailed information but also it needs to be apparent at first sight what is the environmental situation in their country or region.

There are several institutions on supranational level dealing with construction and evaluation of environmental indicators, such as OECD, UNDP, World Bank or other institutions on national or intergovernmental level. The paper presents another alternative approach of monitoring and evaluating of air pollution development in countries in long term perspective.

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II. SELECTED ENVIRONMENTAL DATA

A. Attributes of indicators

Aggregated indicators generally must meet three keys characteristics which are: relevancy, credibility and legitimacy. Therefore, indicator must be useful and relevant. Its credibility is related to scientific correctness, data correctness and used methodology. Legitimacy refers to the perceptions of the indicator, construction method and author´s eligibility [4].

Beyond mentioned attributes such as correctness of the data, aggregated indicator should be comparable meaning the data and methodology of data collection should be comparable across countries and also if other data are coherent and accessible [5].

The meaning of the indicator must be defined clearly and unambiguously. It needs to reflect the particular occurrence or process complexly. It may not be in conflict with other indicators and cannot duplicate any other indicator.

The particular occurrence or process might be divided into parts while it is necessary that the parts are delimited clearly, must be disjunctive and their unification has to cover the full occurrence or process.

The place is defined by the area which the indicator is related to. This area might be e.g. a country, region as well as geographical area etc. [6].

Time or Period

The indicator describes an entity which belongs to a certain time or time period. The time stamp is marked with a symbol, e.g. "t" or "s". The symbols defining time are usually attached to the indicator in the form of index.

A time period is a time interval $\langle s, t)$ determined by two time s and t where $s<t$. The time s is the beginning of the period while time t is the end. The length of a period is the difference between the end and the beginning, i.e. t-s.

Sometimes the interval $\langle t-1,t)$ is denoted also as t. The fact whether it is talked about time or interval is visible from the circumstances. Interval $\langle t,t+dt)$ with a very short length of dt is call infinitesimal interval.

B. Undesirable phenomena and activities

Undesirable phenomena and activities are a result of human activities which are mainly connected to business. Air quality is affected by various factors; also proportion of harmful substances emissions can be different from place to place [7], [8].

As was mentioned before, in general, increasing stringency in environmental regulations across developed countries can be observed. These are more and more supported by other voluntary tools to encourage responsible behaviour of entrepreneurs whose environmental impact is significant, but also general public which contribute to pollution and connected damages as well.

Let us use a simple example of heat production to demonstrate the complexness of the undesirable activity quantification.

Heat production is connected to activities which depend on: necessary amount of heat, burnt substance, utilizing waste from burning, waste disposal, cost of equipment for burning, operational cost, cost related to damages from combustion processes, cost for reducing damages, price of heat and yield from operations, price of emission damages, sustainability of activity, repeatability, etc.

In regards to the damages, following should be monitor:

- harm impacting different groups of population
- lower border of acceptable damage extent
- size of damage when 50% of population is impacted by harmful results
- dependence of the polluting activity and damage size
- absorption ability of the environment for the limiting emitted harmful substances.

Apart from apparent damages there are activities which produce future danger. Apart from rain forest cutting which erodes the climatic balance and global warming jeopardizing large sea side cities, these activities are not given sufficient attention. A question arises whether there are no other such activities. It might be air transport which might trigger other climatic undesirable phenomena in long term.

C. Economics of the environment

The economics of environment is an indispensable part of economics science. Fundamental challenge lies in defining of the value of damaged environment, cost to remove pollution and preserve nature and - more generally - puts emphasis on the need of an efficient policy to protect the environment.

Question arises when the economics of the environment tries to reconcile economy and ecology:

- How to produce and not irreparably destroy natural resources?
- Is it possible to determine the cost of nature in all its form in a monetary form?
- How to internalize the cost of the nature to the internal cost of a production subject?

Hereby are mentioned tools with how to answer these questions:

Utilizing Ownership Rights

Ronald Coase, a supporter of neoliberalism, suggests to use the principle of ownership rights (for natural resources). There are two options:

- The producer disrupting or polluting the environment has the right to this behaviour (e.g. ownership) and it depends on the aggrieved to preventively compensate this potential polluter for the loss of profit by not using this right. E.g. a neighbour who is annoyed by noise from a workshop buys silence from the workshop owner and compensates him for not being able to make money.
- The other solution is derived from the assumption that the right for resource ownership is in the hands of the aggrieved. Therefore, the victims will be compensated for loss by the producer disrupting or polluting the environment.

In both cases, it is possible to express the cost monetarily.

On the other hand, this system cannot be used for the global pollution (acid rains, greenhouse effect, damaging the ozone layer etc.) where it is difficult to find the victims as they are often not aware of being the victims (the problem of information availability and transparency on the market of polluting the environment), with difficult evaluation of the cost (how to investigate and put a price tag on increased risk of more frequent occurrence of cancer in regards to other risk factors?), and it is difficult to find the polluters (every person breathes out CO_2) and also the future generations are not able to negotiate about the compensation.

Using Taxes - the Polluter Pays

This principle has been taken from the theoreticians of the welfare economics (A. C. Pigou, A. Marshall) who were the first to analyze externalities. The intervention of the state is mild in this case: the state will prescribe a tax to the polluter in the value of the damage and the polluter is thus motivated to invest into harmless material in order to avoid the tax. The government might also offer subsidies to those who invest to eliminate pollution. It is also difficult to determine the valued of the damages caused by global or long-term pollution.

Approval to Pollute

The state or a specialized agency sets a desirable level of acceptable pollution and provides a licence for the right to damage the environment within an intensity limited by the level which should be reached. Some of these rights might be sold on the market: those who pollute less might sell not used rights to those who pollute more, therefore, the purchase price becomes an internal cost item for the buyer. This system is relatively effective for local pollution but meets obstacles how to set a desirable level of global pollution and distribute the pollution right among countries in a fair way.

Irreversibility and Damages Which Cannot Be Assessed by Market

The price of some damages cannot be estimated as they will take effect only in a long term; their correction has not started yet and therefore, the cost cannot be defined. If these potential costs are not taken into account in the economic calculations, it may happen once, when they can be evaluated, they are irreversible. There is also another problem: how to put a price tag on non-market resources (e.g. plant and animal species which humankind does not use but destroys them)?

Environment Protection Policy

A method using cost and advantages is used. Every policy has its costs (cost for planned repairs of damages in the environment). The costs are compared with obtained advantages (cost for protection against pollution). The state chooses a policy where the advantages will be higher or equal to the costs. Also a problem with financial evaluation of long term pollution can be seen (greenhouse effect in the year 2050).

Actualization Rate

Eliminating pollution today - and so fight against damages in the future - creates the problem of actualization: it is necessary to compare two values which do not correspond in time (assuming their monetary evaluation has been solved).

It cannot be said that the economic theory has or does not have a reliable solution to fight ecological problems. This statement is used to console both, those who believe in market mechanism as well as those who doubt and suggest various moratoriums, regulations or even bans on activities which contribute to global pollution.

Monitoring of arising externalities is necessary with regards to the originator (polluter pays principle). To fulfil this basic principle is not trivial and the way from detecting emissions and pollutant dispersion to identifying the impacts, their natural valuation and finally monetary valuation is extremely complicated. The result is connected not only with high cost for the analysis itself but also with many uncertainties:

- Emission analysis itself (flow of the pollutants to the environment). The uncertainty here is relatively low, especially for basic polluting substances as many types of emissions are continuously or at least periodically measured based on the air protection law. Considering the influence of polluted air and origin of pollutants it is necessary to take into account very different local ratio of the polluting sources. For imission values (state of pollution) it is necessary to expect significant deviations from the reality according to areas and at estimates of the basic background concentration.
- Behaviour and cycle of the pollutant in the environment - dispersion (transformation) of pollutants. The mechanism of the impact of atmospheric reactions to the state of the atmosphere is extremely complex and not fully explained yet.
- Dose - response function. The most frequent materials are epidemiological studies. For health evaluation as the most important segment of damages, reactions to some substances are not known yet. The function is usually assumed as linear due to missing LV (Limit Value) and the current concentration for other than basic polluting substances. Different outcomes of the influence may be expected for different ways of defining pollutant concentration. In contrary to our practice, emphasis has been recently put on ozone for impact on harvest, for nitrogen oxides the fertilizing function is assumed also for secondary compounds. Our local conditions also consider other agricultural crops as sensitive than in e.g. comparable conditions in Europe. Used methods for calculating damages in agricultural production, above all forests, in the Czech Republic form a standalone chapter. So far the area of burden accumulation and especially problem of synergic impact of more pollutants has been researched very little. This latency of impact might be very significant e.g. for toxicological impacts.
- Economic valuation. Economic valuation is very significant for decision making; it reflects the scarcity and uses market prices, if they are known, readiness to pay, national economic accounting and the principle of external effect internalisation. The techniques used for evaluation in economic terms may be divided into two groups - those where functional relation between pollutant dose and environmental response (damage function) can be found and those where a different solution needs to be found. Results of damage impact calculations derived from knowing natural damage have been published in the literature. Most frequently these were damages from running energetic facilities, especially damages from air pollution and its impact on health of population and production (agriculture, forestry etc.). There are also newer methodical procedures based on population behaviour. This may be in form of direct methods: method of political referendum or contingent valuation method or indirect approaches: method of individual substitution, method of travel cost and method of hedonic price.

The problem of damage estimate also includes a link to the sustainable development principles. These principles take the form of proposing not only narrowly defined utility value (either direct or indirect) but also existential value (given by the existence of the nature itself) and option value (ensuring the value of the nature for the future) in its wider conception about using all types of resources.

Damage may be also defined in cost necessary to remedy or prevent the damage. This alternative solution of the calculation based on the cost (real cost of the damage will not be found) may have various form: cost of avoiding the source of burden, cost of damage remedy, cost to prevent the damage, targeted prohibitive cost, defensive cost etc.

The text above was supposed to outline possible methods for quantification of activities and occurrences which concern humankind as a whole. It is the problem of emission quantification, pollution and its impacts. In the end, a few comments concerning this problem should be outlined.

It has shown that the weak spot is monitoring the undesirable phenomena on one hand and publishing needed data in classification and structures which will have needed explicitness [9], [10]. The methodology of collection and processing such data is at its beginning. For the viewpoint of sustainable development, following principles need to be developed:

- Renewable sources should be used up to the speed at which they can renew
- Non-renewable resource should be used maximally at the speed of building their alternatives which should be continuously used to replace them
- The intensity of pollution must not exceed the assimilation capacity of the environment
- A part of the current technologies should be invested to reduce pollution, waste and increasing efficiency (of products, energy, production processes)

III. THEORETICAL APPROACH

A. *Value, quantity, unit value*

Let us consider a complex (total) phenomenon or process which consists of n partial disjoint parts i, i= 1, 2, …, n. The unification of all parts gives the total [5], [11]. E.g. aggregate pollution consists of pollution caused by CO_2, SO_2, etc.[12], [13], [14].

Let us consider value of individual h_i for any part i. Let us assume that variables h_i are addable i.e. they have the same meaning and it is possible to sum them. We will call them values.

We state

$$H = \sum_{i=1}^{n} h_i \tag{1}$$

Let us define variables $q_{i,}$ which correspond quantity (amount) of an item i. The meaning of these variables can be various. So, their sum may not make sense. These variables will be called quantity.

So, for any item i we have two variables: value and quantity.

For any item i we define

$$p_i = \frac{h_i}{q_i}. \tag{2}$$

The variable p_i means the value of item i per unit of quantity i. If the value is expressed in monetary units, these variables represent prices. Generally, it may not be so, therefore, term unit value will be used.

For any h_i and positive q_i we obtain

$$h_i = \frac{h_i}{q_i}\, q_i \tag{3}$$

$$h_i = p_i\ q_i. \tag{4}$$

Values h_i, i=1,2, …, n form vector h.

Values q_i, i=1,2, …, n form vector q.

Values p_i, i=1,2, …, n form vector p.

Coordinates of these vectors correspond to the item i, i=1,2, …, n. The sum H is the total value in chosen time t or time period (t-1,t>

$$H = \sum_{i=1}^{n} h_i = \sum_{i=1}^{n} p_i\, q_i\ . \tag{5}$$

A question arises whether it is possible to define numbers P and Q which correspond to vectors p and q so that

$$H = P\ Q\ . \tag{6}$$

P and Q represent scalar representatives of vectors p and q.

B. *Data changes*

In this paragraph we will consider continuous time and infinitesimal time interval < t, t + dt).

From relations written above it follows

$$dH = \sum_{i=1}^{n} dh_i = \sum_{i=1}^{n} \frac{dh_i}{h_i}\, h_i = \sum_{i=1}^{n} d(p_i\, q_i) = \sum_{i=1}^{n} (q_i\, dp_i\ + p_i\, dq_i) \tag{7}$$

$$\frac{dH}{H} = \sum_{i=1}^{n} \frac{dh_i}{H} = \sum_{i=1}^{n} \frac{dh_i}{h_i}\frac{h_i}{H} = \sum_{i=1}^{n} \frac{dh_i}{h_i}\, w_i \tag{8}$$

$$\frac{dH}{H} = = \sum_{i=1}^{n} \frac{d(p_i, q_i)}{p_i\ q_i}\ w_i = \sum_{i=1}^{n} \frac{q_i\, dp_i\ + p_i\, dq_i}{p_i\ q_i}\ w_i = \sum_{i=1}^{n} \left(\frac{dp_i}{p_i} + \frac{dq_i}{q_i}\right)\ w_i\ , \tag{9}$$

where weight w_i fulfils

$$w_i = \frac{h_i}{H}. \tag{10}$$

From it follows

$$\mathrm{d}\ln H = \sum_{i=1}^{n} (\mathrm{d}\ln p_i + \mathrm{d}\ln q_i\ )\ \ w_i \tag{11}$$

This relative change of the total value is equal to weighted sum of relative changes of the values for individual i.

Therefore, infinitesimal growth of value is equal to sum of weighted infinitesimal changes of unit value and quantity [2],[11].

In order to express infinitesimal growth we assume dependency of the variables on parameter (e.g. time) t which will be assigned as an index to considered variables. Therefore, we can write

$$H_{t+dt} = H_t + dH_t \tag{12}$$

$$\frac{H_{t+dt}}{H_t} = 1 + \frac{dH_t}{H_t}$$

(13)

It arises from here that the index reduced by a unit represents relative change of variable H.

$$\ln\frac{H_{t+dt}}{H_t} = \ln H_{t+dt} - \ln H_t = \sum_{i=1}^{n} w_i(\ln q_{it+dt} - \ln q_{it}) + \sum_{i=1}^{n} w_i\,(\ln p_{it+dt} - \ln p_{it})$$

(14)

Assume that weights $w_i\ i = 1, 2, \dots, n$ do not depend on parameter t.

From which arises

$$\ln\frac{H_{t+dt}}{H_t} = \sum_{i=1}^{n} w_i\left(\ln\frac{q_{it+dt}}{q_{it}} + \ln\frac{p_{it+dt}}{p_{it}}\right) = \sum_{i=1}^{n} w_i\ \ln\frac{q_{it+dt}}{q_{it}}\,\frac{p_{it+dt}}{p_{it}}$$

(15)

$$\ln\frac{H_{t+dt}}{H_t} = \sum_{i=1}^{n}\ln\left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i}\left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i}$$

(16)

$$\ln\frac{H_{t+dt}}{H_t} = \ln\prod_{i=1}^{n}\left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i}\left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i}$$

(17)

$$\frac{H_{t+dt}}{H_t} = \prod_{i=1}^{n}\left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i}\left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i}$$

(18)

Let us define

$$Q_t = K_Q\, q_{1t}^{w_1}\, q_{2t}^{w_2} \dots q_{nt}^{w_n}$$

(19)

$$P_t = K_P\, p_{1t}^{w_1}\, p_{2t}^{w_2} \dots p_{n}^{w_n}\ ,$$

(20)

where K_Q and K_P are positive constant such that

$$K_Q\, K_P = 1.$$

(21)

So, we can see

$$\frac{H_{t+dt}}{H_t} = \frac{Q_{t+dt}}{Q_t}\,\frac{P_{t+dt}}{P_t}.$$

(22)

For time t;

P_t may be considered as aggregate variable of unit values, i.e. as level of unit value;

Q_t may be considered as aggregate variable of value.

IV. METHOD APPLICATION

Before method application on real data measured in the Czech Republic, essential information about air pollution situation in the Czech Republic is provided.

A. *Air Pollution in the Czech Republic*

Following graph gives an overview about air pollution in the Czech Republic as is monitored by Czech Hydro-meteorological Institute. Each pollutant includes data from large, medium, small stationary and mobile air pollution sources. Unfortunately, data before 2000 are not available in a comparable methodology.

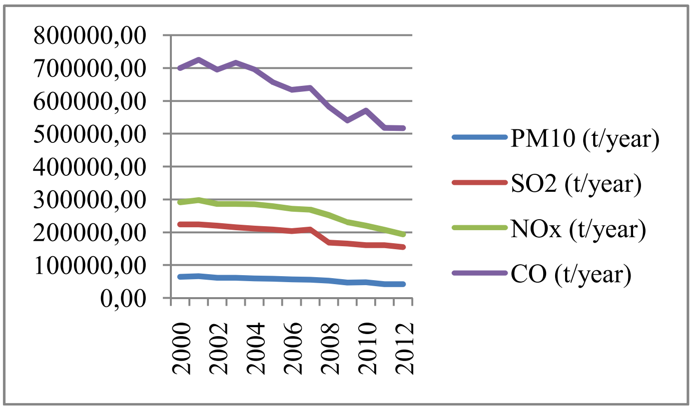

Fig. 1. Air pollution in the Czech Republic. Source: own, based on data retriever from: [15].

Situation in the Czech Republic has slightly improved and there is continuous positive trend over past decade. This improvement in reducing negative impact of the economy on the environment was supported by economic recession in previous years.

The greatest degrease pressures on the environment occurred in the sector of energy, manufacturing and transportation. On the other hand, significant influence of household consumption on the environment remains, particularly influence of local heating [16].

For better illustration of long term development, authors chose emissions of large stationary sources.

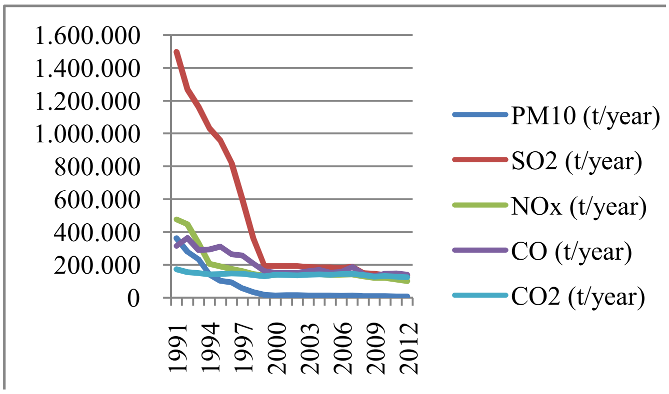

Fig. 2. Large stationary air pollution sources. Source: own, based on data retriever from: [12], [17].

Significant decrease of level of sulphur dioxide before 2000 was caused by extensive investments in power plants denitrification.

It is nothing surprising that especially in regions with high concentration of industry, there are exceeded limits of benzopyrene and particulate matter (less than or equal to a nominal 10 and 2,5 microns). Highest concentration of PM_{10} is traditionally in Moravian-Silesian Region in the east of the Czech Republic. Sustained high concentration and often exceeded permitted limits of PM_{10} is mainly caused by extra large and large industrial sources of pollution, then local heating and traffic. It has been shown [18] that due to weather conditions, situation in air pollution is often negatively influenced by undefined sources, especially large industrial sources from Polish Silesian voivodship. But as Bollen in [19] declare, with upcoming structural changes, structural changes in emission can be expected.

Air quality of the Czech Republic is not improving but only fluctuates. One third of Czech population lives in area with excessive pollution limits for PM_{10}, two thirds are exposed to high concentration of carcinogenic benzopyrene.

B. *Pollution and its clasification*

Environmental data are available in the various classifications of pollution and other harmful effects [13], [20].

For the purposes of interpretation, it is sufficient to consider the information relating to one year and the types of pollution, which are in the following table.

Tab. 1 Selected air pollutants

(n)	Type of pollution	
1	PM_{10}	Particulate Matters
2	SO_2	Sulphur dioxide
3	NO_x	Oxides of nitrogen
4	CO_2	Carbon dioxide

Source: [21]

For further analysis, selected data for the year 2011 will be used [21].

In general, you can assume n types of pollution.

C. *Quantity of Pollution*

Default data are considered types of pollution emissions in tons per year (t/year) that are listed in the following table

Tab. 2 Emitted amount per year

(n)	q	kt/year
1	PM_{10}	48,4207345
2	SO_2	170,180470
3	NO_x	225,308640
4	CO_2	107 991,12

Sources: [17], authors, [20]

Costs of pollution per unit of quantity.

The amount of emissions we will mark the kind of q_i. These quantities are not possible to sum.

The next table shows data about the cost, which the issuer must incur pollution. Costs relating to the nature of the pollution and the unit of quantity (1 ton) we mark p_i. Their size is EUR per ton (€/t).

Tab. 3 Estimating of damage for pollutants unit

(n)	p	€/t
1	PM_{10}	11 000
2	SO_2	4 000
3	NO_x	4 000
4	CO_2	19 000

Source: [22]

The value of the pollution of the kind i we denoted by h_i. We get this value by using the relation

$$h_i = \ p_i \ q_i. \tag{23}$$

Dimension values h_i we determine the size of the values of p_i q_i. Applies

$$[€/Year] = [€/t] \ [t/ \ Year]]. \tag{24}$$

So, the values h_i are possible to sum for i = 1,2, ..., N.

Tab. 4 Damage for year in €

(n)	p * q	€/year
1	PM_{10}	532 628 079,5
2	SO_2	680 721 880,0
3	NO_x	901 234 560,0
4	CO_2	2 051 831 280 000,0
Sum		2 053 945 864 519,5

It is possible to define nonnegative weights prom the formulae

$$w_i = \frac{h_i}{H} \qquad i = 1,2,3,4 \tag{25}$$

So, we receive the table

Tab. 5 Weights calculation

n	Item	h_i=p_i*q_i (€/Year)	w_i
1	PM_{10}	532 628 079,5	0,000259319
2	SO_2	680 721 880,0	0,000331422
3	NO_x	901 234 560,0	0,000438782
4	CO_2	2 051 831 280 000,0	0,998970477
Sum		2 053 945 864 519,5	1,000000

From relations

$$P = p_1^{w_1}\ p_2^{w_2} \dots p_n^{w_{nt}} \tag{26}$$

$$Q = q_1^{w_1}\ q_2^{w_2} \dots q_n^{w_n} \tag{27}$$

We determine the value of P and Q and their product.

In the calculation we use logarithms. Indeed

$$\ln(P) = \ln(p_1^{w_1}) + \ln(p_2^{w_2}) + \dots + \ln(p_n^{w_n}) \tag{28}$$

$$\ln(P) = w_1 \ln(p_1) + w_2 \ln(p_2) + \dots + w_n \ln(p_n) \tag{29}$$

$$\ln(Q) = \ln(q_1^{w_1}) + \ln(q_2^{w_2}) + \dots + \ln(q_n^{w_n}) \tag{30}$$

$$\ln(Q) = w_1 \ln(q_1) + w_2 \ln(q_2) + \dots + w_n \ln(q_n) \tag{31}$$

We come out of table by the logarithms of the values

Tab. 6 Logarithms of the values

n	Item	h_i=p_i*q_i (€/Year)	w_i
1	PM_{10}	532 628 079,5	0,000259319
2	SO_2	680 721 880,0	0,000331422
3	NO_x	901 234 560,0	0,000438782
4	CO_2	2 051 831 280 000,0	0,998970477
Σ		2 053 945 864 519,5	1,000000

Continue of Tab. 6

n	Item	$\ln(h_i)$	$w_i \ln(h_i)$	$w_i \ln(h_i)$ and Σ
1	PM_{10}	20,0933340	0,005210592	0,005210592
2	SO_2	20,3386644	0,006740671	0,006740671
3	NO_x	20,6192761	0,009047368	0,009047368
4	CO_2	28,3497538	28,32056709	28,32056709
Σ		28,3507839	28,35078387	28,34156573

Consider the aggregate numbers

Tab. 7 Aggregate numbers

n	It corresponds to		Exp(Σ)
1	$\Sigma_i\ w_i = 1$	28,3507839	2 053 945 864 520
2	$\Sigma_i\ w_i \ln(h_i)$	28,3415657	2 035 099 290 843

Let's calculate the values of P and Q. We proceed from tables

D. The values and weights

Tab. 8 Given values and weights

n	Item	p_i (€/t)	q_i (t/year)
1	PM_{10}	11 000	48 420,7345
2	SO_2	4 000	170 180,4700
3	NO_x	4 000	225 308,6400
4	CO_2	19 000	107 991 120,00

Continue of Tab. 8

n	Item	h_i=p_i*q_i (€/year)	w_i
1	PM_{10}	532 628 079,5	0,000259319
2	SO_2	680 721 880,0	0,000331422
3	NO_x	901 234 560,0	0,000438782
4	CO_2	2 051 831 280 000,0	0,998970477
Σ		2 053 945 864 519,5	1,000000

Tab. 9 Logarithm of values

n	Item	$\ln(p_i)$	$\ln(q_i)$	$\ln(h_i)=$ $=\ln(p_i)+\ln(q_i)$
1	PM_{10}	9,30565055	10,78768	20,09333395
2	SO_2	8,29404964	12,04461	20,33866438
3	NO_x	8,29404964	12,32523	20,61927611
4	CO_2	9,85219426	18,49756	28,34975382

Tab. 10 Logarithm of the value multiplied by weights and the sum of its

n	Item	$w_i \ln(p_i)$	$w_i \ln(q_i)$	$w_i \ln(h_i)$ $w_i (\ln(p_i)+\ln(q_i))$
1	PM_{10}	0,002413	0,002797	0,005211
2	SO_2	0,002749	0,003992	0,006741
3	NOx	0,003639	0,005408	0,009047
4	CO_2	9,842051	18,478516	28,320567
Σ		9,850852	18,490713	28,341566
EXP(Σ)		18 975	107 254 308	2 035 099 290 843

Easy to see that it is

2 035 099 290 843 = 18 975*107 254 308

$P\,Q = {}_wH$

So,

value of **18 975** can express scalar values representative of p_i, which denoted P

value of **107 254 308** can express scalar values representative of Q_i, which we denote Q;

value **2 035 099 290 843** can express scalar representative h_i, which denote ${}_wH$

At the same applies

$P\,Q = {}_wH$

The problem is that the value ${}_wH$ does not match the

$$H = \sum_{i=1}^{n} h_i\,, \tag{32}$$

which is the default value for the weights.

Insert one of the options for addressing the problem [23], [24]. Let us start from the definition of weights. For all *i* is valid

$$h_i = w_i\,H\,. \tag{33}$$

Therefore

$$\ln(h_i) = \ln(w_i) + \ln(H) \tag{34}$$

$$w_i \ln(h_i) = w_i \ln(w_i) + w_i \ln(H) \tag{35}$$

Adding the obtained relationship over all i, we get

$$\ln({}_wH) = \sum_{i=1}^{n} w_i \ln(w_i) + \ln(H) \sum_{i=1}^{n} w_i\,. \tag{36}$$

Since the sum of the weights is equal to one, we get

$$\ln({}_wH) = \sum_{i=1}^{n} w_i \ln(w_i) + \ln(H) \sum_{i=1}^{n} w_i\,. \tag{37}$$

Hence

$${}_wH = E\,H\,, \tag{38}$$

where

$$E = \exp\left[\sum_{i=1}^{n} w_i \ln(w_i)\right] \tag{39}$$

Hence

$$H = \frac{{}_wH}{E}\,. \tag{40}$$

Consideration is illustrated on the example

Tab. 11 Results of process

n	Item	$w_i \ln(p_i\,q_i)$	$w_i \ln(w_i)$	$w_i \ln(H)$
1	PM_{10}	0,005211	-0,00214	0,007351909
2	SO_2	0,006741	-0,00266	0,009396060
3	NO_x	0,009047	-0,00339	0,012439815
4	CO_2	28,320567	-0,00103	28,32159609
Σt		28,341566	-0,00922	28,35078387
EXP(Σ)		2 035 099 290 843	0,990824	2 053 945 864 520

Easy to see that

$$2\,053\,945\,864\,520 = \frac{2\,035\,099\,290\,843}{0{,}990824} \tag{41}$$

From the relation

$$H = \frac{{}_wH}{E}. \quad (42)$$

we get

$$H = \frac{P\,Q}{E}. \quad (43)$$

Since P is the proportion indicator, it is advisable to perform a correction for indicator Q. So get decomposition

$$H = P\,\frac{Q}{E}. \quad (44)$$

It can however be selected for the correction value P E_P correction value Q E_Q, so that it applies

$$E = E_P\ E_Q\,. \quad (45)$$

The numerical value is obtained for P=18 975 and Q=107 254 308.

If P is not corrected, the corrected Q is equal to 108 247 565,04 as is true

$$2\,053\,945\,864\,520 = 18\,975 * \frac{107\,254\,308}{0{,}990824} \quad (46)$$

$$\mathbf{2\,053\,945\,864\,520 = 18\,975 * \ 108\,247\,565{,}04}\,. \quad (47)$$

The resulting number is a dimensionless quantity related to a particular year. Therefore, it is suitable to use it as an indicator for monitoring and evaluating of the long term development and for comparing the value over years as well as for international ranking. Based on the development of the indicator over time, policy makers can apply appropriate policies and tools to eliminate negative development in the environment.

V. PROS AND CONS OF AGGREGATED INDICATOR

Although issues related to the quality of the outputs of aggregated indicators are widely discussed by experts, on the following lines advantages and disadvantages of aggregated indicators are presented.

Aggregated indicators in general can serve as a useful tool for policy makers for policy analysis and for other interested parties. The aim is to simplify the problem effectively and to avoid distortion [25]. According to [5], aggregated indicators should be a starting point for initiating overall discussion, attracting public interest and draw attention to particular issues.

As an advantage can be easier interpretation and identifying future performance trend than identifying trends in various separate indicators. Indicator can help to policy makers to set policy priorities. In case of simplicity, relevancy, credibility and legitimacy, indicator can be used as benchmark or can be used for comparison of national performance, eventually for assessing progress within countries.

Disadvantages can include misinterpretation of poorly constructed indicator where important performance issues are ignored [5]. If the process of constructing is not transparent, there is a possibility of influencing the choice of indicators or weights by stakeholders. Aggregated indicator is also demanding for good quality data and users may face a problem with lack of data. Replace the missing data by a qualified estimation is one of the ways how to deal with the problem, however, this method brings specific risks too.

Beside the appropriate selection of input indicators with reliable data, there is another source of uncertainty: weights. Weights, of course, can have a significant effect on the final ranking. In the field of air pollution, weighting reflects different harmfulness of each pollutant to the environment.

On the one hand, using aggregated indicators helps to describe reality in a more simple way. On the other hand, this can result into simplified undesirable conclusions, misinterpretation and misleading policy. Despite all the disadvantages, properly defined indicator can be valuable and helpful for decision makers at all levels and for comparing different entities.

CONCLUSION

The article indicated one of the possible methods of obtaining aggregated indicator and its correction. Commonly used features are analyzed in terms of their explanatory power and in terms of accuracy. On this issue article implicitly points.

Generally, when examining structured variables, it is necessary to examine the influence of factors weighing on aggregate variables. This must include in the survey and related analyzes.

Next viewpoints should be considered concern the sensitivity to changes, and the characteristics of the variables that determine them. In other words, when constructing aggregated indicator, sensitivity analysis and uncertainty analysis should be better taken into consideration in order to contribute to the overall quality of the indicator.

In terms of particulate matter, only a selection of pollutants of PM_{10} are taken into account. Due to the lack of data caused by the fact, the indicator is measured only in certain places and it is not monitored throughout the territory. Work therefore operates with the sum of available data. For the level of PM_{10} an expert estimation was used according to which 80% of particulate matter consists of PM_{10}.

Formulas used in the description and definition of the characteristics might be expressing by different way e.g. d ln(x) or statements dx / x have the same meaning.

This article draws attention to other aspects of description and quantification of phenomena and processes. The result should be considered a rough estimate with advantages and disadvantages of aggregate indicator.

These possibilities are illustrated on the example of problematic quantification of environmental damage. This procedure could help to improve the quality and acceleration of data for decision-making processes.

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