Day-Ahead Demand Management in Multi-Supplier Power Grid under Transmission Constraints*

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***Abstract*—Ever-increasing energy consumption and growing penetration of renewable energy sources stimulate the development of new power grid models and architectures. Since the decentralization of power grids raises the unreliability of power supply, it is crucial to switch to a production-oriented consumption in order to provide the stability of the grid. In this work, we describe a multi-supplier power grid model with day-ahead time span planning. We formulate and study a set of consumer cost minimization problems under flow distribution constraints. Finally, we consider an example illustrating the applicability of this model.**

***Index Terms*—power grid, load flow, demand management**

I. Introduction

Traditionally, power grids have a central structure with a clear hierarchy. There are few power plants that produce and supply energy to a large area using transmission and distribution networks, and these power plants respond to a changing demand of consumers. However, due to the fast renewable energy development of recent decades, this situation is starting to change. New power grid architectures need to be created and studied in order to integrate smaller local renewable generators into the power grid while maintaining sustainability of the system.

One of the main challenges is providing balance between production and consumption in the network, especially taking into account the uncontrollable weather-dependent nature of main renewable energy sources (i.e., solar and wind energy). A possible solution is switching to production-oriented consumption, when consumers respond to changes in available generation capacities rather than producers to changing demands. This concept, also known as demand response management, includes different measures, but the goal is the same: to motivate consumers to change their strategies and to coordinate consumption with generators. Another issue is to maintain the transmission network and to avoid overloads in its links. This generally non-trivial problem becomes even more complicated for a decentralized system in the presence of multiple energy producers.

In this paper, we formulate and consider a multi-supplier power grid model, where consumers need to conclude bilateral contracts with suppliers over a day-ahead period of time divided in several time slots (e.g., 24 hours). The distribution of flows in the network deserves special attention, since it is crucial for preventing overloads and other disturbances in transmission lines. We describe consumers' costs as functions of their contract profiles, formulate a competitive game of consumers, and discuss possible schemes of demand response management for this model.

The topic of demand response was intensively studied in recent years. Demand management using pricing mechanisms for systems with a single generator and several competitive consumers are formulated in [10], [14]. Work [14] considers two-level piecewise linear cost functions, whereas in [10] functions are quadratic. Models with multiple generators and storage systems with quadratic costs are studied in [2], [3], where equilibria are found using variational inequalities. The ideas of cooperative game theory and coalition formation can also be applied for demand management (e.g., [1], [9]).

However, these works do not consider the flow distribution in the transmission network that depends on the topology of the network, whereas fulfilling lines capacities constraints is necessary for stability of the power grid. Some authors study networks with simple topologies: a set of parallel links [7], a network with a star-shaped structure [12], a network with a link for each producer-consumer pair [6]. In this work, we formulate a model with a general network topology and discuss challenges arising in this setting.

Flow distribution in electrical networks outside game theoretic scope is a well-studied topic, starting in the nineteenth century with formulation of Kirchhoff's current laws. The problem of finding flow distribution was first formulated as a mathematical program in the middle of the twentieth century [4], [5]. It is known that methods of transportation cannot be applied to power flow distribution, since there are some critical differences between information and electricity, e.g., flows in electric networks cannot be routed directly [8]. However, one can note that Kirchhoff's laws and conditions of user equilibrium in the non-atomic routing setting are similar, and respective optimization problems take similar forms (see, e.g., Ch.2.6.3 in [11]).

The remainder of this work has the following structure. Section 2 describes the multi-supplier power grid model and the flow distribution in the transmission network. Section 3 formulates consumer cost minimization game and discusses

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the existence of equilibria. An example of demand response technics for the model is considered in Section 4, following by simulation results for a specific network. Finally, Section 5 concludes the paper and discusses future work.

II. Model Description

This section describes the structure of a power grid and discusses the power flow distribution in the grid. We also formulate a (generally non-linear) optimization problem for finding a power flow vector.

A. Network

A network is represented by a directed graph (V, A), where V is a set of nodes and A is a set of arcs. Let us enumerate nodes in V in the following manner: $V_Q = \{1, \ldots, m\}$ is a set of m energy consumers, $V_P = \{m+1, \ldots, m+n\}$ is a set of n producers, and $V_O = \{m+n+1, \ldots, |V|\}$ is a set of all other nodes.

In this work, we consider a day-ahead planning period divided into H intervals. Each consumer concludes bilateral energy purchase contracts with several producers for each time interval. By e_{ij}^h we denote an amount of energy to be delivered from producer $j \in V_P$ to consumer $i \in V_Q$ during the time interval $h \in \mathbf{H} = \{1, 2, \ldots, H\}$. We also use the following notation:

$$\mathbf{e}_i^h = (e_{i(m+1)}^h, \ldots, e_{i(m+n)}^h)^T \tag{1}$$

for a vector of i's contracts at a time interval h, and

$$\mathbf{E}_i = (\mathbf{e}_i^1, \ldots, \mathbf{e}_i^H) \tag{2}$$

for a matrix of all i's contracts.

Consumers need to meet their energy demands, both total for the whole day and minimal for each time interval $h \in \mathbf{H}$. We denote the total demand of consumer i by $D_i \geq 0$, and the minimal demand of the same consumer for a time interval h by $d_i^{min}(h) \geq 0$. Therefore, we can write the demand constraints for $\mathbf{E}_i$:

$$\begin{aligned} \mathbf{1}_n^T \cdot \mathbf{e}_i^h &\geq d_i^{min}(h), \\ \mathbf{1}_n^T \cdot \mathbf{E}_i \cdot \mathbf{1}_H &= D_i, \end{aligned} \tag{3}$$

where $\mathbf{1}_k = (1, 1, \ldots, 1)^T \in \mathbb{R}^k$.

Let us define energy balance b_k^h in a node $k \in V$ for a time interval $h \in \mathbf{H}$:

$$\begin{aligned} b_k^h &= -\mathbf{1}_n^T \cdot \mathbf{e}_k^h, && k \in V_Q, \\ b_k^h &= \sum_{i=1}^{m} e_{ik}^h, && k \in V_P, \\ b_k^h &= 0, && k \in V_O. \end{aligned} \tag{4}$$

This value reflects the amount of energy injected or withdrawn in a node during a specific time interval. It is negative for consumers and non-negative for producers, while we assume all other intermediate nodes to have zero energy balance.

Now we describe the flow distribution in the power grid for given energy balances.

B. Flow Distribution

Energy flows in a power grid are distributed according to Kirchhoff's laws, and we can find this distribution for a given set of energy balances and knowing parameters of grid links.

By $f_{kl}^h \geq 0$ we denote a flow in arc $(k, l) \in A$ at time interval h, and set $\mathbf{f}_h = \{f_{kl}^h, (k, l) \in A\}$ is a flow profile of all links at time h. We also define the following two subsets of V

$$\begin{aligned} W_k^{in} &= \{l \in V | (l, k) \in A\}, \\ W_k^{out} &= \{l \in V | (k, l) \in A\}, \end{aligned}$$

and the first Kirchhoff's law can be written as follows:

$$\sum_{l \in W_k^{out}} f_{kl}^h - \sum_{l \in W_k^{in}} f_{lk}^h = b_k^h, \quad \forall k \in V. \tag{5}$$

Let $\Theta_{kl}(f_{kl}^h)$ be a voltage function for an arc $(k, l) \in A$, and by $\pi_k(h)$ denote an electric potential in a node $k \in V$ at time interval h. The second Kirchhoff's law takes the following form:

$$\pi_k(h) - \pi_l(h) = \Theta_{kl}(f_{kl}^h), \quad \forall (k, l) \in A. \tag{6}$$

The flow profile $\mathbf{f}_h$ can be found as a solution of a non-linear optimization problem (as in [11])

$$\begin{aligned} \underset{\mathbf{f}_h}{\text{minimize}} \quad & \sum_{(k,l) \in A} \int_0^{f_{kl}^h} \Theta_{kl}(s) ds, && (7) \\ \text{subject to} \quad & \sum_{l \in W_k^{out}} f_{kl}^h - \sum_{l \in W_k^{in}} f_{lk}^h = b_k^h, \forall k \in V && (8) \\ & f_{kl}^h \geq 0, \quad \forall (k, l) \in A. && (9) \end{aligned}$$

Since a set of contract vectors $\mathbf{E}^h = \{\mathbf{e}_1^h, \ldots, \mathbf{e}_m^h\}$ defines the energy balances $\{b_k^h, k \in V\}$ which are parameters of the constraint set (8), we denote the solution of minimization problem (7)-(9) by $\mathbf{f}_h(\mathbf{E}^h)$. This mapping is generally non-linear, and contract changes of a single consumer affect the flow distribution in the whole grid.

III. Game of Consumers

This section formulates and studies a consumer game as a model of interactions in the grid. First, we describe cost functions of consumers and formulate a game as a set of coupled cost minimization problems. In the second part of the section, the existence of Nash equilibria for the described game is discussed.

A. Consumer Cost Minimization

Each consumer tries to minimize their total costs over time span $\mathbf{H}$. These costs consist of two parts: generation costs and transmission costs. Generation costs can be assigned proportionally to the contracts between respective agents, while it is non-trivial to define the shares for use of transmission network.

More specifically, let $\alpha_j^h(b_j^h)$ denote a generation cost of a unit of energy at node $j \in V_P$ during time interval h. It is a function of total energy b_j^h to be generated at node j according to contracts with consumers $\mathbf{E}^h$. Hence, generation cost of consumer i during interval h can be determined in the following way:

$$G_i^h(\mathbf{E}^h) = \sum_{j=m+1}^{m+n} e_{ij}^h \cdot \alpha_j^h(b_j^h). \quad (10)$$

Transmission costs depend on the flow distribution $\mathbf{f}_h(\mathbf{E}^h)$. We define transmission cost for an arc $(k,l) \in A$ as a function $\beta_{kl}^h(f_{kl}^h)$ of the amount of flow using this arc. We call a set of functions $\Delta = \{\delta_{kl}^{i,h}(\mathbf{E}^h)\}$ a cost sharing rule, if it fulfills the following conditions:

$$\begin{aligned} \delta_{kl}^{i,h}(\mathbf{E}^h) \geq 0, &\quad \forall (k,l) \in A, i \in V_Q, h \in \mathbf{H}, \\ \sum_{i=1}^{m} \delta_{kl}^{i,h}(\mathbf{E}^h) = 1, &\quad \forall (k,l) \in A, h \in \mathbf{H}. \end{aligned} \quad (11)$$

For a given cost sharing rule Δ the transmission cost of consumer i at interval h takes the form:

$$T_i^h(\mathbf{E}^h) = \sum_{(k,l)\in A} \delta_{kl}^{i,h}(\mathbf{E}^h) \cdot \beta_{kl}^h(f_{kl}^h(\mathbf{E}^h)). \quad (12)$$

Hence, the total cost of consumer i is

$$C_i(\mathbf{E}) = \sum_{h=1}^{H} \left(G_i^h(\mathbf{E}^h) + T_i^h(\mathbf{E}^h) \right), \quad (13)$$

where $\mathbf{E} = \{\mathbf{E}^1, \ldots, \mathbf{E}^H\}$ is a total profile of all consumer contracts over the whole time span $\mathbf{H}$, and where the calculation of each transmission cost $T_i^h(\mathbf{E}^h)$ requires solution of problem (7)-(9) for a respective time interval h.

We now formulate the game of consumers:

$$\begin{aligned} \underset{\mathbf{E}_i}{\text{minimize}} \quad & C_i(\mathbf{E}), \quad 1 \leq i \leq m, & (14)\\ \text{subject to} \quad & \mathbf{1}_n^T \cdot \mathbf{E}_i \cdot \mathbf{1}_H = D_i, \quad \forall i \in V_Q, & (15)\\ & \mathbf{1}_n^T \cdot \mathbf{e}_i^h \geq d_i^{min}(h), \quad \forall i \in V_Q, \forall h \in \mathbf{H}, & (16)\\ & e_{ij}^h \geq 0, \quad \forall i \in V_Q, \forall j \in V_P, \forall h \in \mathbf{H}. & (17) \end{aligned}$$

In this game, contract matrix $\mathbf{E}_i$ is a strategy of consumer i. We denote by $\mathbf{\Sigma}_i$ a set of all i's feasible strategies, i.e., a set of all matrices $\{\mathbf{E}_i\}$ fulfilling the conditions (15)–(17).

B. Existence of Nash Equilibria

The idea of Nash equilibrium proved to be the most appropriate solution concept for competitive games. A set of agents' strategies is in Nash equilibrium, if none of agents may reduce their total cost by unilaterally changing their strategy. In our model, a total profile $\mathbf{E}^*$ is in Nash equilibrium, if the following conditions are fulfilled:

$$C_i(\mathbf{E}^*) \leq C_i(\mathbf{E}_i, \mathbf{E}_{-i}^*), \forall \mathbf{E}_i \in \mathbf{\Sigma}_i, \quad (18)$$

where $\{\mathbf{E}_i, \mathbf{E}_{-i}^*\}$ is a total profile that differs from $\mathbf{E}^*$ only in component $\mathbf{E}_i$.

The existence of Nash equilibria in a consumer game strong-ly depends on the form of cost functions $\{\alpha_j^h(\cdot)\}$, $\{\beta_{kl}^h(\cdot)\}$ and the cost sharing rule Δ. Moreover, arguments of $\{\beta_{kl}^h(\cdot)\}$ are flows in the corresponding arcs, which are in turn components of a solution of non-linear optimization problem (7)-(9). Hence, establishing the fact of equilibrium's existence is a non-trivial task.

Theorem 1: Assume that a network contains no cycles, functions $\{\alpha_j^h(\cdot)\}$ are convex and increasing, functions $\{\beta_{kl}^h(\cdot)\}$ are convex, and transmission costs are shared according to rule (7)–(9). Then game (14)–(17) has a Nash equilibrium contract profile $\mathbf{E}^*$.

Proof. According to ([13]), an equilibrium exists for any n-person game with concave payoff functions. Since we consider cost functions rather than payoff functions, the same statement is true for games with convex cost functions. Therefore, we need to check whether a cost function $C_i(\mathbf{E}) = C_i(\mathbf{E}_1, \ldots, \mathbf{E}_m)$ is convex in $\mathbf{E}_i$ for each consumer $i \in V_Q$.

Function $C_i(\mathbf{E})$ consists of several summands:

$$C_i(\mathbf{E}) = \sum_{h=1}^{H} \left(G_i^h(\mathbf{E}^h) + T_i^h(\mathbf{E}^h) \right).$$

If we show that each summand in this sum is convex, convexity of the whole sum will be established as well. First, we study function $G_i^h(\mathbf{E}^h)$:

$$G_i^h(\mathbf{E}^h) = \sum_{j=m+1}^{m+n} e_{ij}^h \cdot \alpha_j^h(b_j^h). \quad (19)$$

When we fix the contract profiles of all consumers except i, function $\alpha_j^h(b_j^h + \lambda)$ remains convex and increasing, and function in (19) is convex as a product of two non-negative increasing convex functions.

Second, we rewrite function $T_i^h(\mathbf{E}^h)$ with fixed contract profiles of all consumers except i applying Proposition 3.1:

$$T_i^h(\mathbf{E}^h) = \sum_{(k,l)\in A} \delta_{kl}^{i,h}(\mathbf{E}^h) \cdot \beta_{kl}^h(f_{kl}^h(\mathbf{E}^h)). \quad (20)$$

The argument of $\beta_{kl}(\cdot)$ in (20) is a linear combination of $\{e_{ij}, j \in V_P\}$, components of consumer i's contract profile. Therefore, $\beta_{kl}^i(\mathbf{E}^h)$ remains convex in $\mathbf{E}_i^h$, as well as $T_i^h(\mathbf{E}^h)$.

The convexity of cost functions in respective arguments is established, that completes the proof.

IV. EXAMPLE

Consider a network with 7 nodes that is depicted in Figure 1. There are 3 consumers (red nodes), 3 producers (green nodes) and one intermediate node. Therefore, $V_Q = \{1, 2, 3\}$, $V_P = \{4, 5, 6\}$, and $V_O = \{7\}$.

All nodes are located in the same local area except for node 4 that depicts a conventional energy generator, e.g. a power plant. Hence, arc (4, 2) is longer than all other arcs, and transmission costs are higher for this arc.

Since there are no cycles in the network, we only need to check the first Kirchhoff's law (5). A flow on each arc is a linear combination of $\{e_{ij}^h\}$, $h = \{1, 2, 3, 4\}$:

$$\hat{f}_{25}^h = e_{34}^h + e_{36}^h - e_{15}^h - e_{25}^h, \quad \hat{f}_{53}^h = e_{34}^h + e_{35}^h + e_{36}^h,$$
$$\hat{f}_{42}^h = e_{14}^h + e_{24}^h + e_{34}^h, \quad \hat{f}_{27}^h = e_{14}^h + e_{15}^h - e_{26}^h - e_{36}^h,$$
$$\hat{f}_{71}^h = e_{14}^h + e_{15}^h + e_{16}^h, \quad \hat{f}_{67}^h = e_{16}^h + e_{26}^h + e_{36}^h.$$

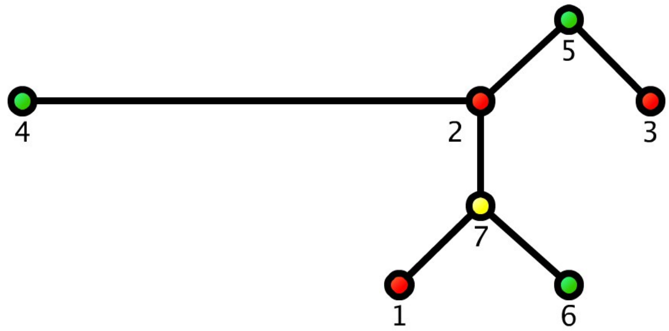

Fig. 1. 7-node network with no cycles

The direction of flow in arcs (2, 5) and (2, 7) may differ depending on the values $\{e_{ij}^h\}$. If $\hat{f}_{25}^h < 0$, we assign $\hat{f}_{25}^h = 0$ and $\hat{f}_{52}^h = -\hat{f}_{25}^h$. The same is true for $\hat{f}_{27}^h$.

Let us assume that functions $\{\alpha_j^h(\cdot)\}$ and $\{\beta_{kl}^h(\cdot)\}$ have the following form:

$$\begin{aligned} \alpha_j^h(x) &= \lambda_j^h \cdot x^{1+\epsilon} + \mu_j^h, \quad \forall j \in V_P, \\ \beta_{kl}^h(x) &= \lambda_{kl}^h \cdot x^{1+\zeta}, \quad \forall (k,l) \in A, \end{aligned} \tag{21}$$

where all coefficients are non-negative. We are ready now to solve the problem (14)–(17) with specific values of demands and coefficients in (21), and evaluate the total cost reduction. Actually, it is clear that the problem is a computationally difficult. Indeed, the presence of four time periods makes us to compare numerous combinations of different contracts. Thus, we are dealing with combinatorial optimization and the problem could be NP-hard. In future works we will investigate these questions carefully.

V. Conclusion

In this work, we have introduced new model for multi-supplier power grid under transmission constraints. Our model studies daily energy dynamics. The game of consumers was formulated and the existence result was established given specific properties of cost functions. There are several directions to improve and generalize the methods discussed in this work, and we name only few of them. First, real-world production and transmission costs, as well as voltage change functions, should be further studied in order to provide realistic representation of the network. Secondly, one can investigate a setting with dynamic network topology. Though power grid structures are relatively constant, there might be different applications of this model, e.g. for planning an optimal modification of a grid, or for maintaining the stability in a case of emergency such as blackouts.

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