

A Stochastic Approach for Ground Motion Simulation

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Abstract—In the paper a statistical method is built up for simulating the propagation of the seismic signal over the territory. If the basic data relevant to some ground motions really occurred in a region, as i.e. the epicenter intensity and location, the site intensity, the magnitude value, the time duration, the occurrence period, the peak acceleration, etc., are known the probability density function of the seismic intensity at site can be solved by the statistical elaboration of the recorded data. After building the seismic intensity function the propagation of the signal over territory can be simulated for some expected earthquakes. A sample of the method was developed for an Italian area by statistically elaborating the data relevant to a series of historical earthquakes and has shown a good correspondence with the real seismic propagation in the area.

Keywords—Seismic engineering; seismic macro-zoning; signal simulation; Gaussian function; historical earthquakes.

I. INTRODUCTION

THE problem of protecting civil structures with regards to seismic events involves some features relevant to seismic forecasts [1],[2], which is particularly attractive, especially with reference to existing structures and monumental constructions, besides the development of analysis tools [3]-[17], and protection strategies [18]-[28].

The propagation of the seismic signal over the territory during an earthquake is often solved by using probabilistic methodologies that consider the different seismic parameters characterizing an earthquake from a macroscopic point of view as random variables.

By statistically elaborating the recorded data of historical earthquakes occurred in an geographic area, the hazard function at the site can be evaluated.

Some of the fundamental seismic parameters such as the magnitude, the peak intensity, the epicentral and site's intensity, the time duration of the earthquake, the occurrence period, etc. can be correlated to each other by means of simple functions that allow to distinguish the primitive entities from the dependent ones; in this way one sets up a model where, once some basic variables are fixed, the seismic phenomenon can be looked at as a stochastic process. Evaluation of seismic risk follows from probabilistic treatment of such model, and can be approached at different degrees of detail (Fig.1).

For example, the statistical elaboration of some of the fundamental parameters characterizing a seismic event, like the magnitude M , the time duration T_0 , the epicentral location E , etc., endowed with a model for propagation and attenuation of the seismic signal over the territory, allows to trace some macroseismic maps; from the maps one can read the distribution of the seismic hazard on the territory and, in details, the hazard at the considered site, which, combined with the vulnerability and the exposition, produce an evaluation of the seismic risk at the site.

Some deeper investigations can be developed from the hazard maps, identifying some other macro-parameters, like the epicentral intensity and the intensity at the site I , the peak acceleration a_p , the occurrence time T ; these parameters can be related to every other macro-parameter, such as the factor of local response, the coefficient of seismic amplification, the seismic response function, etc.; coupling with a higher-level vulnerability or with some particular engineering analysis and controls, up to the maximum details, can be carried on.

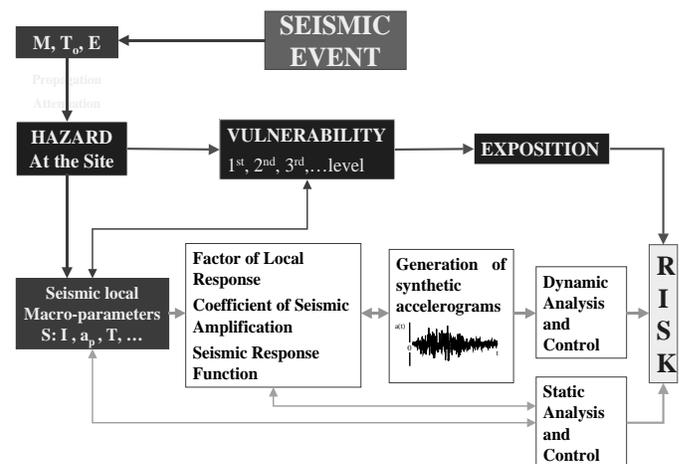


Fig. 1. Flow scheme for the evaluation of the seismic risk.

The generation of synthetic site-compatible accelerograms, which are mandatory for full nonlinear structural analysis, can, thus, be performed. The combination between the calculated accelerograms and some specific engineering analysis and control allows to determine the local value of the seismic risk.

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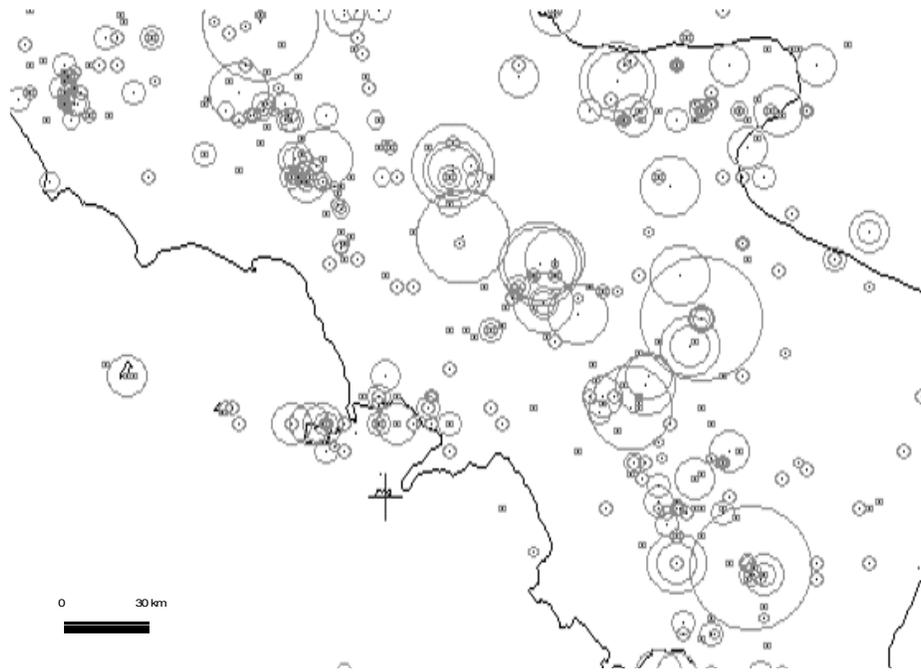


Fig.2. Distribution of the 542 selected historical epicenters (the circles show the magnitude values relevant to the seismic event) in South of Italy in the time period 1000-1997.

Starting from the classical models (see e.g. Cornell [29]) for the probabilistic analysis of the basic macroseismic parameters, i.e. the seismicity law (see e.g. the Gutenberg-Richter law), the occurrence process (e.g. the Poisson process), the attenuation law and, possibly, other ones, and by exploiting the fundamentals of Statistics and Probabilities, a stochastic field is built by means of an original methodology specifically developed for this research and applied to a sample area of the central-southern Italy.

II. EPICENTER DISTRIBUTION OVER THE TERRITORY

The data of historical earthquakes occurred in a sample area are elaborated in a number of original computer calculation codes, purposely built up by the authors for this scope.

The sample area is included in Central-Southern Italy and the data necessary to the elaboration consist of the fundamental macroseismic parameters, as epicentral intensity, magnitude, intensity at the site, distance between the site and the epicenter. The data of historical earthquakes with varying intensities and magnitude values larger than 4 and occurred in the area in the time period between the years 1000-1997, have been collected from the principal seismic catalogues i.e. the Catalogue of the strong motions in Italy [30] and the Macro-seismic Bulletins of Italy [31] and selected on the basis of the availability and reliability of the historical data (Fig.2).

If a seismic event with epicenter E and magnitude M has occurred, the geographical location of the epicenter \tilde{E} for an expected earthquake can be looked at as a two-dimensional random vector ruled by a Joint Probability Density Function (JPDF) of the epicentral location as

$$\begin{aligned}
 P(E|M) &= \text{Prob}\{\tilde{E} = E\} = \\
 &= \sum_{j=1}^n k_j p_j(x_E, y_E, \bar{x}_{Ej}, \bar{y}_{Ej}, \sigma_{xj}, \sigma_{yj}, \rho_j)
 \end{aligned}
 \quad (1)$$

where k_j denote some combination coefficients which depend on the magnitude, and p_j are some probability functions which depend on the geographical position (x_E and y_E) of the possible epicenters in the area, and on some other probabilistic parameters.

Such unknown parameters (medium values $\bar{x}_{Ej}, \bar{y}_{Ej}$, standard deviations σ_{xj}, σ_{yj} , correlation coefficients ρ_j and combination parameters k_j), which are necessary to solve the probability functions of epicentral location, can be deduced by the statistical elaboration of the recorded data of historical earthquakes occurred in the sample area over a fixed time period.

By fixing a number of magnitude intervals and distributing the earthquakes in each class of magnitude, the epicentral density is modeled by means of a linear convex combination of a number of bivariate Gaussian distributions, whose basic variables are the geographical coordinates of the epicenter.

The parameters that identify the Gaussian distributions, as mean values, standard deviations, correlation coefficients, and the relevant combination coefficients are calibrated by two different procedures.

The procedure "A" (Fig.3.a) fixes a grid for each class of magnitude and solves the various JPDFs for each grid's

quadrant, then the JPDF of the total area is given by the sum of these distributions. The procedure “B” (Fig.3.b) solves the unknown parameters of the Gaussian distribution by minimizing the difference between the simulated moments and the moments calculated by elaboration of the data of historical earthquakes already occurred in the sample area (see [1] for the extended procedure).

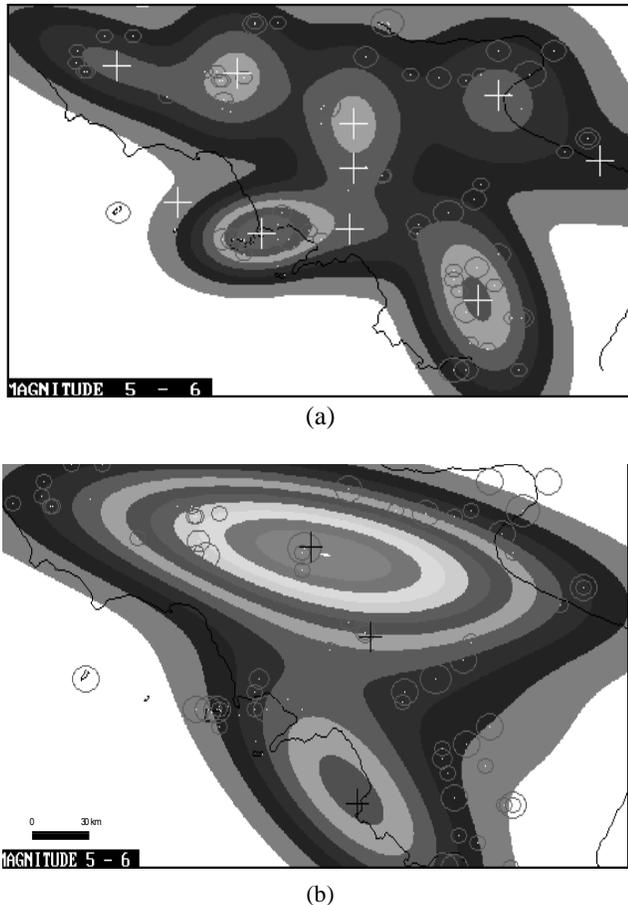


Fig. 3. (a) Procedure “A”: macroseismic map solved with a prefixed grid for a selection of historical earthquakes having a magnitude value of 5 6, (b) Procedure “B”: macroseismic map solved for selected historical earthquakes with magnitude of 5-6.

The stability of both procedures is demonstrated by checking the variation of the epicentral JPDF’s results for different time ranges; which also demonstrates the robustness of the data used in conjunction with the proposed procedure.

With respect to other statistical methodologies, both procedures for the elaboration of seismic maps, although simplified, present the advantage to be prompt and to be based on the fundamental macroseismic data, which are easily available.

Some disadvantage can be found in the necessity of elaborating a large quantity of basic data, which, anyway is a common characteristic of any probabilistic study (see [2] for the results of the extended procedure). On the other side, an important difference with respect to the classical models so far available, is that both the procedures “A” and “B” do not let

the choice of the seismogenetic area to the subjectivity of the operator.

III. SEISMIC PROPAGATION OVER THE TERRITORY

The identification of a distribution function of the epicenters is followed by a large series of developments. The first of them is a model for the propagation of the seismic signal over the territory.

During an earthquake the energy is dissipated as seismic waves that spread along radial paths from the seismic source through the earth crust beneath the territory.

Generically a seismic signal $\tilde{s}(r, \alpha)$ that propagates from the epicenter E to the recording station S walking a distance r, can be expressed by a probabilistic function as the sum of a deterministic function of r, with randomness lumped in the process $\tilde{\theta}(\alpha)$ at the exponent, and of a function that depends randomly both on r and the polar angle α (or the anomaly with respect to the horizontal x-axis) as

$$\tilde{s}(r, \alpha) = s_0 \cdot e^{-\tilde{\theta}(\alpha)r} [1 + \tilde{d}(r)] \tag{2}$$

with s_0 the epicentral value of the signal, $\tilde{\theta}(\alpha)$ a random function of the polar angle α , $\tilde{d}(r)$ a stationary random function of the distance r of the site from the epicentre E.

The character of both the processes $\tilde{\theta}(\alpha)$ and $\tilde{d}_1(r)$ can be investigated by elaborating the available data from historical earthquakes.

For example, the propagation of a generic seismic wave along a particular direction from the epicenter E to the current recording station S at increasing distance r is shown in Fig.4, where one can distinguish in the signal $\tilde{s}(r, \alpha)$ the deterministic and the random components.

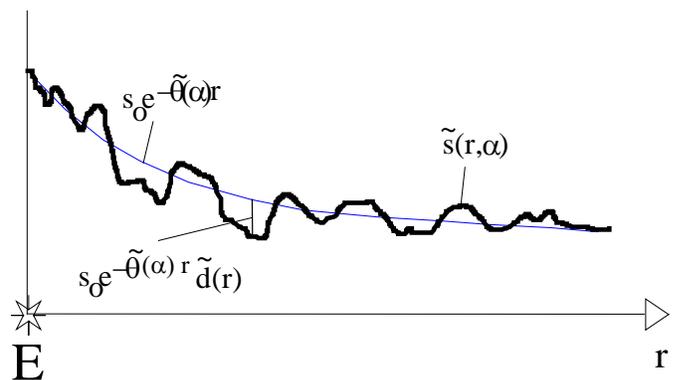


Fig.4. Generic propagation of a seismic impulse from the epicenter E along the direction r.

In a first approximation it can be considered the probabilistic function of the seismic propagation as exclusively expressed by the deterministic component of the signal where the exponential term includes all its random character. So the random component can be completely neglected and the

probability density function of the seismic intensity at the site I_s –conditioned by the given value of magnitude and by the epicenter location as r and α – can be easily expressed by means of the epicenter intensity I_o as

$$I_s(r, \alpha) = I_o e^{-\tilde{\theta}(\alpha) r} \tag{3}$$

where the random process $\tilde{\theta}(\alpha)$ gets the directionality of the propagation of the seismic signal on the territory and can be investigated by elaborating the available data from historical earthquakes.

If one considers the “characteristic distance” $\tilde{R}(\alpha, I_o)$ as “the distance at which the intensity is reduced by one degree”

$$\tilde{R}(\alpha, I_o) = \frac{-\ln(I_o - 1/I_o)}{\tilde{\theta}(\alpha)} \tag{4}$$

it is easy to realize that the characteristic distance represents the attenuation of a seismic signal over the territory produced along the radial directions from the epicenter during an

earthquake.

By solving the attenuation law for some ground earthquakes (i.e $N_T=12$ for the Italian area considered in the sample) for all the selected ground motions it is possible to notice that: 1) the radial diagrams of the process $\tilde{\theta}(\alpha)$ present a strongly random character dependent on the angle α , 2) the mean quadratic values have a quasi-circular form, and the mean quadratic errors along all directions from the epicenter appear as very small curves, as shown in Fig. 5.

In conclusion the sample demonstrates that since the exponent $\tilde{\theta}(\alpha)$ is strongly random, by contrast, the exponential law for the seismic waves’ propagation is practically deterministic, and the propagation of the seismic signal over the territory can be correctly expressed by Equation (3).

Finally by the radial representation of the inverse function of the $\tilde{\theta}(\alpha)$ coefficient plotted on an Italian area it is possible to have the local intensity function’s propagation on the territory for a series of historic earthquakes (Fig.6).

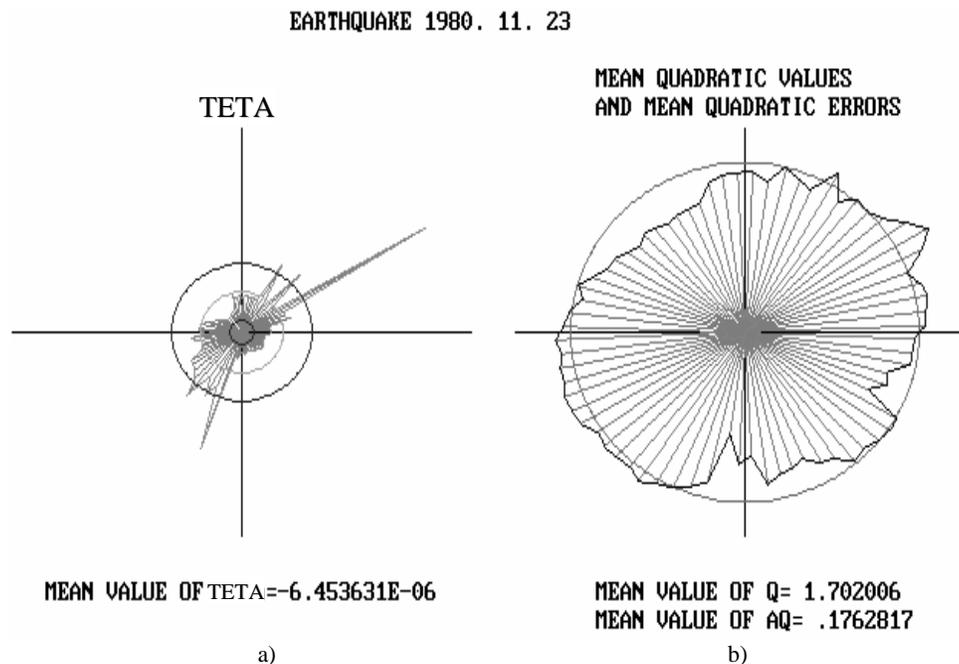


Fig. 5. Sample: a) radial distributions of the process $\tilde{\theta}(\alpha)$ for the 1980.11.23 Irpinia earthquake; b) relevant mean quadratic values (external line) and mean quadratic errors in the exponential approximation of the radial attenuation (internal line).

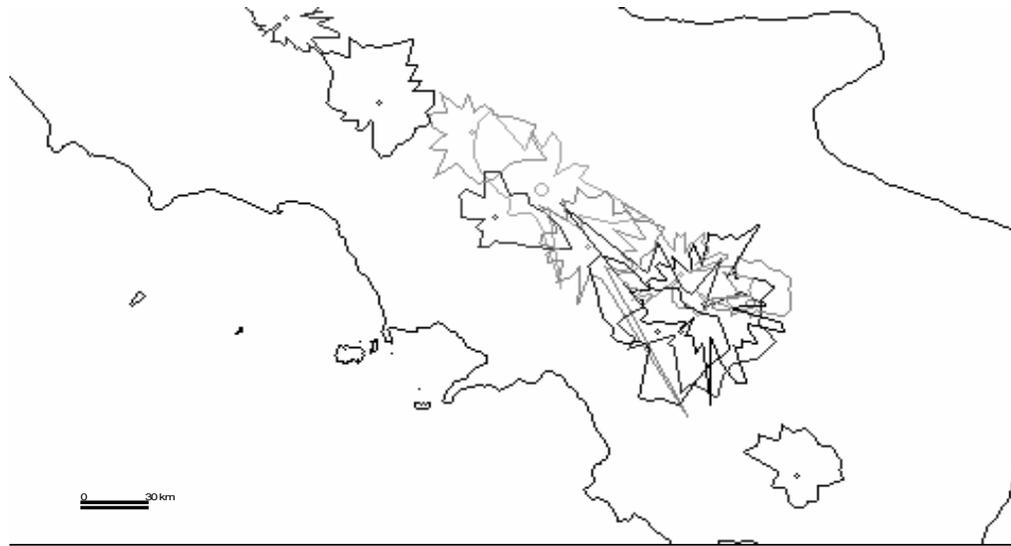


Fig. 6. Attenuation of the seismic signal over the territory produced along the radial directions from some historic ground motions.

IV. ATTENUATION LAW

By the previous considerations the random function $\tilde{\theta}(\alpha)$ can be considered as a periodic process, so that it can be expressed as a sum of n harmonic functions (by means of a Fourier series' development truncated to a limited number of terms) affected by random coefficients as

$$\tilde{\theta}(\alpha) = \sum_{k=1}^n (\tilde{c}_k \cos k\alpha + \tilde{c}_{k+n} \sin k\alpha) \tag{5}$$

where the period of the k-harmonic function is given by

$$T_k = \frac{1}{f_k} = \frac{2\pi}{\omega_k} = \frac{1}{k} \tag{6}$$

In order to calculate the statistics of the \tilde{c}_k , for any \tilde{c}_k sample values c_{kj} (with $j= 1, \dots, N_T$) can be given by developing in a Fourier sum the radial propagation at any historic epicentral location

$$\theta_j(\alpha) = \sum_{k=1}^n (c_{kj} \cos k\alpha + c_{k+n,j} \sin k\alpha) \tag{7}$$

Then by means of such sample coefficients $c_{\ell j}$ (with $\ell = 1, \dots, 2n$, $j=1, \dots, N_T$ and $N_T= 12$) their probabilistic parameters, as the mean values' vector \bar{c}_i and the correlation matrix C_{rs} of the coefficients c_{ij} can be solved as

$$\bar{c}_i = E[\tilde{c}_i] = \frac{1}{N_T} \sum_{j=1}^{N_T} c_{ij} \tag{8}$$

$$C_{rs} = \text{Cor}[\tilde{c}_r, \tilde{c}_s] = \frac{1}{N_T} \sum_{j=1}^{N_T} c_{rj} c_{sj} \tag{9}$$

with $r, s = 1, \dots, 2n$

where the coefficient j is characteristic of the historical epicentre, r and s are characteristic of the order of the harmonic function.

Finally that the probabilistic parameters of the coefficient $\tilde{\theta}(\alpha)$, as the mean value function and the autocorrelation function, can be solved as

$$\bar{\theta}(\alpha) = E[\tilde{\theta}(\alpha)] = \sum_{k=1}^n (\bar{c}_{kj} \cos k\alpha + \bar{c}_{k+n,j} \sin k\alpha) \tag{10}$$

$$R_{\beta}(\alpha, \gamma) = \text{Cor}[\tilde{\theta}(\alpha), \tilde{\theta}(\alpha + \gamma)] = \sum_{k=1}^n \sum_{h=1}^n \left(\begin{aligned} &C_{kh} \cos k\alpha \cos h(\alpha + \gamma) + \\ &C_{k+n,h+n} \sin k\alpha \sin h(\alpha + \gamma) + \\ &C_{k,h+n} \cos k\alpha \sin h(\alpha + \gamma) + \\ &C_{k+n,h} \sin k\alpha \cos h(\alpha + \gamma) \end{aligned} \right) \tag{11}$$

The approximation between the $\tilde{\theta}(\alpha)$ curves calculated by means of the seismic records and the $\bar{\theta}(\alpha)$ curves calculated by means of the Fourier series represents a good check of this methodology, as shown in Fig.6.

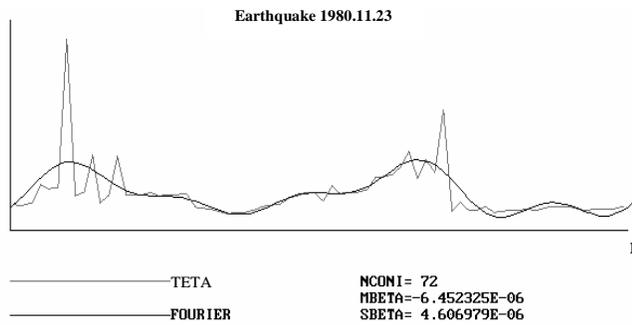


Fig.7. Comparison between the $\theta(\alpha)$ values for the recorded sites and the $\theta(\alpha)$ function simulated by means of the Fourier series for the Irpinia earthquake on 1980.11.23.

V. SIMULATION OF THE PROPAGATION PROCESS

Assuming that the coefficients \tilde{c}_k in Equation (5) are Gaussian, the process $\tilde{\theta}(\alpha)$ is Gaussian as well.

After discretizing the time period $(0,2\pi)$ in n-steps of α the random variables $\tilde{\theta}_1 = \tilde{\theta}(\alpha_1), \dots, \tilde{\theta}_n = \tilde{\theta}(\alpha_n)$ are representative of $\tilde{\theta}(\alpha)$. Since the $\tilde{\theta}_i$ are jointly Gaussian it is possible to simulate n-dimensional sample vectors by means of the medium values and of the co-variances of the $\tilde{\theta}_i$.

If one considers the n-transformed random variables

$$\tilde{\mathbf{y}} = \mathbf{U} \tilde{\mathbf{c}} \tag{12}$$

they become independent if \mathbf{U} is the eigenvector matrix of the co-variances of the $\tilde{\theta}_i$.

So after simulation of n-independent normal values y_i , the corresponding simulated values $\tilde{\theta}_i$ are

$$\hat{\theta}_i = \mathbf{U}^T \mathbf{y} \tag{13}$$

which allows for simulation of the directional propagation of the seismic intensity over the territory by generating independent random vectors y and transforming through Equation (13).

Finally the simulation of the process $\tilde{\theta}(\alpha)$ yields a simulated propagation of the seismic intensity during an earthquake over the territory.

A comparison between the real distribution of the seismic signal over the territory during the earthquake and the propagations simulated by means of this procedure has been performed for the historical earthquakes ($N_T=12$ in the selected area).

For each earthquake a series of simulation has been produced and some of these have very different forms; however, some of the samples produced for each shock appear

very similar to the real seismic propagation.

For example (Fig.8) by the comparison between the simulated seismic map of the Irpinia's earthquake on 1980.11.23 and the seismic map shown in the "Atlas of the Iseismic Maps for Italian Earthquakes" (edited by the Research National Council) [32], it is possible to notice that the simulation reproduces the fundamental characters of the signal's propagation: the principal direction from SE to NW and the form, roughly a triangle, with two preferential propagations to the northern direction.

On the other side, the randomness of the seismic propagation over the territory is a characteristic of the seismic propagation over the territory. Actually it can be often observed that the recorded data of real ground motions occurred in the same epicentral area with the same magnitude value show very different seismic maps. However the macroseismic character of the area is conserved so that, as example for the studied area, the general Apennines form is conserved.

Many other simulations can be developed for an expected epicenter of the macroseismic maps in Fig.2 and for a prefixed magnitude that reproduce the waves' propagation on the territory. So, as example, the comparison between the isosists recorded during the 1694.09.08 earthquake occurred in Irpinia-Basilicata (South of Italy) with an intensity of 10,5, and two possible simulations for the same epicenter are shown in Fig.9.

VI. CONCLUSION

In the present research a statistical methodology is developed for the evaluation of the seismic propagation of the signal over the territory. Starting from the classical models for the statistical analysis of the fundamental macroseismic parameters (i.e. the seismic risk analysis at the site of the Cornell model, the seismicity law of Gutenberg-Richter, the occurrence process of Poisson) a stochastic propagation field is built up.

The first step consists of the elaboration of the macro-scale maps of seismogenetic areas by building up the epicentral probability density function.

The advantage of the method is that the maps of the epicentral probabilistic distribution are obtained automatically by simply elaborating in a proper way the known seismic parameters of the historically occurred earthquakes, by assuming that the propagation law is expressed by an exponential random function of the intensity where the exponential coefficient includes the random character of the function.

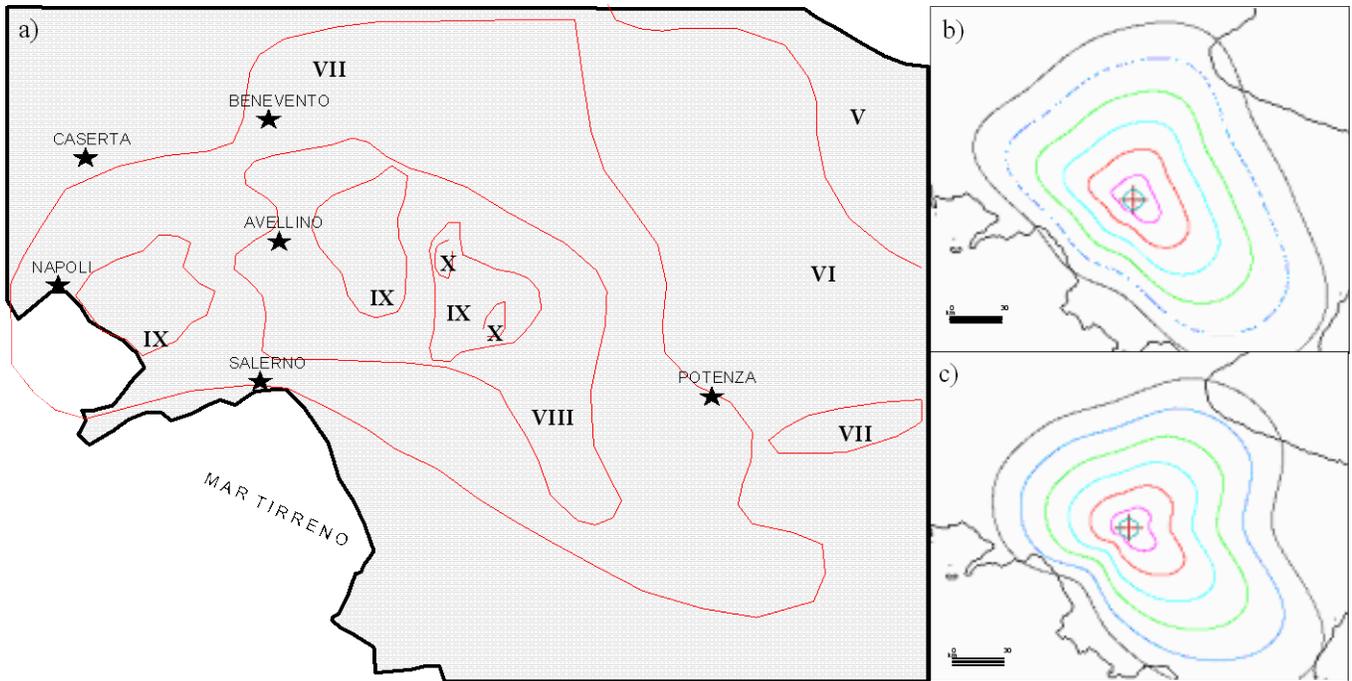


Fig.8. Comparison between: a) the recorded waves' propagation during the Irpinia earthquake of 1980.11.23, and b)-c) two simulated waves' propagation for an epicentral intensity of 10.

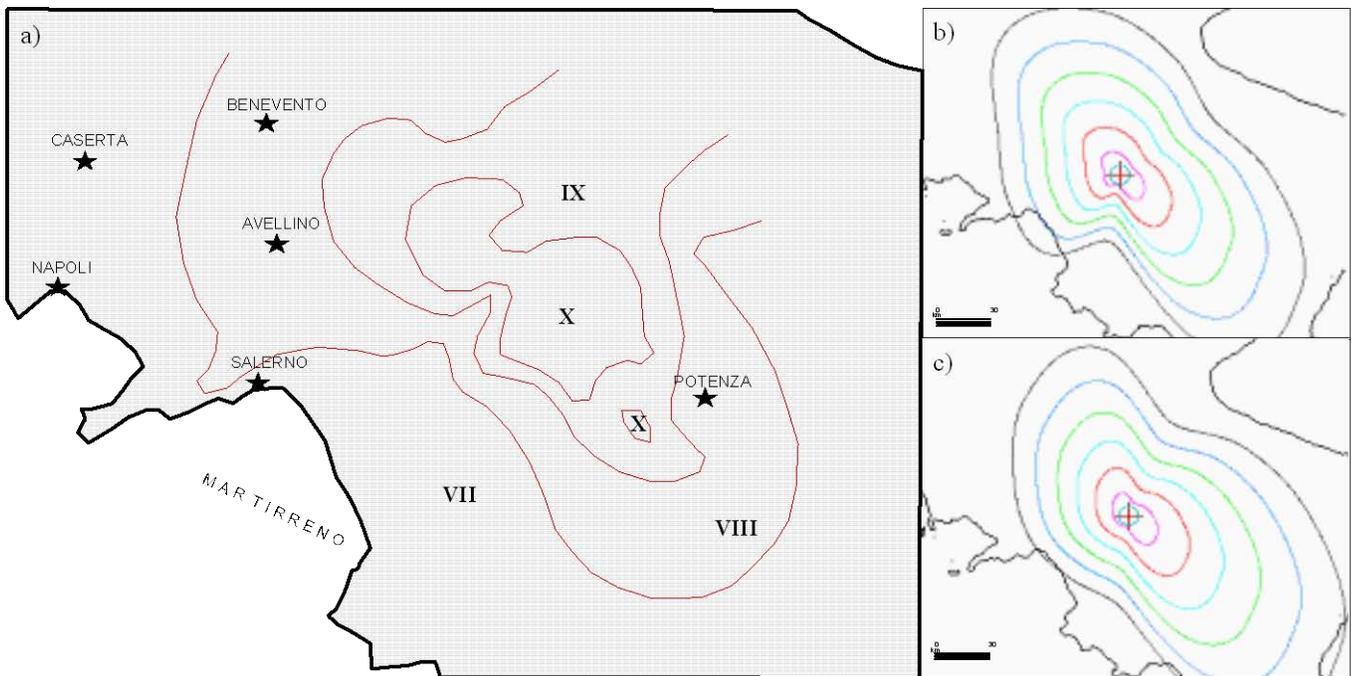


Fig.9. Comparison between: a) the recorded waves' propagation during the Irpinia earthquake of 1694.09.08, and b)-c) two simulated waves' propagation for an epicentral intensity of 10,5.

The results of the application of the methodology presented in the paper to a sample Italian area demonstrate a good agreement with this assumption.

In the subsequent step, the stochastic field of the seismic attenuation is built for some ground motions occurred in the sample area and the comparison between the simulated events and the real situations during same historical earthquakes shows that the procedure yields pretty satisfactory results, thus validating the proposed procedure, which can be recognized to be effective and reliable.

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