Ranking Accident Blackspots With Reference To Cost Of Accidents Using Hierarchical Bayesian Approach

Noorizam Daud, Kamarulzaman Ibrahim

Abstract— Road accident is an unfortunate event which is a matter of serious concern to the authority. A proactive measure taken in reducing the rate of accidents is to identify hazardous locations for treatment. In order to allocate resources wisely when treating accident locations, engineers usually rank accident locations based on the mean number of accidents observed over a period of time. Identification, ranking and selecting hazardous accident locations from a group under consideration is a fundamental goal for traffic safety researchers. The search of a better method to carry out such tasks is the main aim of this study in order to improve road safety in the country. The number of accident varies within and between locations, hence making Bayesian hierarchical model suitable to be applied when allowing for these two stages of variation. This study will illustrates the use of posterior mean to rank accident blackspots.

Keywords— Hierarchical Bayesian, Posterior mean, Rankings, Accident Blackspots

I. Introduction

Since 1990 Ministry of Science, Technology and Environment have been funding research programmes to improve the accident data collection and analysis system in Malaysia. The programmes also aim to encourage wider usage of the system to assist in the identification of accident blackspots prior to any effective treatment given in order to improve road safety in the country (Radin 1998). The current method used in the country to identify such hazardous location is not based on specific probabilistic approach. Since accidents are random and multi factor events, the use of probability and tools of statistics in such road safety research is more appropriate. Recently, Empirical Bayes methods have been used in road safety studies to identify dangerous locations arguing that adjusting historical data by statistical estimates yields improved predictability (Elvik 1997; Miaou 1994). Furthermore, the recent use of ranking procedures based on a hierarchical Bayes approach has been proposed in literature (Geurts 2005; MacNab 2003; Schlüter 1997) since this method can handle uncertainty and variability in accident data by producing a probabilistic ranking of the accident locations. This paper will highlight the use of Bayesian hierarchical approach to produce an alternative ranking method in identifying the hazardous accident locations. The hierarchical Bayesian method proposed by Schlüter et al. (1997) is reviewed and some adjustments has been made by including the fatal and serious injury accident categories and also the cost ratio of fatal accidents as compared to serious injury accidents.

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II. Data

The Royal Malaysian Police has classified accident into four types: fatal, serious injury, slight injury and damage to the vehicles or properties only (Baguley 1995). Due to the problem of misclassification for the type of injury accidents, only the accident data for the fatal and serious injury accidents are considered in this study. The analysis made to illustrate the propose ranking method are based on accident data collected over 3-year period from 1996 to 1998 for 30 locations. Since the details pertaining to cost of a particular accident may not be readily available, a sample of insurance claims for fatal and serious injury accidents are used for estimating the accident cost and the ratio between these insurance claims will be used as a scaling factor, treated as the nuisance factor in the Bayesian modelling.

III. Ratio of Cost of Fatal Relative to Serious Injury Accident

Assuming that the insurance claim for fatal accident (C_1) and the insurance claim for serious injury accident (C_2) are following Gamma distributions, denoted as

$C_1 \sim Gama(\gamma_1, \eta_1)$ and $C_2 \sim Gama(\gamma_2, \eta_2)$

Consider a new variable $A = \frac{C_1}{C_2}$. By using the method of transformation of variables, the conditional probability of a given $\gamma_1, \gamma_2, \eta_1, \eta_2$ could be written as

$$f(a \mid \gamma_1, \gamma_2, \eta_1, \eta_2) = \frac{\Gamma(\gamma_1 + \gamma_2)\, \eta_1^{\gamma_1}\ \eta_2^{\gamma_2}\ \ a^{\gamma_1 - 1}}{\Gamma(\gamma_1)\,\Gamma(\gamma_2)\ \ [a\eta_1 + \eta_2]^{\gamma_1 + \gamma_2}}\ ; a > 0 \tag{3.1}$$

Based on the maximum likelihood method, the estimates for the parameters $\gamma_1, \gamma_2, \eta_1, \eta_2$ are obtained where, $\hat{\gamma}_1 = 1.0951$, $\hat{\gamma}_2 = 1.1825$, $\hat{\eta}_1 = 12481.8508$ and $\hat{\eta}_2 = 704.1688$. These values are been substituted into (3.1), and by numerical method the median and mean values are found to be 1.31 and 8.6 respectively (Noorizam 2007).

IV THE HIERARCHICAL BAYESIAN MODEL

Consider two discrete random variables X_{ij} dan Y_{ij}, each representing the number of fatal accidents and the number of non-fatal accidents occurring at locations i =1,2,…, k in j = 1,2,…, t_i years. Since each location observed two accident categories, so random variables X_{ij} and Y_{ij} are assumed to have mean number of accident per year of λ_{1i} and λ_{2i} respectively. Since both X_{ij} and Y_{ij} satisfy the characteristics of a Poisson process, so it is reasonable to assume that both variables are following Poisson distribution with mean λ_{1i} and λ_{2i}, respectively.

Let the number of fatal accidents occurring in location i in the period of t_i year be denoted as $X_i = \sum_{j=1}^{t_i} X_{ij}$, and the number of serious injury accidents occurring at location i in t_i year be denoted as $Y_i = \sum_{j=1}^{t_i} Y_{ij}$. Hence, random variable X_i and Y_i respectively are assumed to be having Poisson distribution with mean number of accidents of $t_i\lambda_{1i}$ and $t_i\lambda_{2i}$ given as follows:

$$f(x_i \mid \lambda_{1i}) = \frac{(t_i\, \lambda_{1i})^{x_i}\ exp(-t_i\lambda_{1i})}{x_i!}, \quad x_i = 0,1,2....$$

and $$f(y_i \mid \lambda_{2i}) = \frac{(t_i\, \lambda_{2i})^{y_i}\ exp(-t_i\lambda_{2i})}{y_i!}, \quad y_i = 0,1,2,... \tag{4.1}$$

for i=1,2,…, k where $\lambda_{1i} > 0$ and $\lambda_{2i} > 0$.

In the following explanation to obtain the mean posterior, t will be excluded since it is regarded as a constant term.

Assuming that the uncertainties in the mean number of fatal and serious injury accidents are modeled as Gamma distributions which are also commonly known as conjugate priors can be given by

$$f(\lambda_{1i} \mid \alpha_1, \beta_1) = \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \lambda_{1i}^{\alpha_1 - 1}\ exp(-\beta_1\, \lambda_{1i}) \tag{4.2a}$$

where $\lambda_{1i} > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$,

and $$f(\lambda_{2i} \mid \alpha_2, \beta_2) = \frac{\beta_2^{\alpha_2}}{\Gamma(\alpha_2)} \lambda_{2i}^{\alpha_2 - 1}\ exp(-\beta_2\, \lambda_{2i}) \tag{4.2b}$$

where $\lambda_{2i} > 0$, $\alpha_2 > 0$ and $\beta_2 > 0$.

Based on the elicitation of expert opinions and referring to some previous studies (see Geurts et. al 2005; Al-Masaeid et. al 1999; Downing 1997 for example), it reveals that expected cost for fatal accident is more than the expected cost of serious injury accident. Since the true cost of each type of accident is not known, we consider that the ratio of claims for fatal accident and serious injury accident as obtained in equation (3.1) could be suitably be used as a scaling factor for scaling up the expected number of fatal accidents, thus adjusting the hazardous level of each location.

The joint posterior distribution of λ_{1i}, λ_{2i} and a conditional on X_i, Y_i could be obtained through the Bayes theorem mechanism given as

$$f(\lambda_{1i}, \lambda_{2i}, a \mid X_i, Y_i) \propto f(X_i, Y_i \mid a, \lambda_{1i}, \lambda_{2i})\ f(a, \lambda_{1i}, \lambda_{2i})\,. \tag{4.3}$$

Since the parameters λ_{1i}, λ_{2i}, and a are assumed independent, then (4.3) could be simplified as follows

$$f(\lambda_{1i},\lambda_{2i},a \mid X_i,Y_i) \propto$$
$$f(X_i \mid a,\lambda_{1i})\, f(Y_i \mid \lambda_{2i})\, f(\lambda_{1i} \mid \alpha_1,\beta_1)$$
$$.\ f(\lambda_{2i} \mid \alpha_2,\beta_2)\, f(a). \tag{4.4}$$

Alternatively, equation (4.4) could be rewritten as

$$f(\lambda_{1i},\lambda_{2i},a \mid X_i,Y_i)$$
$$\propto \frac{(a\lambda_{1i})^{x_i}\ e^{-a\lambda_{1i}}}{x_i!}\frac{(\lambda_{2i})^{y_i}\ e^{-\lambda_{2i}}}{y_i!}\frac{\beta_1^{\alpha_1-1}}{\Gamma(\alpha_1)}$$
$$.\ \lambda_{1i}^{\alpha_1-1}e^{-\beta_1\lambda_{1i}}\frac{\beta_2^{\alpha_2-1}}{\Gamma(\alpha_2)}\lambda_{2i}^{\alpha_2-1}e^{-\beta_2\lambda_{2i}}\ f(a). \tag{4.5}$$

Let $\lambda_i{}' = a\,\lambda_{1i} + \lambda_{2i}$ represents the prioritized score to be used in ranking the accident locations. Hence, *a* will be regarded as a nuisance factor and it should be integrated out.

The posterior mean of $\lambda_i{}'$ which is the required prioritized score could be obtained as

$$E(\lambda_i' \mid X_i,Y_i)$$
$$= E(a\lambda_{1i}+\lambda_{2i} \mid X_i,Y_i)$$
$$\propto \int_{\lambda_{2i}}\int_{\lambda_{1i}}\int_{a}(a\lambda_{1i}+\lambda_{2i})f(\lambda_{1i},\lambda_{2i},a \mid X_i,Y_i)\,da\,d\lambda_{1i}\,d\lambda_{2i}$$
$$= \int_{a}\int_{\lambda_{1i}} a\lambda_{1i}\Big(\int_{\lambda_{2i}} f(\lambda_{1i},\lambda_{2i},a \mid X_i,Y)\,\partial\lambda_{2i}\Big)\,d\lambda_{1i}\,da)$$
$$+ \int_{\lambda_{2i}}\lambda_{2i}\Big(\int_{\lambda_{1i}}\int_{a} f(\lambda_{1i},\lambda_{2i},a \mid X_i,Y_i)\,da\,d\lambda_{1i}d\lambda_{2i}$$

$$= \Big(\int_{a} a\int_{\lambda_{1i}} f(\lambda_{1i},a \mid X_i,Y_i)\,d\lambda_{1i}\,da\Big)$$
$$.\ \Big(\int_{\lambda_{1i}}\lambda_{1i}\int_{a} f(\lambda_{1i},a \mid X_i,Y_i)\,da\ \ d\lambda_{1i}\Big)$$
$$+ \int_{\lambda_{2i}}\lambda_{2i}\ f(\lambda_{2i} \mid X_i,Y_i)\,d\lambda_{2i}$$
$$= \Big(\int_{a} a\ f(a \mid X_i,Y_i)\,da\Big)\Big(\int_{\lambda_{1i}}\lambda_{1i}f(\lambda_{1i} \mid X_i,Y)\,d\lambda_{1i}\Big)$$
$$+ \int_{\lambda_{2i}}\lambda_{2i}\ f(\lambda_{2i} \mid X_i,Y_i)\,d\lambda_{2i}$$
$$= E(a \mid X_i,Y_i)\,E(\lambda_{1i} \mid X_i,Y_i) + E(\lambda_{2i} \mid X_i,Y_i)\,. \tag{4.6}$$

On the other hand, if the factor a is regarded as a constant, then (4.6) will be further reduced as follows

$$E(\lambda_i' \mid X_i,Y_i)$$
$$\propto \int_{\lambda_{2i}}\int_{\lambda_{1i}}(a\lambda_{1i}+\lambda_{2i})f(\lambda_{1i},\lambda_{2i} \mid X_i,Y_i)\,d\lambda_{1i}\ d\lambda_{2i}$$
$$= aE(\lambda_{1i} \mid X_i,Y_i) + E(\lambda_{2i} \mid X_i,Y_i). \tag{4.7}$$

For comparison on the sensitivity of the results to the choice of prior distributions, four different prior distributions are considered. The prior distributions are:

(i) Prior 1: Ratio of two Gamma distributions (see equation 3.1)

(ii) Prior 2: Improper Prior ($f(a) = \frac{1}{a} \quad , a > 0$)

(iii) Prior 3: Median value (1.31)

(iv) Prior 4: Mean value (8.6).

V DISCUSSION OF RESULTS

From Table 5.1 and Table 5.2, it appears that the results slightly change when different prior distributions are used. As expected, the results based on the choice of improper prior, as given in Table 5.4, are similar to those obtained based on not allowing for factor *a* in the model as shown in Table 5.7. When factor *a* is considered as a constant, with allowance of the median value of *A* =1.31 and mean of *A*=8.6 respectively, it is found that the uncertainty of the estimated posterior mean are much larger in the case when the later prior are used (refer Table 5.5 and Table 5.6). Thus, when the two measures of *A* are compared, median is a better choice. On the average, it is found that the posterior standard deviation for the estimated posterior mean based on prior 1 are smaller compared to when other priors are used. (see Table 5.3, Table 5.4, Table 5.5, Table 5.6 and Table 5.7). We believe that allowance for cost of accident is a prudent way of ranking of blackspots.

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Table 5.1 Posterior Mean $E(\lambda' \mid X_i, Y_i, a)$ for 30 locations using several priors for factor *a*

			Prior for Factor *a*				Without factor *a*
Location	**X**	**Y**	**Prior 1**	**Prior 2**	**Prior 3**	**Prior 4**	
L[1]	8	12	4.808	5.383	5.679	7.491	5.367
L[2]	4	19	6.135	6.625	6.837	8.107	6.598
L[3]	6	17	5.883	6.379	6.621	8.193	6.354
L[4]	11	10	4.579	5.259	5.627	7.819	5.262
L[5]	3	16	5.307	5.766	5.968	7.066	5.766
L[6]	3	17	5.555	5.996	6.195	7.334	6.021
L[7]	8	13	5.09	5.642	5.927	7.761	5.639
L[8]	4	19	6.164	6.638	6.818	8.119	6.62
L[9]	9	12	4.907	5.521	5.807	7.799	5.513
L[10]	8	10	4.315	4.895	5.18	6.994	4.904
L[11]	3	15	5.082	5.522	5.698	6.834	5.518
L[12]	3	17	5.531	5.999	6.222	7.36	6.005
L[13]	3	19	6.041	6.515	6.656	7.831	6.493
L[14]	4	12	4.442	4.9	5.122	6.401	4.9
L[15]	8	10	4.33	4.916	5.192	6.987	4.913
L[16]	5	18	6.001	6.511	6.767	8.131	6.494
L[17]	2	19	5.899	6.376	6.528	7.549	6.388
L[18]	3	14	4.809	5.246	5.487	6.642	5.271
L[19]	2	18	5.656	6.125	6.298	7.341	6.145
L[20]	2	18	5.686	6.137	6.323	7.29	6.131
L[21]	5	9	3.801	4.296	4.546	5.973	4.312
L[22]	0	20	5.827	6.374	6.496	7.261	6.381
L[23]	4	12	4.451	4.919	5.111	6.415	4.917
L[24]	2	16	5.188	5.633	5.822	6.819	5.648
L[25]	2	17	5.421	5.869	6.068	7.08	5.881
L[26]	3	13	4.566	5.022	5.235	6.359	5.052
L[27]	8	5	3.095	3.681	3.973	5.777	3.684
L[28]	1	17	5.268	5.768	5.924	6.794	5.777
L[29]	3	15	5.057	5.517	5.72	6.824	5.518
L[30]	4	13	4.808	5.152	5.368	7.491	5.177

Prior 1: Ratio of two Gamma distributions (see equation 3.1)

Prior 2: Improper Prior ($f(a) = \frac{1}{a}$, $a > 0$)

Prior 3: Median value (a = 1.31)

Prior 4: Mean value (a = 8.6)

Table 5.2 Ranking for 30 accident locations based on the posterior mean using different factor *a*

	Prior for factor *a*				Without *a*
	Prior 1	**Prior 2**	**Prior 3**	**Prior 4**	
Ranking			**Location**		
Rank 1	L[8]	L[8]	L[2]	L[3]	L[8]
Rank 2	L[2]	L[2]	L[8]	L[16]	L[2]
Rank 3	L[13]	L[16]	L[16]	L[8]	L[16]
Rank 4	L[16]	L[13]	L[13]	L[2]	L[13]
Rank 5	L[17]	L[17]	L[3]	L[13]	L[17]
Rank 6	L[3]	L[22]	L[17]	L[4]	L[22]
Rank 7	L[22]	L[3]	L[22]	L[9]	L[3]
Rank 8	L[20]	L[19]	L[20]	L[7]	L[19]
Rank 9	L[19]	L[20]	L[19]	L[17]	L[20]
Rank 10	L[6]	L[6]	L[12]	L[1]	L[6]
Rank 11	L[12]	L[12]	L[6]	L[30]	L[12]
Rank 12	L[25]	L[25]	L[25]	L[12]	L[25]
Rank 13	L[5]	L[28]	L[5]	L[19]	L[28]
Rank 14	L[28]	L[5]	L[7]	L[6]	L[5]
Rank 15	L[24]	L[24]	L[28]	L[20]	L[24]
Rank 16	L[7]	L[7]	L[24]	L[22]	L[7]
Rank 17	L[11]	L[11]	L[9]	L[25]	L[11]
Rank 18	L[29]	L[29]	L[29]	L[5]	L[29]
Rank 19	L[9]	L[9]	L[11]	L[10]	L[9]
Rank 20	L[18]	L[1]	L[1]	L[15]	L[1]
Rank 21	L[1]	L[18]	L[4]	L[11]	L[18]
Rank 22	L[30]	L[4]	L[18]	L[29]	L[4]
Rank 23	L[4]	L[30]	L[30]	L[24]	L[30]
Rank 24	L[26]	L[26]	L[26]	L[28]	L[26]
Rank 25	L[23]	L[23]	L[15]	L[18]	L[23]
Rank 26	L[14]	L[15]	L[10]	L[23]	L[15]
Rank 27	L[15]	L[10]	L[14]	L[14]	L[10]
Rank 28	L[10]	L[14]	L[23]	L[26]	L[14]
Rank 29	L[21]	L[21]	L[21]	L[21]	L[21]
Rank 30	L[27]	L[27]	L[27]	L[27]	L[27]

Table 5.3 Posterior mean $E(\lambda' \mid X_i, Y_i, a)$ and other related information for 30 locations using prior 1 for factor *a*

Location	Mean Posterior	Standard deviation	MC Error	2.50%	median	97.50%
L[1]	4.808	1.018	0.01029	3.076	4.707	7.087
L[2]	6.135	1.196	0.01537	4.047	6.041	8.751
L[3]	5.883	1.148	0.01555	3.869	5.78	8.361
L[4]	4.579	0.9431	0.01261	2.953	4.515	6.589
L[5]	5.307	1.108	0.01624	3.364	5.239	7.67
L[6]	5.555	1.127	0.01441	3.583	5.493	8.033
L[7]	5.09	1.024	0.01382	3.278	5.036	7.298
L[8]	6.164	1.208	0.01478	4.051	6.071	8.733
L[9]	4.907	1.012	0.01407	3.146	4.844	7.03
L[10]	4.315	0.9609	0.01242	2.669	4.23	6.397
L[11]	5.082	1.087	0.01434	3.204	4.997	7.396
L[12]	5.531	1.122	0.01513	3.598	5.445	7.98
L[13]	6.041	1.18	0.01406	3.972	5.96	8.542
L[14]	4.442	0.9889	0.01206	2.703	4.38	6.564
L[15]	4.33	0.9336	0.01078	2.731	4.249	6.383
L[16]	6.001	1.178	0.01432	3.963	5.91	8.599
L[17]	5.899	1.193	0.01517	3.807	5.799	8.47
L[18]	4.809	1.027	0.01452	3.02	4.72	7.027
L[19]	5.656	1.138	0.01623	3.641	5.576	8.106
L[20]	5.686	1.157	0.01544	3.646	5.614	8.183
L[21]	3.801	0.9069	0.01255	2.261	3.721	5.756
L[22]	5.827	1.198	0.01404	3.722	5.741	8.39
L[23]	4.451	1.02	0.01381	2.672	4.366	6.669
L[24]	5.188	1.109	0.01487	3.26	5.11	7.654
L[25]	5.421	1.138	0.01602	3.457	5.327	7.854
L[26]	4.566	1.031	0.0123	2.801	4.476	6.847
L[27]	3.095	0.7742	0.01153	1.819	3.016	4.79
L[28]	5.268	1.113	0.01635	3.363	5.191	7.682
L[29]	5.057	1.074	0.01292	3.183	4.972	7.419
L[30]	4.685	1.009	0.01222	2.956	4.61	6.914

Jadual 5.4 Posterior mean $E(\lambda' \mid X_i, Y_i, a)$ and other related information for 30 locations using prior 2 for factor *a*

Location	Mean Posterior	Standard deviation	MC Error	2.50%	median	97.50%
L[1]	5.383	1.055	0.01054	3.56	5.306	7.659
L[2]	6.625	1.212	0.01171	4.45	6.555	9.2
L[3]	6.379	1.177	0.01113	4.335	6.298	8.888
L[4]	5.259	1.011	0.009002	3.469	5.194	7.393
L[5]	5.766	1.126	0.01095	3.786	5.695	8.154
L[6]	5.996	1.156	0.01122	3.994	5.929	8.464
L[7]	5.642	1.087	0.01122	3.714	5.573	8.014
L[8]	6.638	1.22	0.01098	4.48	6.568	9.259
L[9]	5.521	1.059	0.01047	3.664	5.444	7.817
L[10]	4.895	1.005	0.01059	3.122	4.816	7.062
L[11]	5.522	1.108	0.009834	3.603	5.438	7.879
L[12]	5.999	1.136	0.01249	3.969	5.946	8.362
L[13]	6.515	1.203	0.0107	4.374	6.449	9.067
L[14]	4.9	1.027	0.01084	3.081	4.837	7.12
L[15]	4.916	0.9927	0.01126	3.195	4.846	7.051
L[16]	6.511	1.188	0.0117	4.396	6.439	9.039
L[17]	6.376	1.201	0.01132	4.245	6.291	8.977
L[18]	5.246	1.079	0.01147	3.344	5.165	7.605
L[19]	6.125	1.18	0.01253	4.074	6.037	8.704
L[20]	6.137	1.191	0.01289	4.029	6.051	8.683
L[21]	4.296	0.9425	0.008829	2.687	4.222	6.366
L[22]	6.374	1.225	0.01228	4.175	6.286	8.974
L[23]	4.919	1.028	0.008886	3.112	4.839	7.171
L[24]	5.633	1.125	0.01085	3.638	5.557	8.035
L[25]	5.869	1.166	0.0122	3.814	5.791	8.35
L[26]	5.022	1.045	0.01094	3.183	4.943	7.258
L[27]	3.681	0.8393	0.008091	2.251	3.602	5.507
L[28]	5.768	1.147	0.01051	3.736	5.698	8.208
L[29]	5.517	1.107	0.0119	3.558	5.448	7.908
L[30]	5.152	1.045	0.01059	3.305	5.08	7.4

Table 5.5 Posterior mean $E(\lambda' \mid X_i, Y_i, a)$ and other related information for 30 locations using prior 3 for factor *a*

Location	Mean Posterior	Standard deviation	MC Error	2.50%	median	97.50%
L[1]	5.679	1.07	0.01476	3.813	5.594	7.977
L[2]	6.837	1.247	0.01429	4.658	6.735	9.58
L[3]	6.621	1.225	0.01452	4.424	6.559	9.223
L[4]	5.627	1.069	0.01609	3.733	5.546	7.904
L[5]	5.968	1.148	0.01618	3.937	5.882	8.37
L[6]	6.195	1.169	0.01709	4.065	6.126	8.691
L[7]	5.927	1.111	0.01678	3.948	5.864	8.296
L[8]	6.818	1.238	0.01799	4.653	6.714	9.444
L[9]	5.807	1.095	0.01655	3.826	5.747	8.164
L[10]	5.18	1.044	0.01505	3.345	5.129	7.391
L[11]	5.698	1.144	0.01606	3.683	5.623	8.155
L[12]	6.222	1.199	0.01874	4.139	6.144	8.864
L[13]	6.656	1.242	0.02242	4.433	6.582	9.332
L[14]	5.122	1.046	0.01425	3.275	5.063	7.394
L[15]	5.192	1.047	0.01428	3.369	5.107	7.495
L[16]	6.767	1.218	0.01645	4.565	6.68	9.354
L[17]	6.528	1.202	0.01653	4.394	6.457	9.077
L[18]	5.487	1.127	0.01637	3.506	5.419	7.889
L[19]	6.298	1.198	0.01858	4.187	6.221	8.914
L[20]	6.323	1.203	0.01632	4.205	6.249	8.896
L[21]	4.546	0.9781	0.01399	2.89	4.472	6.599
L[22]	6.496	1.231	0.01856	4.354	6.416	9.173
L[23]	5.111	1.054	0.0152	3.281	5.017	7.364
L[24]	5.822	1.153	0.0167	3.814	5.755	8.318
L[25]	6.068	1.155	0.01517	4.044	5.98	8.539
L[26]	5.235	1.078	0.01254	3.376	5.147	7.554
L[27]	3.973	0.8903	0.01107	2.441	3.908	5.938
L[28]	5.924	1.168	0.0167	3.876	5.872	8.378
L[29]	5.72	1.112	0.01696	3.766	5.647	8.105
L[30]	5.368	1.089	0.01618	3.435	5.311	7.768

Table 5.6 Posterior mean $E(\lambda' \mid X_i, Y_i, a)$ and other related information for 30 locations using prior 4 for factor *a*

Location	Mean Posterior	Standard deviation	MC Error	2.50%	median	97.50%
L[1]	7.491	1.383	0.01892	5.062	7.387	10.41
L[2]	8.107	1.435	0.02117	5.576	7.991	11.16
L[3]	8.193	1.463	0.0246	5.628	8.091	11.34
L[4]	7.819	1.45	0.01804	5.205	7.737	10.93
L[5]	7.066	1.344	0.0205	4.758	6.965	9.911
L[6]	7.334	1.345	0.0185	4.936	7.221	10.22
L[7]	7.761	1.414	0.02055	5.207	7.677	10.78
L[8]	8.119	1.422	0.01888	5.505	8.033	11.09
L[9]	7.799	1.428	0.02035	5.164	7.74	10.75
L[10]	6.994	1.35	0.02111	4.548	6.916	9.881
L[11]	6.834	1.352	0.02006	4.503	6.737	9.82
L[12]	7.36	1.364	0.01843	4.976	7.278	10.34
L[13]	7.831	1.393	0.02061	5.295	7.749	10.79
L[14]	6.401	1.303	0.01746	4.158	6.341	9.18
L[15]	6.987	1.324	0.01614	4.643	6.887	9.841
L[16]	8.131	1.433	0.02271	5.544	8.08	11.17
L[17]	7.549	1.376	0.01836	5.091	7.47	10.44
L[18]	6.642	1.303	0.01761	4.34	6.563	9.454
L[19]	7.341	1.34	0.01939	4.917	7.284	10.15
L[20]	7.29	1.359	0.02019	4.896	7.218	10.14
L[21]	5.973	1.251	0.01818	3.823	5.897	8.586
L[22]	7.261	1.341	0.01871	4.823	7.193	10.1
L[23]	6.415	1.304	0.01541	4.119	6.316	9.286
L[24]	6.819	1.338	0.01982	4.487	6.737	9.667
L[25]	7.08	1.343	0.02012	4.714	6.98	9.895
L[26]	6.359	1.275	0.01661	4.115	6.284	9.102
L[27]	5.777	1.247	0.01885	3.593	5.7	8.496
L[28]	6.794	1.318	0.01827	4.474	6.701	9.623
L[29]	6.824	1.332	0.01847	4.454	6.739	9.605
L[30]	6.644	1.296	0.01974	4.382	6.541	9.387

Table 5.7 Posterior mean $E(\lambda' \mid X_i, Y_i, a)$ and other related information for 30 locations without factor *a*

Location	Mean Posterior	Standard deviation	MC Error	2.50%	median	97.50%
L[1]	5.367	1.065	0.01175	3.531	5.278	7.665
L[2]	6.598	1.214	0.01151	4.453	6.527	9.17
L[3]	6.354	1.168	0.01137	4.271	6.286	8.839
L[4]	5.262	1.02	0.009984	3.464	5.196	7.479
L[5]	5.766	1.124	0.01098	3.806	5.691	8.191
L[6]	6.021	1.167	0.01169	3.995	5.945	8.527
L[7]	5.639	1.096	0.01193	3.736	5.559	8.019
L[8]	6.62	1.213	0.01154	4.459	6.544	9.184
L[9]	5.513	1.053	0.01082	3.644	5.44	7.75
L[10]	4.904	0.9993	0.009363	3.147	4.843	7.025
L[11]	5.518	1.112	0.01223	3.588	5.432	7.927
L[12]	6.005	1.164	0.01267	3.963	5.903	8.54
L[13]	6.493	1.213	0.01176	4.33	6.406	9.072
L[14]	4.9	1.025	0.01107	3.094	4.831	7.132
L[15]	4.913	1.006	0.01048	3.139	4.844	7.084
L[16]	6.494	1.202	0.01275	4.379	6.42	9.033
L[17]	6.388	1.2	0.01262	4.29	6.308	8.993
L[18]	5.271	1.073	0.01143	3.404	5.196	7.607
L[19]	6.145	1.179	0.01102	4.057	6.06	8.692
L[20]	6.131	1.182	0.01202	4.059	6.065	8.663
L[21]	4.312	0.9612	0.008964	2.684	4.238	6.44
L[22]	6.381	1.208	0.01083	4.228	6.312	8.939
L[23]	4.917	1.026	0.008804	3.152	4.838	7.149
L[24]	5.648	1.13	0.01162	3.69	5.572	8.091
L[25]	5.881	1.151	0.01068	3.832	5.813	8.356
L[26]	5.052	1.057	0.009331	3.187	4.979	7.366
L[27]	3.684	0.8322	0.008346	2.217	3.612	5.471
L[28]	5.777	1.158	0.01187	3.725	5.691	8.284
L[29]	5.518	1.1	0.01015	3.615	5.438	7.864
L[30]	5.177	1.05	0.01217	3.315	5.108	7.428