OWA – type Fuzzy Aggregations in a Decision Making Regarding the Selection of Investments

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Abstract **—** The Ordered Weighted Averaging (OWA) operator was introduced by R.R. Yager (Yager, 1988) to provide a method for aggregating inputs that lie between the Max and Min operators. In this article a new generalization of the OWA aggregation operator - AsFPOWA is presented in the environment of possibility uncertainty. For the illustration of the applicability of the new aggregation operator - AsFPOWA an example of the fuzzy decision making regarding optimal selection of investment is considered. Several variants of the new aggregation operator are used for the comparing of decision making results.

Keywords **—** *Decision Making System, Expert Evaluation, Fuzzy Number, Information Measures of an Aggregation Operator, OWA Aggregation operator, Possibility Uncertainty, Selection of investments.*

I. Introduction

It is well recognized that decision making systems (DMS) and technologies have been playing an important role in improving almost every aspect of human society. In this type of problem the decision making person (DMP) has a collection $D = \{d_1, d_2, ..., d_n\}$ of possible uncertain alternatives from which he/she must select one or perform ranking of decisions by some expert's preference relation values. Associated with this problem as a result is a variable of characteristics, activities, symptoms and others, acting on the decision procedure. This variable normally called the state of nature, which affects the payoff, utilities, valuations, etc of DMP's preferences or subjective activities. This variable is assumed to take its values from some set $S = \{s_1, s_2, ..., s_m\}$. As a result the DMP knows that if he/she selects d_i and the state of nature assumes the value s_j then his/her payoff (valuation, utility and so on) is a_{ij}. The objective of the decision is to select the "best" alternative and get the biggest payoff. But in DMS the selection procedure becomes more difficult. In this case each alternative can be seen as corresponding to a row vector of possible payoffs. To make a choice the DMP must compare these vectors, a problem which generally doesn't lead to a compelling solution. Assume d_i and d_k are two alternatives such that for all $j, j = 1, 2, ..., m$ $a_{ij} \geq a_{kj}$. In this case there is no reason to select d_i. In this situation we shall say d_i dominates d_k $(d_i \succeq d_k)$. Furthermore if there exists one alternative (optimal decision) that dominates all the alternatives then it will be *Pareto optimal solution*. Faced with the general difficulty of comparing vector payoffs we must provide some means of comparing these vectors. Our focus in this work is on the construction of aggregation operator F that can take a collection of m values and convert it into a single value, $F : R^m \Rightarrow R^1$.

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II. Preliminary Concepts

In [14] R.R. Yager introduced a class of mean aggregation operators called Ordered Weighted Averaging (OWA) operator.

Definition 1 ([14]): An OWA operator of dimension m is mapping $OWA : R^m \Rightarrow R^1$ that has an associated weighting vector W of dimension m with $w_j \in [0;1]$ and $\sum_{j=1}^{m} w_j = 1$, such that

$$OWA(a_1,....,a_m) = \sum_{j=1}^{m} w_j b_j \ , \quad (1)$$

where b_j is the j-th largest of the $\{a_i\}, i = 1,2,...,m$.

The fuzzy numbers (FN) have been studied by many authors ([3] and others). It can be represented in a more complete way as an imprecision variable of the incomplete information because it can consider the maximum and minimum and the possibility that the interval values may occur.

Definition 2 ([4]): $\tilde{a}(t): R^1 \to [0;1]$ is called the FN which can be considered as a generalization of the interval number:

$$\tilde{a}(t) = \begin{cases} 1 & if \ \ t \in [a_2'; a_2''] \\ \dfrac{t - a_1}{a_2' - a_1} & if \ \ t \in [a_1, a_2'] \\ \dfrac{a_3 - t}{a_3 - a_2''} & if \ \ t \in [a_2'', a_3] \\ 0 & otherwise \end{cases}, \quad (2)$$

where $a_1 \le a_2' \le a_2'' \le a_3 \in R^1$.

In the following, we are going to review the triangular FN (TFN) ([4]) arithmetic operation as follows in (2) ($a_2' = a_2''$). Let $\tilde{a}$ and $\tilde{b}$ be two TFNs, where $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$. Then

1: $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

2: $\tilde{a} - \tilde{b} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

3: $\tilde{a} \times k = (ka_1, ka_2, ka_3), \ k > 0$

4: $\tilde{a}^k = (a_1^k, a_2^k, a_3^k), \ k > 0, a_i > 0$

5: $\tilde{a} \cdot \tilde{b} = (a_1 b_1, a_2 b_2, a_3 b_3), a_i > 0, b_i > 0$

6: $\tilde{b}^{-1} = \left\{\dfrac{1}{b_3}, \dfrac{1}{b_2}, \dfrac{1}{b_1}\right\}, \ b_i > 0$

7: $\tilde{a} > \tilde{b}$ *if* $a_2 > b_2$ *and if* $a_2 = b_2$ *then* $\tilde{a} > \tilde{b}$ *if* $\dfrac{a_1 + b_3}{2} > \dfrac{b_1 + b_3}{2}$ *otherwise* $\tilde{a} = \tilde{b}$.

The set of all TFNs is denoted by Ψ and positive TFNs ($a_i > 0$) by Ψ^+. Note that other operations and ranking methods could be studied ([1-5] and others).

Now we consider some extensions of the OWA operator, mainly developed by J.M. Merigo ([4]), because in this paper we are concerned with extensions of Merigo's aggregation operators constructed on the basis of the OWA operator.

Definition 3 ([4]): Let Ψ be the set of TFNs. A fuzzy OWA operator - FOWA of dimension m is a mapping $FOWA: \Psi^m \Rightarrow \Psi$ that has an associated weighting vector W of dimension m with $w_j \in [0,1]$, $\sum_{j=1}^{m} w_j = 1$ and

$$FOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m) = \sum_{j=1}^{m} w_j \tilde{b}_j \ , \quad (3)$$

where $\tilde{b}_j$ is the j-th largest of the $\{\tilde{a}_i\}_{i=1}^m$ and $a_i \in \Psi$, $i = 1,2,...,m$.

The FOWA operator is an extension of the OWA operator that uses imprecision information in the arguments represented in the form of TFNs. The reason for using this aggregation operator is that sometimes the available information presented by the DMP and formalized in payoffs (valuations, utilities and others) can't be assessed with exact numbers and it is necessary to use other techniques such as TFNs. So, in this aggregation incomplete information is presented by imprecision variable of experts reflections and formalized in TFNs. Sometimes the available information presented by the DMP (or expert) also has an uncertain character, which is presented by the probability distribution on the states of nature consequents on the payoffs of the DMP.

The fuzzy probability aggregations based on the OWA operator was constructed by J. M. Merigo. One of the variants we present here:

Definition 4 ([4]): A probabilistic OWA operator - POWA of dimension m is a mapping $POWA: R^m \Rightarrow R^1$ that has an associated weighting vector W of dimension m such that $w_j \in [0,1]$ and $\sum_{j=1}^{m} w_j = 1$ according to the following formula:

$$POWA(a_1, a_2, ..., a_m) = \sum_{j=1}^{m} \hat{p}_j b_j \ , \quad (4)$$

where b_j is the *j*-th largest of the $\{a_i\}, i = 1,2,...,m$; each argument a_i has an associated probability p_i with $\sum_{i=1}^{m} p_i = 1$, $0 \le p_i \le 1$, $\hat{p}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0,1]$ and p_j is the probability p_i ordered according to b_j, that is according to the *j*-th largest of the a_i.

Note that if $\beta = 0$, we get the usual probabilistic mean aggregation (mathematical expectation - E_p with respect to probability distribution $\{p_i\}_{i=1}^m$), and if $\beta = 1$, we get the OWA operator. Equivalent representation of (4) may be defined as:

$$POWA\ (a_1, a_2, ..., a_m) = $$
$$= \beta \sum_{j=1}^{m} w_j b_j + (1-\beta) \sum_{i=1}^{m} p_i a_i = \quad (5)$$
$$= \beta \cdot OWA\ (a_1, a_2, ..., a_m) + (1-\beta) \cdot E_p\ (a_1, a_2, ..., a_m).$$

We often use fuzzy probabilistic information in the decision making systems and consequently in their aggregation operators. As well-known many fuzzy probabilistic aggregations have been researched in OWA and other operators [4, 6-14] and other authors. In the following we present one of them introduced in [4]:

Definition 5 ([4]): Let Ψ be the set of TFNs. A fuzzy probabilistic OWA operator - FPOWA of dimension m is a mapping $FPOWA: \Psi^m \Rightarrow \Psi$ that associated a weighting vector

W of dimension m such that $w_j \in [0,1]$, $\sum_{j=1}^{m} w_j = 1$, according to the following formula:

$$FPOWA\ (\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m) = \sum_{j=1}^{m} \hat{p}_j \tilde{b}_j\ , \qquad (6)$$

where $\tilde{b}_j$ is the j-th largest of the $\{\tilde{a}_i\}_{i=1}^{m}$ are TFNs and each one has an associated probability $p_i \equiv P(\tilde{a} = \tilde{a}_i)$, with $\sum_{j=1}^{m} p_j = 1,\ 0 \le p_j \le 1,\ \hat{p}_j = \beta w_j + (1-\beta) p'_j,\ \beta \in [0,1]$ and p'_j is the probability ordered according to $\tilde{b}_j \left(p'_j = P(\tilde{a} = \tilde{b}_j) \right)$ that is according to the j-th largest of the $\{\tilde{a}_i\}_{i=1}^{m}$.

Analogously to (5) we present the equivalent form of the FPOWA operator as a weighted sum of the OWA operator and the mathematical expectation - E_p :

$$FPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m) = $$
$$= \beta \sum_{j=1}^{m} w_j \tilde{b}_j + (1-\beta) \sum_{i=1}^{m} p_i \tilde{a}_i = \qquad (7)$$
$$= \beta \cdot OWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m) + (1-\beta) \cdot E_p(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m)\ .$$

III. Possibilistic Aggregations in The FPOWA Operator

It is important that in the aggregation operators POWA and FPOWA the both nature of incomplete information: 1. an uncertain measure (probability distribution $\{p_i\}$) and 2. An imprecision variable (random variable (a) or fuzzy variable ($\tilde{a}$)) are condensed in the outcome values, which get us more credibility for use of these aggregation operators in applications.

In this paper we define new generalization of the FPOWA operator where more general measure of uncertainty – Possibility measure [4] is used instead of probability measure in the role of uncertainty measure. So, we consider possibilistic aggregations based on the OWA operator. Therefore we introduce the definition of a possibility measure:

Definition 6 ([4]): A possibility measure - Pos on 2^S can be uniquely determined by its possibility distribution function $\pi : S \to [0,1]$ $\exists s_0 \in S, \pi(s_0) = 1$, via the formula:

$$\forall A \in 2^S \quad Pos(A) = \max_{s \in A} \pi(s). \qquad (8)$$

Let S_m be the set of all permutations of the set $\{1,2,..,m\}$. Let $\{P_\sigma\}_{\sigma \in S_m}$ be the associated probabilities class of a possibility measure - Pos. Then, we have the following connections between $\{\pi(s)\}_{s \in S}$ and $\{P_\sigma\}_{\sigma \in S_m} : \forall \sigma \in S_m$,

$$P_\sigma(s_{\sigma(i)}) =$$
$$= Pos(s_{\sigma(1)}, s_{\sigma(2)}, ..., s_{\sigma(\nu)}) - Pos(s_{\sigma(1)}, s_{\sigma(2)}, ..., s_{\sigma(\nu-1)}) = \qquad (9)$$
$$= \max_{\nu=1,i} \pi(s_{\sigma(\nu)}) - \max_{\nu=1,i-1} \pi(s_{\sigma(\nu)}),$$

$i = 1,...,m$, for each $\sigma = (\sigma(1), \sigma(2), ..., \sigma(m)) \in S_m$, $\pi(s_{\sigma(0)}) \equiv 0$ (properties of associated probabilities of a monotone (fuzzy) measure see in ([4, 6-13]).

Let $M : \Psi^k \Rightarrow \Psi$ $(k = m!)$ be some mean aggregation function with the following properties - monotonicity, boundedness, idempotency and symmetricity ([14]).

Definition 7: An associated fuzzy probabilistic OWA operator AsFPOWA of dimension m is mapping $AsFPOWA : \Psi^m \Rightarrow \Psi$, that has an associated objective weighted vector W of dimension m such that $w_j \in (0,1)$ and $\sum_{j=1}^{m} w_j = 1$ some possibility measure $Pos : 2^S \Rightarrow [0,1]$, according the following formula:

$$AsFPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_m) = \beta \sum_{j=1}^{m} w_j \tilde{b}_j +$$
$$+ (1-\beta) M\left(E_{P_{\sigma_1}}(\tilde{a}), E_{P_{\sigma_2}}(\tilde{a}), ..., E_{P_{\sigma_k}}(\tilde{a}) \right), \qquad (10)$$

where $\tilde{b}_j$ is the j-th largest of the $\{\tilde{a}_i\}, i = 1,...,m$; $E_{P_{\sigma_i}}(\tilde{a})$ is a Mathematical Expectation of $\tilde{a}$ with respect to associated probability P_{σ_i}; $k = m!$.

Some analytical properties of the AsFPOWA operator for general fuzzy measure g and different mean aggregation function M are proved but are omitted here. We will consider concrete AsFPOWA operators for concrete mean function M : AsFPOWAmin if M=Min, AsFPOWAmax, if M=Max and AsFPOWAmean if M=Mean.

IV. Fuzzy Decision Making Problem Regarding the Selection of Investment

We analyze an illustrative example of the using of the new AsFPOWA operator in a fuzzy decision-making problem regarding selection of investments. The main reason for using our new aggregation operator is that we are able to assess the decision making problem considering possibility distribution and the attitudinal characters of the DMPs. In the following, we study a Company that wants to invest some money in a new market. They consider five alternatives: d_1: “Invest in the Asian market”; d_2: “Invest in the South American market”; d_3: “Invest in the African market”; d_4: “Invest in all three markets”; d_5: “Do not invest money in any market”. In order to analyze these investments, the investor has brought together a group of experts. This group considers that the key factors are the economic situations of the world (external) and country (internal) economy for the next period. They consider 3 possible states of nature that in hole could occur in the future: s_1: “Bad economic situation”; s_2: “Regular economic situation”; s_3: “Good economic situation”. As a result the group of experts gives us union one opinions and results. The results depending on the state of nature s_i and alternative d_k that the company selects, are presented in the Table 1.

Table 1: Expert's valuations in TFNs

D \ S	s_1	s_2	s_3
d_1	(60,70,80)	(40,50,60)	(50,60,70)
d_2	(30,40,50)	(60,70,80)	(70,80,90)
d_3	(50,60,70)	(50,60,70)	(60,70,80)
d_4	(70,80,90)	(40,50,60)	(40,50,60)
d_5	(60,70,80)	(70,80,90)	(50,60,70)

Following the expert's knowledge on the world economy for the next period, experts decided the objective weights (as an external factor) of states of nature let be $W=(0,5;0,3;0,2)$, which the country economy for the next period takes only some possibilities to occur presented states of nature in the country (as an internal factor). So, there exists some possibilities (internal levels), as an uncertainty measure, to occur states of nature in the country. Let be given possibility levels of states of nature (it may be constructed by some method of expert knowledge presentation):

$$poss(\{s_1\}) \equiv \pi_1 = 0,7;$$
$$poss(\{s_2\}) \equiv \pi_2 = 1;$$
$$poss(\{s_3\}) \equiv \pi_3 = 0,5.$$

In this model we have the following weights: $\beta \equiv 0,3$.

For $\tilde{a}=(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3)$ we have:

$$AsFPOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3)= = \beta\sum_{j=1}^{3}\tilde{b}_j w_j + (1-\beta)M\Big(E_{P_{\sigma_1}}(\tilde{a}), E_{P_{\sigma_2}}(\tilde{a}),\ldots,E_{P_{\sigma_6}}(\tilde{a})\Big).$$

It is clear that k=m!=3!=6 and for calculation of the AsFPOWA operator we firstly define the associated probability class $\{P_\sigma^{Pos}\}_{\sigma\in S_3}$ for the $Pos: 2^S \Rightarrow [0,1]$ and mathematical expectations, $\forall\sigma=\{\sigma(1),\sigma(2),\sigma(3)\}\in S_3$

$$E_{P_\sigma}(d)=E_{P^{Pos}{}_\sigma}(\tilde{a})=\sum_{i=1}^{3}P_{\sigma(i)}\cdot\tilde{a}_{\sigma(i)}\ .$$

The results presented in the Table 2.

Table 2: Associated Probability Class - $\{P_\sigma\}_{\sigma\in S_3}$

$\sigma=(\sigma(1),\sigma(2),\sigma(3))$	$P_{\sigma(1)}$	$P_{\sigma(2)}$	$P_{\sigma(3)}$
$(1,2,3)=\sigma_1$	$P_1=0,7$	$P_2=0,3$	$P_3=0$
$(1,3,2)=\sigma_2$	$P_1=0,7$	$P_3=0$	$P_2=0,3$
$(2,1,3)=\sigma_3$	$P_2=1$	$P_1=0$	$P_3=0$
$(2,3,1)=\sigma_4$	$P_2=1$	$P_3=0$	$P_1=0$
$(3,1,2)=\sigma_5$	$P_3=0,5$	$P_1=0,2$	$P_2=0,3$
$(3,2,1)=\sigma_6$	$P_3=0,5$	$P_2=0,5$	$P_1=0$

Following the table 2 we calculate Mathematical Expectations - $\{E_{P_\sigma}(\cdot)\}_{\sigma\in S_3}$ (see Table 3).

Table 3: Mathematical Expectations - $\{E_{P_\sigma}(\cdot)\}_{\sigma\in S_3}$

$E_{P_\sigma}(\cdot)$ σ	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
$E_{P_\sigma}(d_1)$	(54,64,74)	(54,64,74)	(40,50,60)	(40,50,60)	(49,59,69)	(45,55,65)
$E_{P_\sigma}(d_2)$	(39,49,59)	(39,49,59)	(60,70,80)	(60,70,80)	(59,69,79)	(65,75,85)
$E_{P_\sigma}(d_3)$	(50,60,70)	(50,60,70)	(50,60,70)	(50,60,70)	(55,65,75)	(55,65,75)
$E_{P_\sigma}(d_4)$	(61,71,81)	(61,71,81)	(40,50,60)	(40,50,60)	(46,56,66)	(40,50,60)
$E_{P_\sigma}(d_5)$	(63,73,83)	(63,73,83)	(70,80,90)	(70,80,90)	(58,68,78)	(60,70,80)

Now we may calculate the values of different variants of the AsFPOWA operator with respect to different mean operators M (Table 4):

Table 4: Aggregation results

D/Ag. Op.	$AsFPOWA_{min}$	$AsFPOWA_{max}$	$AsFPOWA_{mean}$
d_1	(44,54,64)	(54,64,74)	(49,59,69)
d_2	(45,55,65)	(64,74,84)	(57,66,75)
d_3	(52,62,72)	(56,66,76)	(53,63,73)
d_4	(45,55,65)	(60,70,80)	(51,61,71)
d_5	(60,70,80)	(68,78,88)	(64,74,84)

Calculating numerical values of AsFPOWAmin, AsFPOWAmax, AsFPOWAmean operators we can rank the alternatives from the most preformed to the less preformed. The results are shown in table 5. It is clear that decision d_5 - "Do not invest money in any market" is an optimal solution (decision) in this problem.

Table 5: Ordering of the policies

N	Aggreg. Operator	Ordering
1	$AsFPOWA_{min}$	$d_5 \succ d_3 \succ d_2 = d_4 \succ d_1$
2	$AsFPOWA_{max}$	$d_5 \succ d_2 \succ d_4 \succ d_3 \succ d_1$
3	$AsFPOWA_{mean}$	$d_5 \succ d_2 \succ d_3 \succ d_4 \succ d_1$

V. CONCLUSION

In this work our focus is directed on the construction of a new generalization of the aggregation OWA operator – AsFPOWA in the possibilistic uncertainty environment. For the illustration of the applicability of the new aggregation operator - AsFPOWA an example of the fuzzy decision making problem regarding optimal selection of investments is considered, where we study a Company that is planning to invest some money in a new market.

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